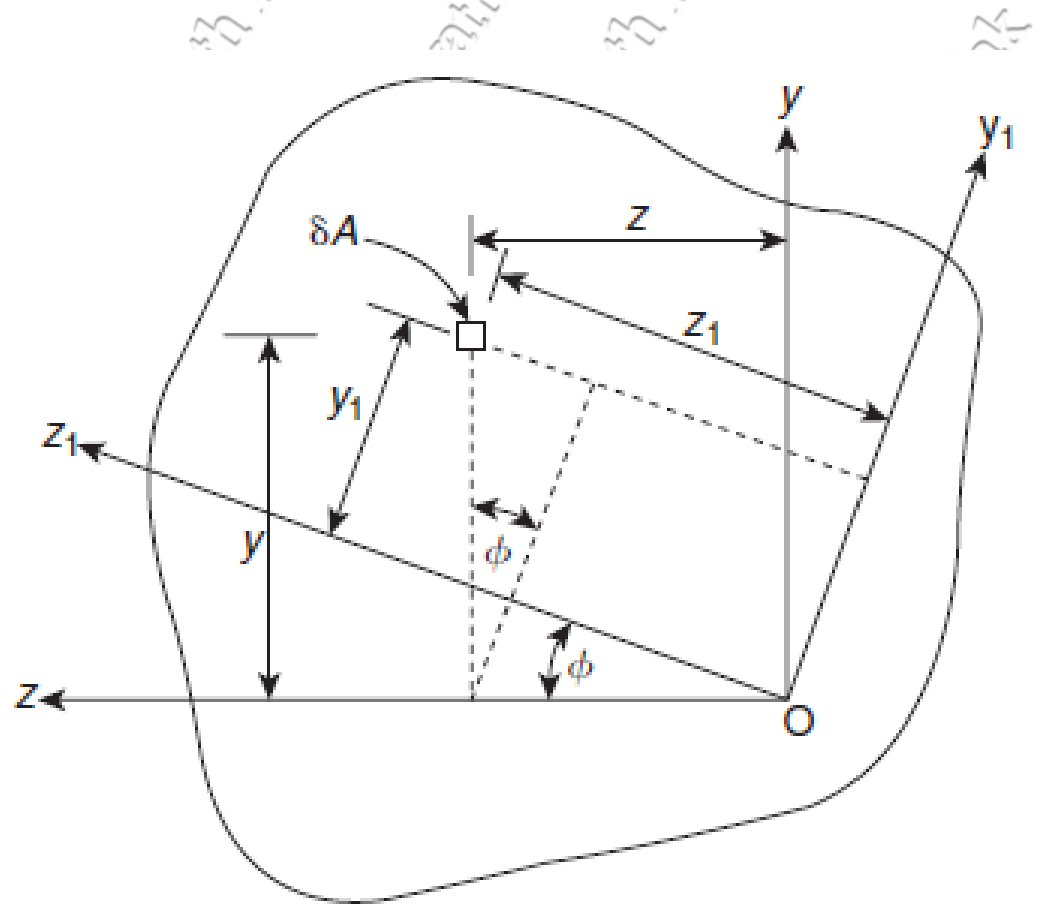


PRINCIPAL AXES AND PRINCIPAL SECOND MOMENTS OF AREA

In any beam section there is a set of axes, neither of which need necessarily be an axis of symmetry, for which **the product second moment of area is zero**. Such axes are known as **principal axes** and the second moments of area about these axes are termed **principal second moments of area**.

