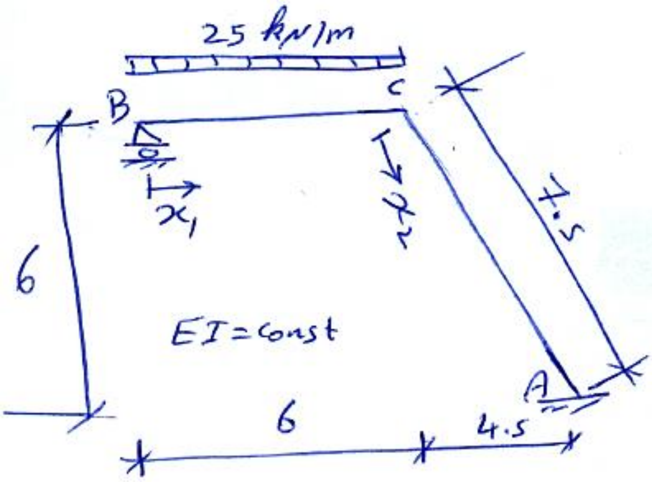
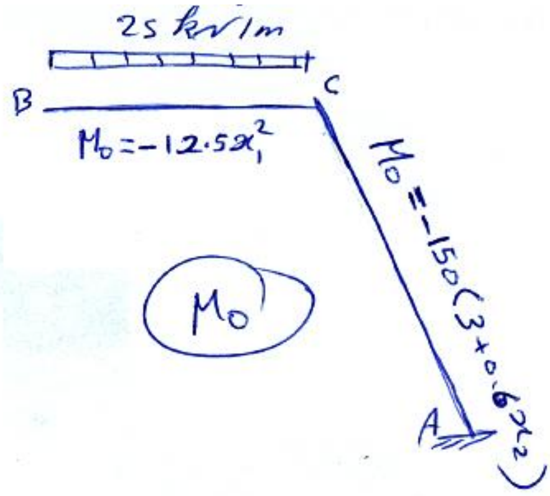
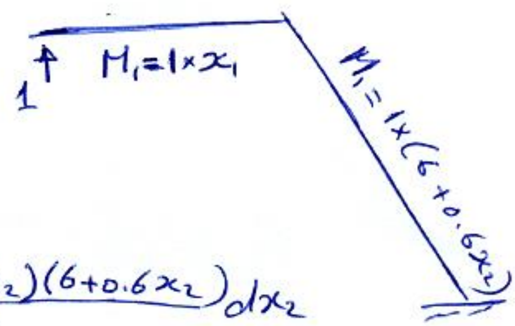
Question:
Use the **virtual work principle** to determine the bending moments for the frame shown and draw it.







$$\Delta_{Bo} = \int \frac{M_o M_1}{EI} dx$$

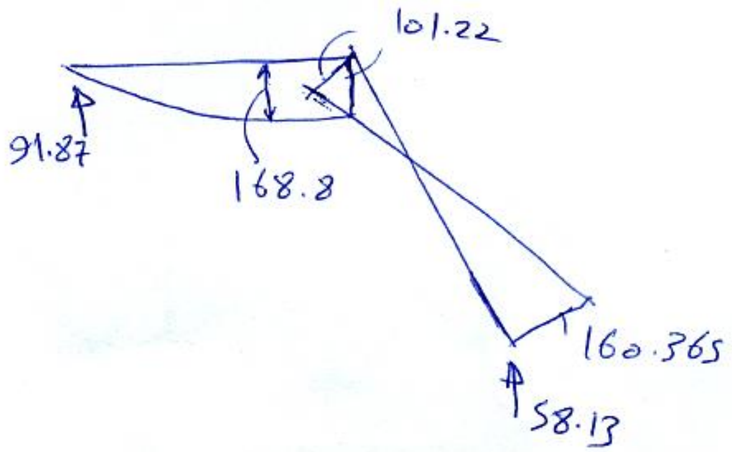
$$\Delta_{Bo} = \int_0^6 \frac{-12.5 x_1^2 \times x_1}{EI} dx_1 + \int_0^{7.5} \frac{-150(3+0.6x_2)(6+0.6x_2)}{EI} dx_2$$

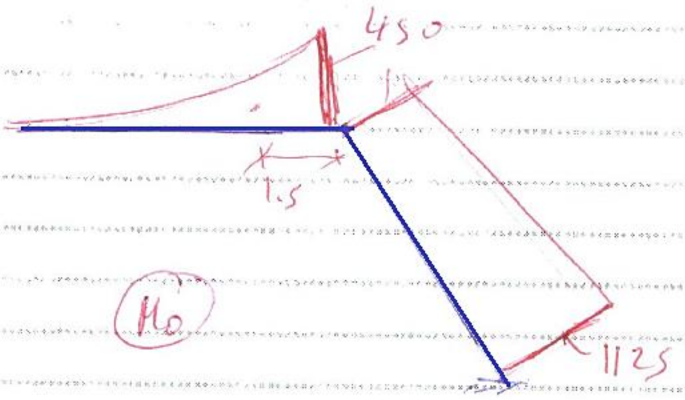
$$\Delta_{Bo} = \frac{-54675 \; (kNm^3)}{EI}$$

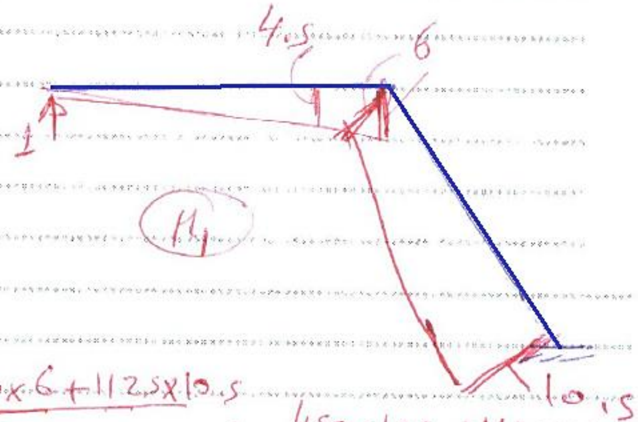
$$f_{BB} = \int \frac{M_1^2}{EI} dx = \int_0^6 \frac{x_1^2}{EI} dx_1 + \int_0^{7.5} \frac{(6+0.6x_2)^2}{EI} dx_2 = \frac{595.125}{EI}$$

$$\Delta_{Bo} + f_{AA} R_B = 0 \Rightarrow R_B = 91.87 \; kN$$

$$M_{final} = M_o + M_1 R_B$$







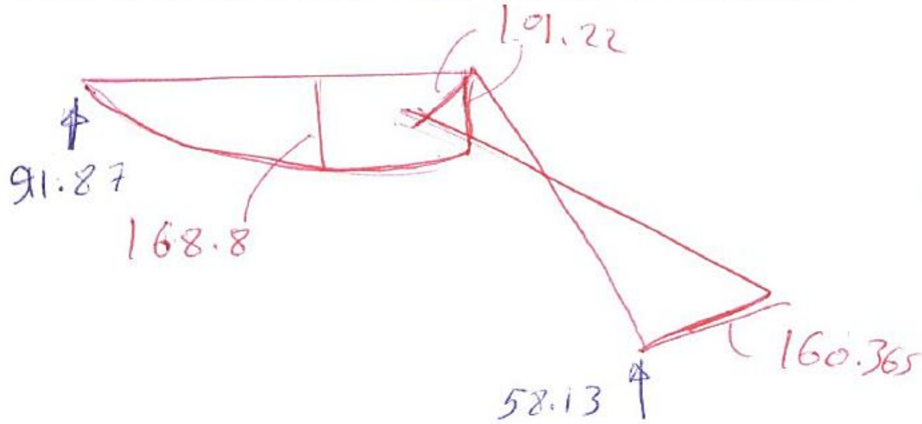
$$\Delta_{BO} = -\frac{1}{EI}\left[\frac{1}{3}\times 450\times 6\times 4.5\right] - \frac{7.5}{EI}\left[\frac{450\times 6+112.5\times 10.5}{3} + \frac{450\times 10.5+112.5\times 6}{6}\right]$$

$$\Delta_{BO} = -\frac{54675\ kNm^3}{EI}$$

$$f_{BB} = \frac{6}{EI}\left[\frac{6\times 6}{3}\right] + \frac{7.5}{EI}\left[\frac{6\times 6+10.5\times 10.5}{3} + \frac{6\times 10.5+10.5\times 6}{6}\right]$$

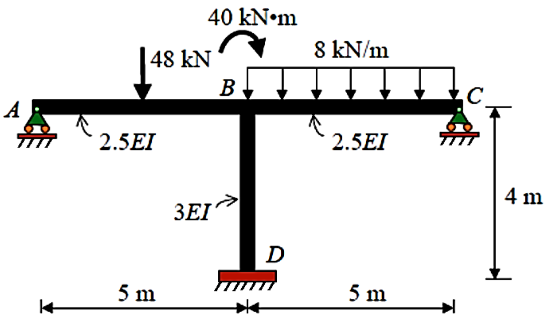
$$f_{BB} = \frac{595.125\ (kNm^3/kN)}{EI}$$

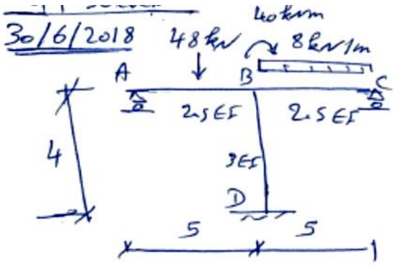
$$\Delta_{BO} + f_{BB} R_B = 0 \Rightarrow R_B = 91.87\ kN$$

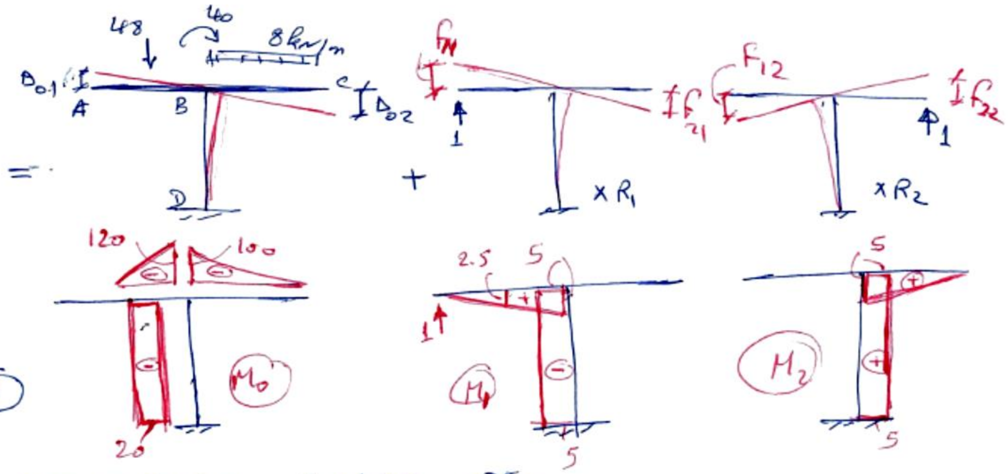


Question 3

For the frame shown use the **virtual work principle** to determine the end moments of the members and draw the bending moment diagram**.**