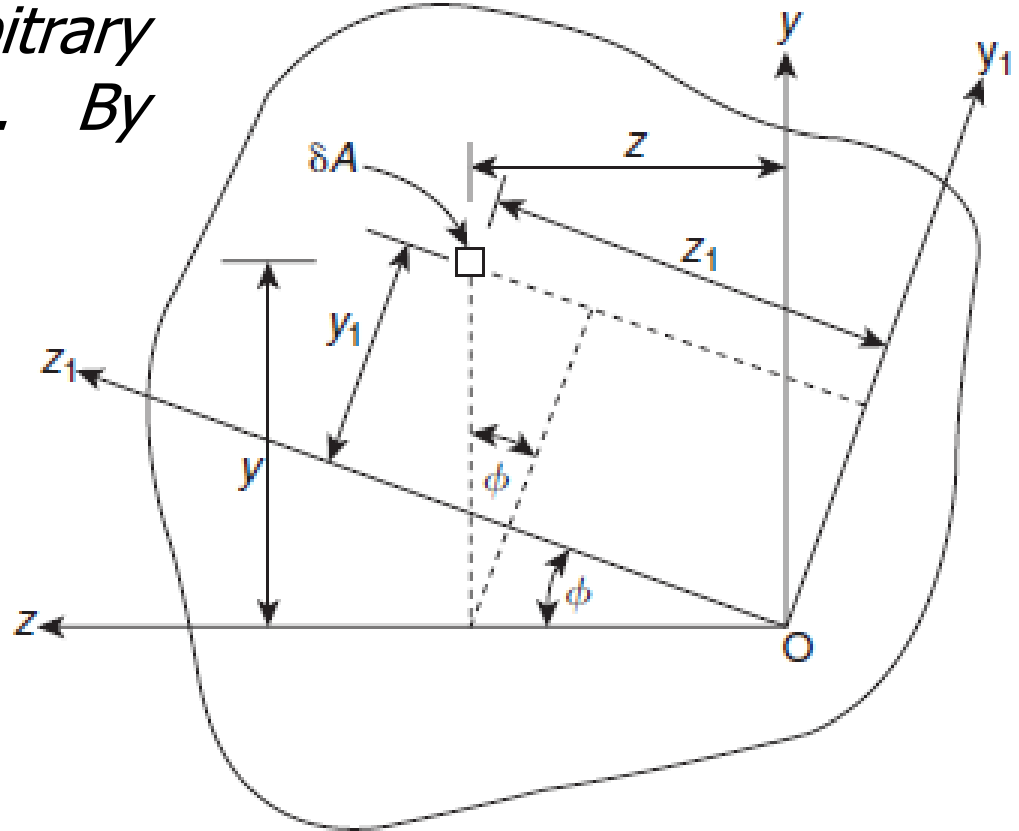


Suppose that the second moments of area I_z , I_y and the product second moment of area, I_{zy} , about arbitrary axes Ozy are known. By definition

$$I_y = \int_A z^2 dA$$

$$I_z = \int_A y^2 dA$$

$$I_{zy} = \int_A y z dA$$



The corresponding second moments of area about axes Oz_1y_1 are

$$I_{y_1} = \int_A z_1^2 dA$$

$$I_{z_1} = \int_A y_1^2 dA$$

$$I_{z_1 y_1} = \int_A y_1 z_1 dA$$

From Fig.