Compatability Eqns:

$$\left.\begin{aligned} \Delta_{01} + f_{11} R_1 + f_{12} R_2 &= 0 \\ \Delta_{02} + f_{21} R_1 + f_{22} R_2 &= 0 \end{aligned}\right\} (1)$$

$$\Delta_{01} = \int \frac{M_0 M_1}{EI} dx = \frac{-2.5}{2.5EI}\left(\frac{0+120\times5}{3} + \frac{0+120\times2.5}{6}\right) + \frac{20\times4\times5}{3EI} = \frac{-350}{3EI}$$

$$\Delta_{02} = \int \frac{M_0 M_2}{EI} dx = \frac{1}{2.5EI}\left(-\frac{1}{3}\times5\times100\times3.75\right) - \frac{1}{3EI}(20\times4\times5) = \frac{-1150}{3EI}$$

$$f_{11} = \int \frac{M_1^2}{EI} dx = \frac{5}{2.5EI}\left(\frac{5\times5}{3}\right) + \frac{1}{3EI}(5\times4\times5) = \frac{150}{3EI}$$

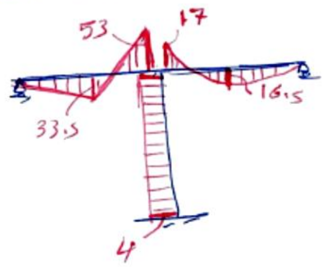
$$f_{22} = \int \frac{M_2^2}{EI} dx = \frac{150}{3EI}$$

$$f_{12} = \frac{-1}{3EI}(5\times4\times5) = \frac{-100}{3EI}$$

Substituting into Eqns (1) ⇒

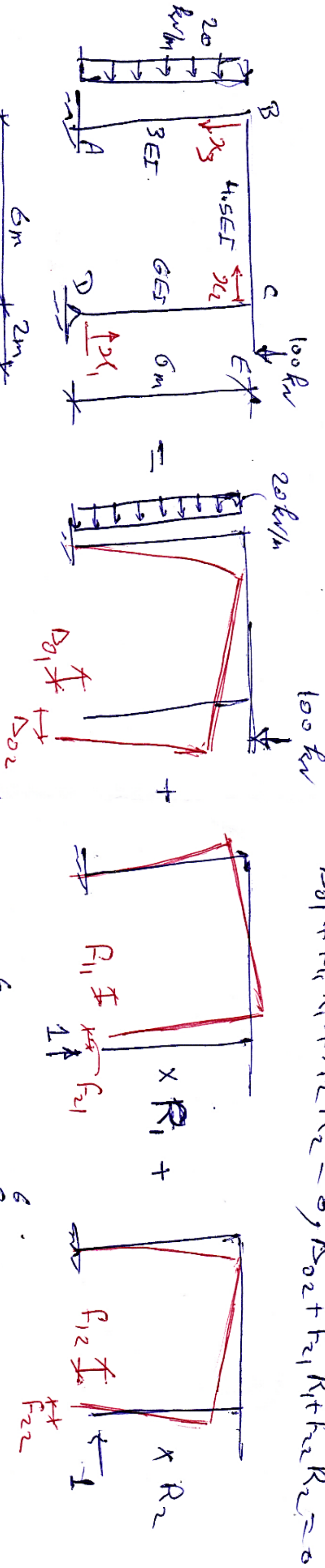
$$\left.\begin{aligned} \frac{-350}{3EI} + \frac{150}{3EI} R_1 - \frac{100}{3EI} R_2 &= 0 \\ \frac{-1150}{3EI} + \frac{100}{3EI} R_1 + \frac{150}{3EI} R_2 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} R_1 &= 13.4 \text{ kN} \\ R_2 &= 16.6 \text{ kN} \end{aligned}$$

$$M_{final} = M_0 + M_1 R_1 + M_2 R_2$$



Question 3

For the frame shown use the **virtual work principle** to determine the end moments of the members and draw the bending moment diagram**.**



$\Delta_{01} + f_{11}R_1 + f_{12}R_2 = 0, \quad \Delta_{02} + f_{21}R_1 + f_{22}R_2 = 0$

Range	M_0	M_1	M_2
DC $x_1: 0 \to 6$	0	0	$-x_1$
CB $x_2: 0 \to 6$	$-100x_2 - 200$	x_2	-6
BA $x_3: 0 \to 6$	$-10x_3^2 - 800$	$+6$	$x_3 - 6$

$$f_{11} = \int_0^6 \frac{M_1^2}{6EI}dx_1 + \int_0^6 \frac{M_1^2}{4.5EI}dx_2 + \int_0^6 \frac{M_1^2}{3EI}dx_3$$

$$f_{11} = 0 + \int_0^6 \frac{(x_2)^2}{4.5EI}dx_2 + \int_0^6 \frac{(-6)^2}{3EI}dx_3 = \frac{88}{EI}$$

$$f_{22} = \int \frac{M_2^2}{EI}dx = \int_0^6 \frac{(-x_1)^2}{6EI}dx_1 + \int_0^6 \frac{(+6)^2}{4.5EI}dx_2 + \int_0^6 \frac{(x_3-6)^2}{3EI}dx_3 = \frac{84}{EI}$$

$$f_{12} = f_{21} = \int \frac{M_1 M_2}{EI}dx = 0 + \int_0^6 \frac{x_2(-6)}{4.5EI}dx_2 + \int_0^6 \frac{(+6)(x_3-6)}{3EI}dx_3 = \frac{-60}{EI}$$

$$\Delta_{01} = \int_0^6 \frac{M_0 M_1}{6EI}dx_1 + \int_0^6 \frac{M_0 M_1}{4.5EI}dx_2 + \int_0^6 \frac{M_0 M_1}{3EI}dx_3$$

$$= 0 + \int_0^6 \frac{(-100x_2-200)x_2}{4.5EI}dx_2 + \int_0^6 \frac{(-10x_3^2-800)(-6)}{3EI}dx_3$$

$$\Delta_{01} = -13440/EI$$

$$\Delta_{02} = \int_0^6 \frac{M_0 M_2}{6EI}dx_1 + \int_0^6 \frac{M_0 M_2}{4.5EI}dx_2 + \int_0^6 \frac{M_0 M_2}{3EI}dx_3$$

$$\Delta_{02} = 0 + \int_0^6 \frac{(-100x_2-200)(+6)}{4.5EI}dx_2 + \int_0^6 \frac{(-10x_3^2-800)(x_3-6)}{3EI}dx_3$$

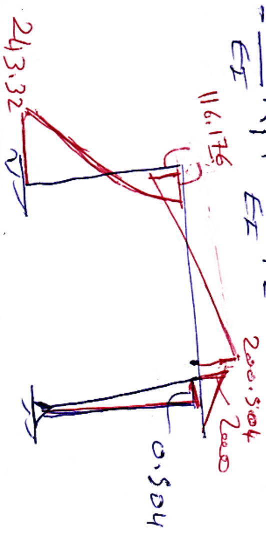
$$\Delta_{02} = 9160/EI$$

$$-\frac{13440}{EI} + \frac{88}{EI}R_1 - \frac{60}{EI}R_2 = 0$$

$$\frac{9160}{EI} - \frac{60}{EI}R_1 + \frac{84}{EI}R_2 = 0$$

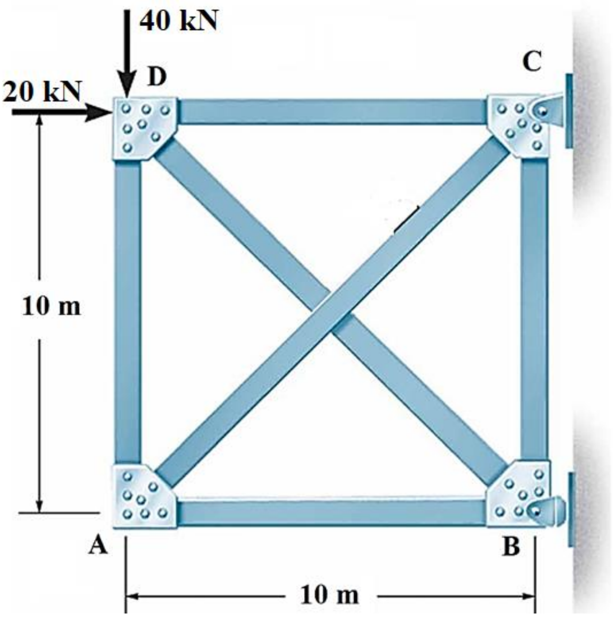
$\Rightarrow R_1 = 152.78,\ R_2 = 0.084$ kN

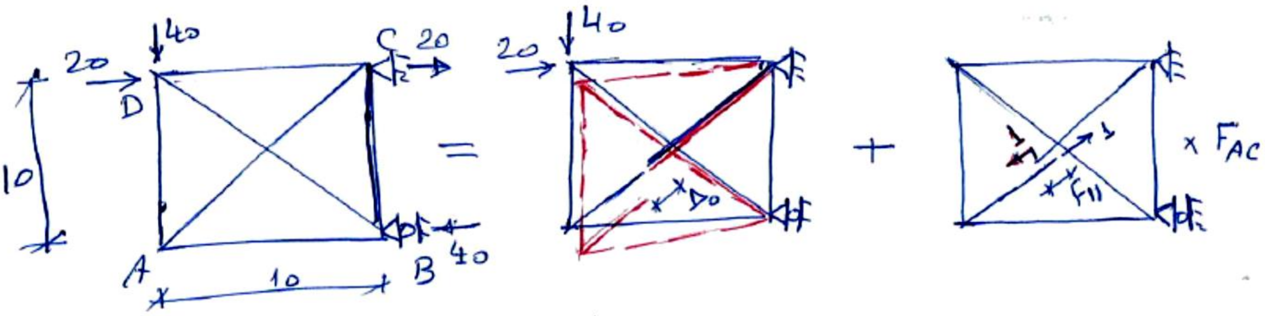
$M_{final} = M_0 + M_1 R_1 + M_2 R_2$



Question 4

Using the **virtual work principle**, determine the forces in members of the truss shown in figure.
AE is constant**.**

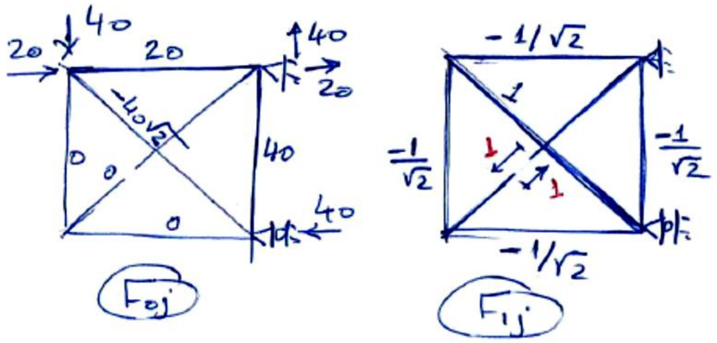




$$\Delta_0 + f_{11} F_{AC} = 0$$

$$\Delta_0 = \sum \frac{F_{0j} F_{1j} \ell_j}{EA_j}$$

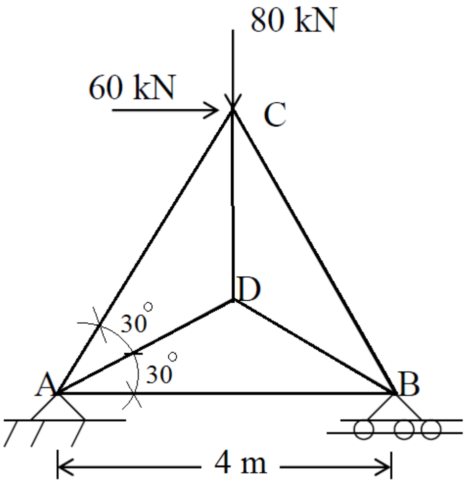
$$f_{11} = \sum \frac{F_{1j}^2 \ell_j}{EA_j}$$