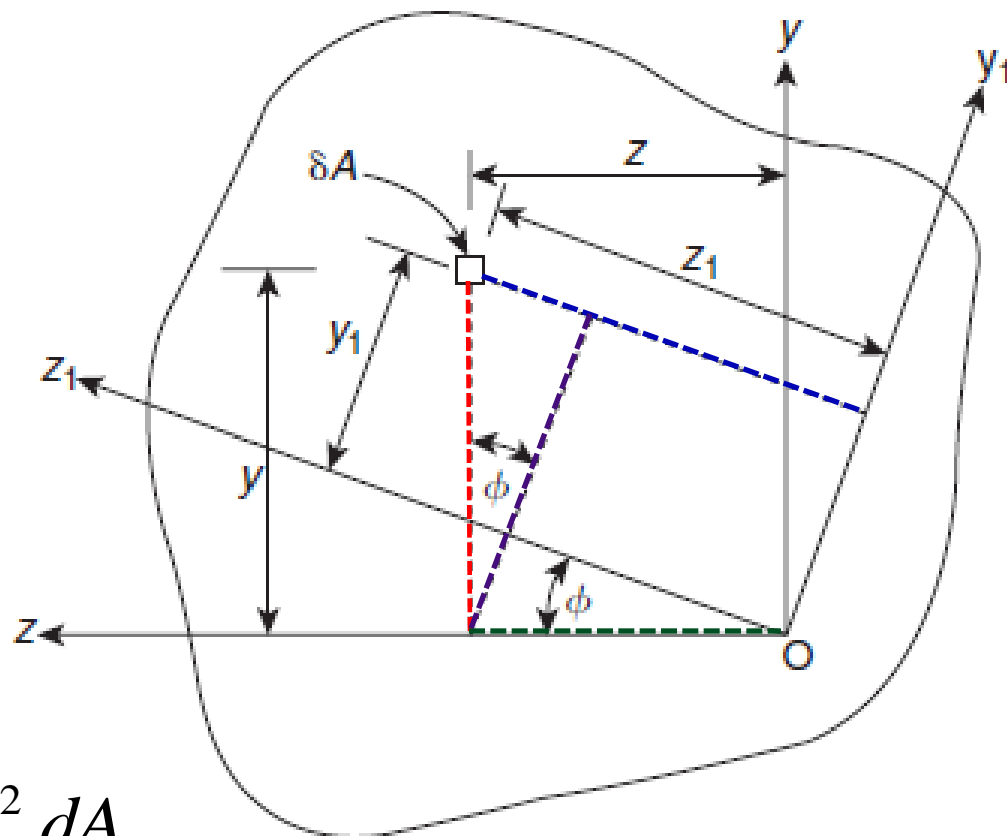
$$z_1 = z \cos \phi + y \sin \phi$$

$$y_1 = y \cos \phi - z \sin \phi$$

$$I_{y_1} = \int_A (z \cos \phi + y \sin \phi)^2 dA$$

$$I_{y_1} = \int_A z^2 \cos^2 \phi dA + \int_A y^2 \sin^2 \phi dA + 2 \int_A y z \sin \phi \cos \phi dA$$

$$I_{y_1} = I_y \cos^2 \phi + I_z \sin^2 \phi + I_{zy} \sin 2\phi \quad (1)$$



Similarly, we find

$$I_{z_1} = I_z \cos^2 \phi + I_y \sin^2 \phi - I_{zy} \sin 2\phi \quad (2)$$

$$I_{z_1 y_1} = \left(\frac{I_z - I_y}{2} \right) \sin 2\phi + I_{zy} \cos 2\phi \quad (3)$$

Equations (1, 2 & 3) give the second moments of area and product second moment of area about axes inclined at an angle ϕ to the z axis. In the special case where Oz_1y_1 are principal axes, $I_{z(p),y(p)} = 0$, $\phi = \phi_p$ and Eqs. (1) and (2) become

$$I_{yp} = I_y \cos^2 \phi_p + I_z \sin^2 \phi_p + I_{zy} \sin 2\phi_p \quad (4)$$

$$I_{zp} = I_z \cos^2 \phi_p + I_y \sin^2 \phi_p - I_{zy} \sin 2\phi_p \quad (5)$$

Since $I_{z_1 y_1} = I_{z_p, y_p} = 0$, Eq. (3) gives

$$0 = \left(\frac{I_z - I_y}{2} \right) \sin 2\phi_p + I_{zy} \cos 2\phi_p \Rightarrow$$

$$\tan 2\phi_p = \frac{2I_{zy}}{I_y - I_z} \quad (6)$$

The angle ϕ_p may be eliminated from Eqs (4) and (5) by first determining $\cos 2\phi_p$ and $\sin 2\phi_p$ using Eq. (6). We have from trigonometry (*double-angle formulas*)

$$\sin \alpha \cdot \cos \alpha = \frac{1}{2} \sin 2\alpha, \quad \cos^2 \alpha = \frac{1}{2} (1 + \cos 2\alpha)$$

$$\sin^2 \alpha = \frac{1}{2} (1 - \cos 2\alpha), \quad \sin 2\alpha = \frac{\tan 2\alpha}{\sqrt{1 + \tan^2 2\alpha}}$$

$$\cos 2\alpha = \frac{1}{\sqrt{1 + \tan^2 2\alpha}}$$

Thus

$$\sin 2\phi_p = \frac{\tan 2\phi_p}{\sqrt{1 + \tan^2 2\phi_p}} = \frac{2I_{zy} / (I_y - I_z)}{\sqrt{1 + 4I_{zy}^2 / (I_y - I_z)^2}} = \frac{I_{zy}}{\sqrt{[(I_y - I_z) / 2]^2 + I_{zy}^2}}$$
$$\cos 2\alpha = \frac{1}{\sqrt{1 + \tan^2 2\alpha}} = \frac{1}{\sqrt{1 + 4I_{zy}^2 / (I_y - I_z)^2}} = \frac{(I_y - I_z) / 2}{\sqrt{[(I_y - I_z) / 2]^2 + I_{zy}^2}}$$