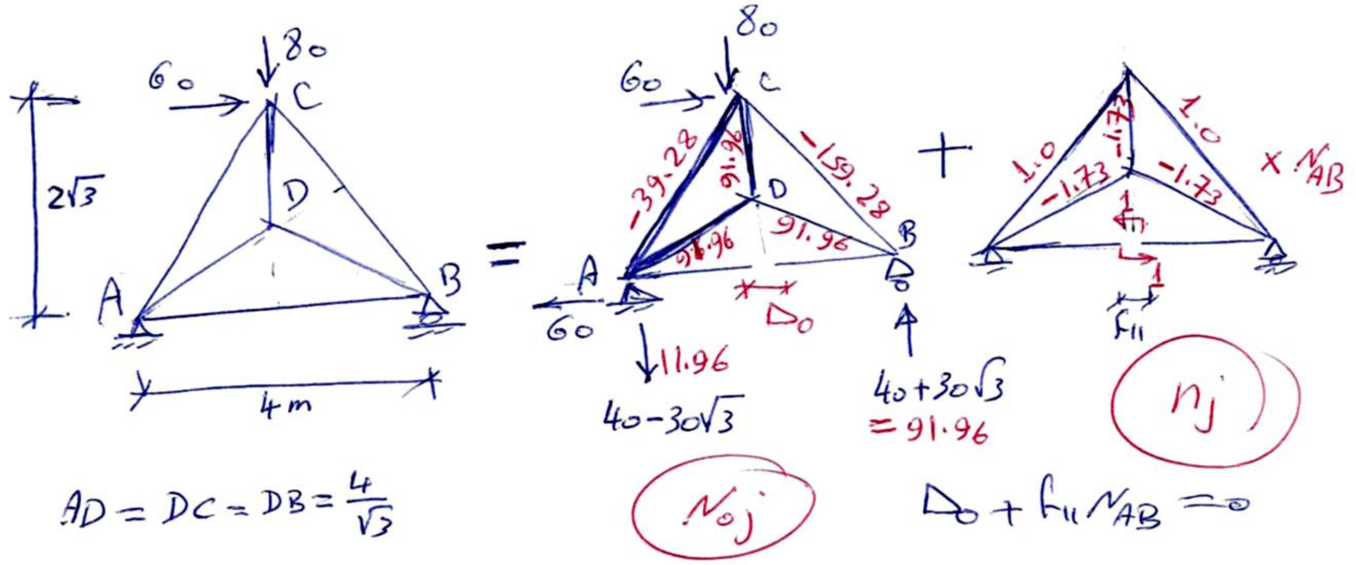


Member	ℓ_j	F_{0j}	F_{1j}	$F_{0j} F_{1j} \ell_j$	$F_{1j}^2 \ell_j$	$F_{j\,Final} = F_{0j} + F_{1j} \times F_{AC}$
AB	10	0	$-1/\sqrt{2}$	0	5	-17.93
BC	10	40	$-1/\sqrt{2}$	$-400/\sqrt{2}$	5	22.07
CD	10	20	$-1/\sqrt{2}$	$-200/\sqrt{2}$	5	2.07
AD	10	0	$-1/\sqrt{2}$	0	5	-17.93
AC	$10\sqrt{2}$	0	1	0	$10\sqrt{2}$	25.355
BD	$10\sqrt{2}$	$-40\sqrt{2}$	1	-800	$10\sqrt{2}$	-31.21
				-1224.264	48.284	

$$\frac{-1224.264}{EA} + \frac{48.284}{EA} F_{AC} = 0 \Rightarrow F_{AC} = 25.355 \text{ kN}$$

Question1:
Using the **virtual work method**, determine the reactions of the truss shown below and find all members forces. *EA is* constant**.**





$AD = DC = DB = \frac{4}{\sqrt{3}}$

$\Delta_0 + f_{11} N_{AB} = 0$

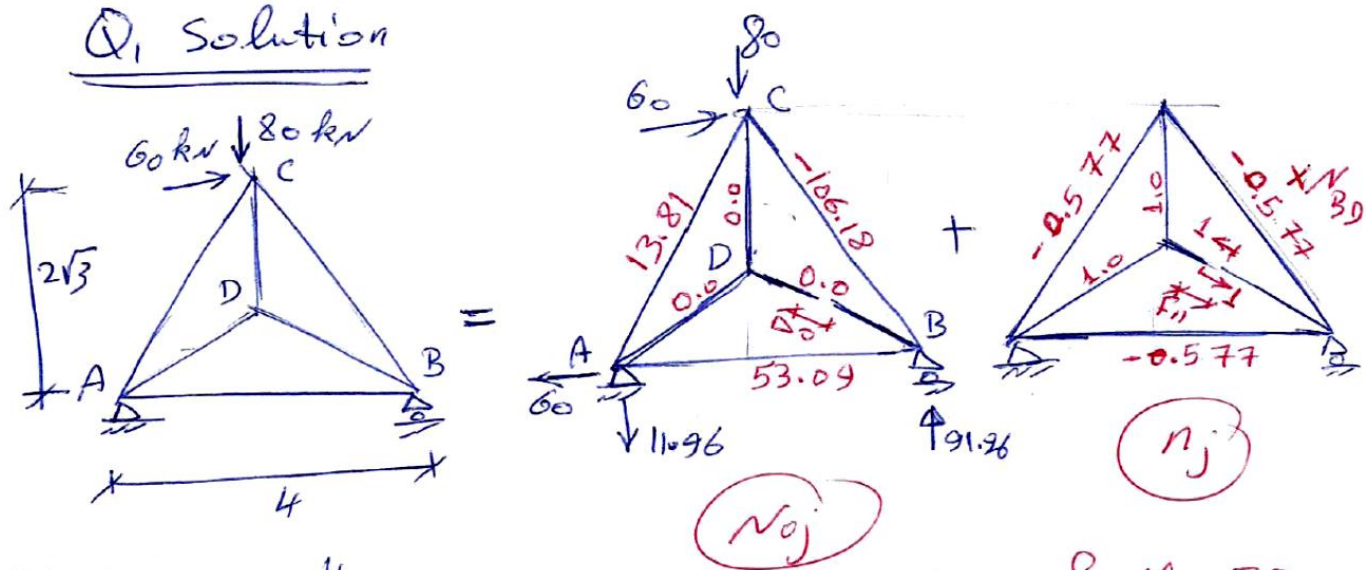
Member	l_j	N_{oj}	n_j	$N_{oj}\, n_j\, l_j$	$n_j^2 l_j$	N_j
AB	4	0	1.0	0	4	57.87
BC	4	−159.28	1.0	−637.12	4	−101.41
CA	4	−39.28	1.0	−157.12	4	18.59
DA	2.31	91.96	−1.73	−367.84	6.93	−8.15
DB	2.31	91.96	−1.73	−367.84	6.93	−8.15
DC	2.31	91.96	−1.73	−367.84	6.93	−8.15
			Σ	−1897.76	32.79	

$$\Delta_0 = \frac{\sum N_{oj} n_j l_j}{EA} = -\frac{1897.76}{EA} \quad , \quad f_{11} = \frac{\sum n_j^2 l_j}{EA} = \frac{32.79}{EA}$$

$$\Delta_0 + f_{11} N_{AB} = 0 \Rightarrow -\frac{1897.76}{EA} + \frac{32.79}{EA} N_{AB} = 0 \Rightarrow N_{AB} = 57.87 \text{ kN}$$

$$N_{j\,final} = N_{oj} + n_j \times N_{AB}$$

Q_1 Solution



$DA = DB = DC = \frac{4}{\sqrt{3}} = 2.31\ m$

$\Delta_o + f_{11} N_{BD} = 0$

Member	l_j	N_{oj}	n_j	$N_{oj} n_j l_j$	$n_j^2 l_j$	N_j
AB	4	53.09	-0.577	-124.4	1.33	57.78
BC	4	-106.18	-0.577	+245.06	1.33	-101.45
CA	4	13.81	-0.577	-31.87	1.33	18.5
DA	2.31	0	1	0	2.31	-8.13
DB	2.31	0	1	0	2.31	-8.13
DC	2.31	0	1	0	2.31	-8.13
				$\Sigma = +88.79$	$\Sigma = 10.92$	

$$\Delta_o = \sum \frac{N_{oj} n_j l_j}{EA} = +\frac{88.79}{EA}, \quad f_{11} = \sum \frac{n_j^2 l_j}{EA} = \frac{10.92}{EA}$$

$$+\frac{88.79}{EA} + \frac{10.92}{EA} \times N_{BD} = 0 \Rightarrow N_{BD} = -8.13\ kN$$

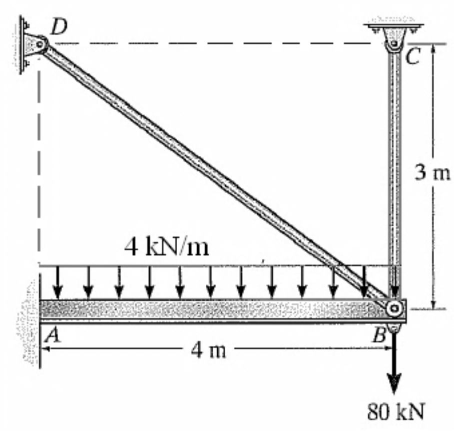
Question

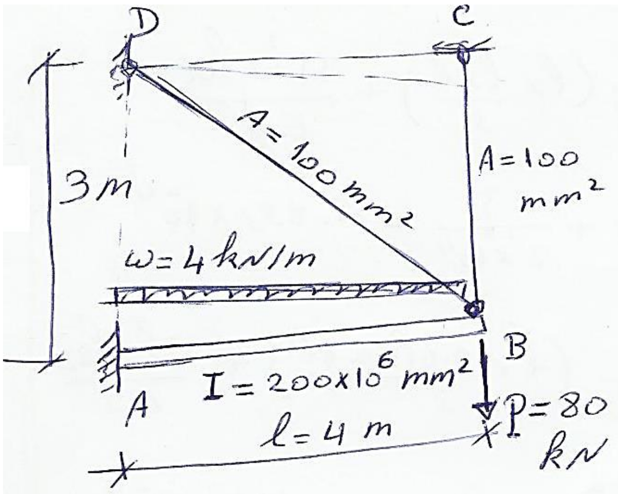
The cantilevered beam *AB* is additionally supported using two tie rods. Determine the force in each of these rods by using the **virtual work principle** Draw the bending moment diagram in the beam. Neglect axial compression and shear in the beam.

For the beam, $I = 200(10^6)\ mm^4$,

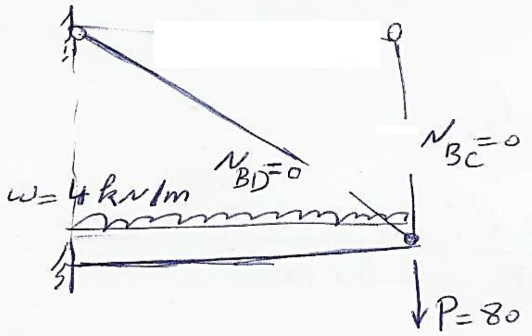
and for each tie rod, $A = 100\ mm^2$.

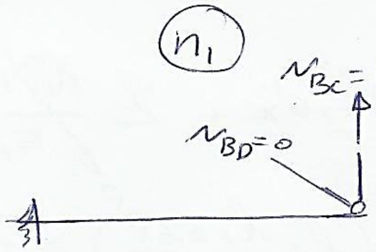
Take $E = 200$ GPa.

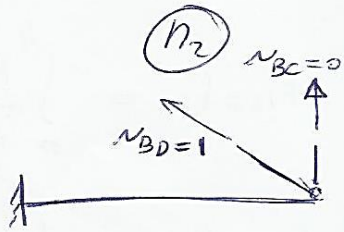


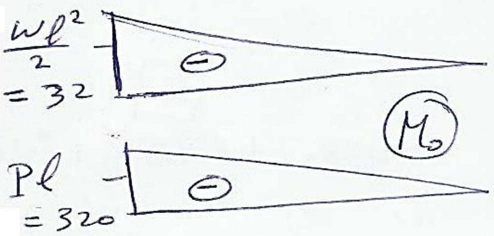


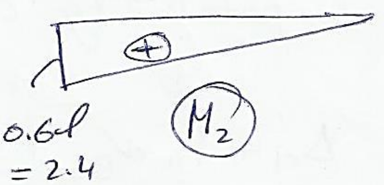
$$EI = 200\times10^{6}\times 200\times10^{6}\times10^{-12}$$
$$= 4\times10^{4}\ kN\,m^{2}$$
$$EA = 200\times10^{6}\times100\times10^{-6} = 2\times10^{4}\ kN$$











$$\Delta_{01} = \int_0^4 \frac{M_0 M_1}{EI}\,dx + \cancel{\sum \frac{N_{0j}\, n_{1j}\, l_j}{EA_j}}^{\,0}$$

$$\Delta_{01} = \frac{1}{EI}\left[-\frac{1}{3}\times\frac{wl^2}{2}\times l\times\frac{3}{4}l - l\left(\frac{Pl\times l}{3}\right)\right] = \frac{-1}{EI}\left(\frac{wl^4}{8}+\frac{Pl^3}{3}\right)$$

$$\Delta_{01} = -\frac{1}{EI}\left(\frac{4\times4^4}{8}+\frac{80\times4^3}{3}\right) = \frac{-1834.7}{EI} = -458.7\times10^{-4}\ m$$

$$\Delta_{02} = \int_0^4 \frac{M_0 M_2}{EI}\,dx + \cancel{\sum \frac{N_{0j}\, n_{2j}\, l_j}{EA_j}}^{\,0}$$

$$\Delta_{02} = \frac{1}{EI}\left[-\frac{1}{3}\times\frac{wl^2}{2}\times l\times0.75l - l\left(\frac{Pl\times0.6l}{3}\right)\right] = \frac{-1}{EI}\left(\frac{0.15wl^4}{2}+0.2Pl^3\right)$$

$$\Delta_{02} = -\frac{1}{EI}\left(\frac{0.15\times4\times4^4}{2}+0.2\times80\times4^3\right) = \frac{-1100.8}{EI} = -275.2\times10^{-4}\ m$$

$$f_{11} = \int_0^{l} \frac{M_1^2}{EI} dx + \Sigma \frac{n_{1j}^2 l_j}{EA_j} = \frac{1}{EI}\left(l \times \frac{l \times l}{3}\right) + \frac{(1)^2 \times l_{BC}}{EA}$$

$$f_{11} = \frac{l^3}{3EI} + \frac{l_{BC}}{EA} = \frac{4^3}{3 \times 4 \times 10^4} + \frac{3}{2 \times 10^4} = 6.833 \times 10^{-4}$$

$$f_{22} = \int_0^4 \frac{M_2^2}{EI} dx + \Sigma \frac{n_{2j}^2 l_j}{EA_j} = \frac{1}{EI}\left(l \times \frac{0.6l \times 0.6l}{3}\right) + \frac{(1)^2 \times l_{BD}}{EA}$$

$$f_{22} = \frac{0.12 l^3}{EI} + \frac{l_{BD}}{EA} = \frac{0.12 \times 4^3}{4 \times 10^4} + \frac{5}{2 \times 10^4} = 4.42 \times 10^{-4}$$

$$f_{12} = f_{21} = \int_0^4 \frac{M_1 M_2}{EI} dx + \underbrace{\Sigma \frac{n_{1j} n_{2j} l_j}{EA_j}}_{0}$$

$$f_{12} = f_{21} = \frac{1}{EI}\left(l \times \frac{l \times 0.6l}{3}\right) = \frac{0.2 l^3}{EI} = \frac{0.2 \times 4^3}{4 \times 10^4} = 3.2 \times 10^{-4}$$

Compatability conditions:

$$\left.\begin{aligned} \Delta_{01} + f_{11} N_{BC} + f_{12} N_{BD} &= 0 \\ \Delta_{02} + f_{21} N_{BC} + f_{22} N_{BD} &= 0 \end{aligned}\right\} \xrightarrow{\text{substitution}} \begin{aligned} -458.7 + 6.833 N_{BC} + 3.2 N_{BD} &= 0 \\ -275.2 + 3.2 N_{BC} + 4.42 N_{BD} &= 0 \end{aligned}$$

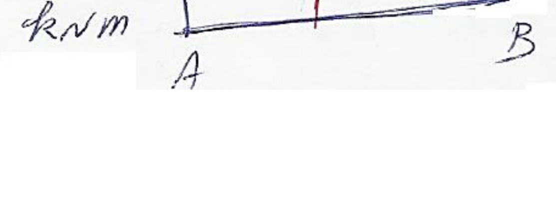
Solving these two eq'ns $\Rightarrow$ $N_{BC} = 57.45\ kN$, $N_{BD} = 20.69\ kN$

$$M_{final} = M_0 + M_1 \times N_{BC} + M_2 \times N_{BD}$$

$$M_{A\,final} = \left(-\frac{wl^2}{2} - Pl\right) + l \times 57.45 + 0.6 l \times 20.69$$

$$M_{A\,final} = \left(-\frac{4 \times 4^2}{2} - 80 \times 4\right) + 4 \times 57.45 + 0.6 \times 4 \times 20.69 = -72.544$$

kNm



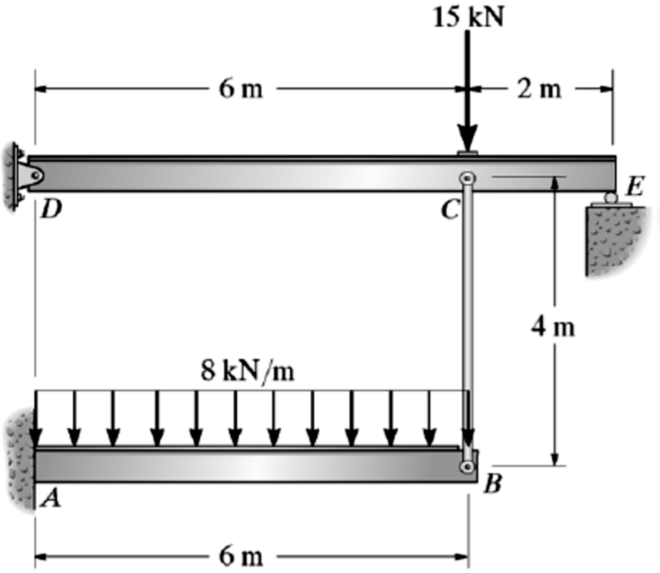
مثال

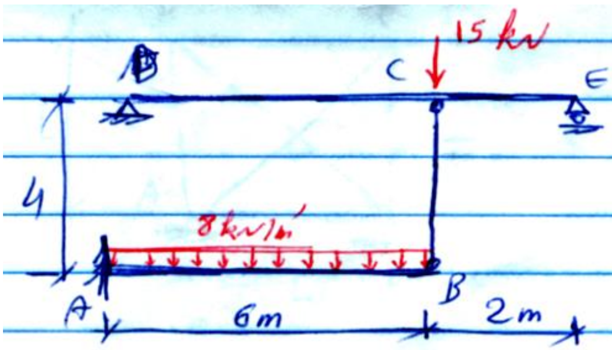
استخدم مبدأ العمل الوهمي لتحديد القوة المحورية في العنصر BC وارسم مخطط عزم الانعطاف لكلا الجائزين

$I=100x10^6$ mm^4 لكل من الجائزين

$A=200$ mm^2 BC للشداد

$E=2x10^5$ MPa لكل العناصر

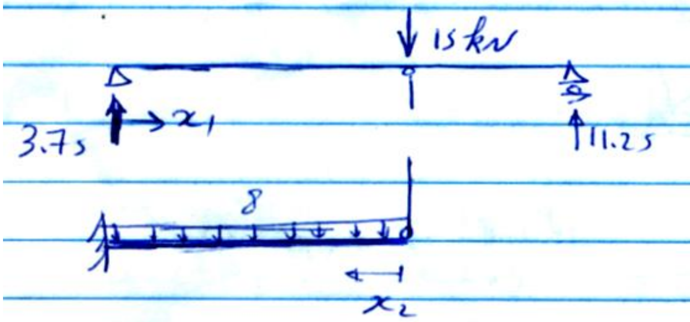




$I = 100\times10^{6}\ mm^4 = 100\times10^{-6}\ m^4$

$A = 200\ mm^2 = 200\times10^{-6}\ m^2$

$G = 200\ Gpa = 200\times10^{6}\ kN/m^2$

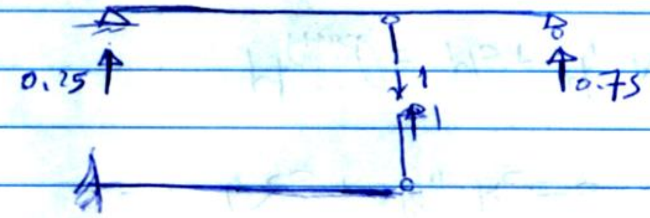


$M_{DC_0} = 3.75x_1 \qquad 6 \geq x_1 \geq 0$

$M_{CE_0} = 3.75x_1 - 15(x_1 - 6)$

$M_{CE_0} = 90 - 11.25x_1 \qquad 8 \geq x_1 \geq 6$

$M_{BA_0} = -4x_2^2 \qquad 6 \geq x_2 \geq 0$



$M_{DC_1} = 0.25x_1 \qquad 6 \geq x_1 \geq 0$

$M_{CE_1} = 6 - 0.75x_1 \qquad 8 \geq x_1 \geq 6$

$M_{BA_1} = 1\times x_2$

$$1\times \Delta_{BC_0} = \frac{1}{EI}\int_0^6 3.75x_1 \times 0.25x_1\,dx_1 + \frac{1}{EI}\int_6^8 (90 - 11.25x_1)(6 - 0.75x_1)\,dx_1$$

$$+ \frac{1}{EI}\int (-4x_2^2)(1\times x_2)\,dx_3 = \frac{-1206}{EI}$$

$$f_{11} = \int \frac{M_1^2}{EI}dx + \frac{n^2 L}{EA}$$

$$= \frac{1}{EI}\int_0^6 (0.25x_1)^2 + \int_6^8 \frac{(6-0.75)^2}{EI}dx_1 + \int_0^6 \frac{(x_2)^2 dx_3}{EI} + \frac{1\times4}{EA}$$

$$= \frac{78}{EI} + \frac{4}{EA}$$

$$\Delta_{BC_0} + f_{11}\times F_{BC} = 0 \Rightarrow \frac{-1206}{EI} + \left(\frac{78}{EI} + \frac{4}{EA}\right)F_{BC} = 0$$

$F_{BC} = 15.1\ kN$