Rewriting Eq. (5) in terms of $\cos 2\phi_p$ and $\sin 2\phi_p$ we have

$$I_{zp} = \frac{I_z}{2} (1 + \cos 2\phi_p) + \frac{I_y}{2} (1 - \cos 2\phi_p) - I_{zy} \sin 2\phi_p$$

Substituting for $\cos 2\phi_p$ and $\sin 2\phi_p$ from the above we obtain

$$I_{zp} = \frac{I_z + I_y}{2} - \frac{1}{2} \sqrt{(I_z - I_y)^2 + 4I_{zy}^2}$$

Similarly

$$I_{yp} = \frac{I_z + I_y}{2} + \frac{1}{2} \sqrt{(I_z - I_y)^2 + 4I_{zy}^2}$$

Note that the solution of Eq. (6) gives two values for the inclination of the principal axes, φ_p and $\varphi_p + \pi/2$, corresponding to the axes Oz_p and Oy_p .

MOMENTS OF INERTIA OF COMPOSITE AREAS

Determine the product of inertia of the right triangle shown (a) with respect to the x and y axes and (b) with respect to centroidal axes parallel to the x and y axes.

$$I_{xy} = \int \bar{x} \cdot \bar{y} \cdot dA$$

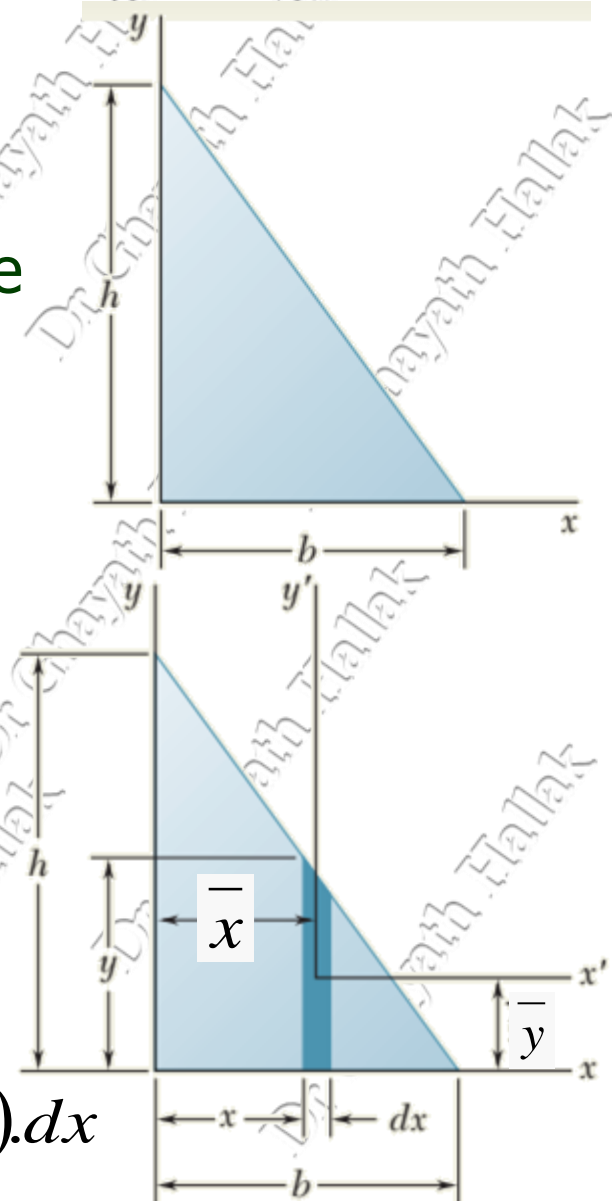
$$dA = y \cdot dx$$

$$\frac{y}{h} = \frac{b-x}{b} \Rightarrow y = (b-x) \cdot \frac{h}{b}$$

$$\therefore dA = (b-x) \cdot \frac{h}{b} \cdot dx$$

$$\bar{x} = x, \quad \bar{y} = \frac{y}{2} = (b-x) \cdot \frac{h}{2b}$$

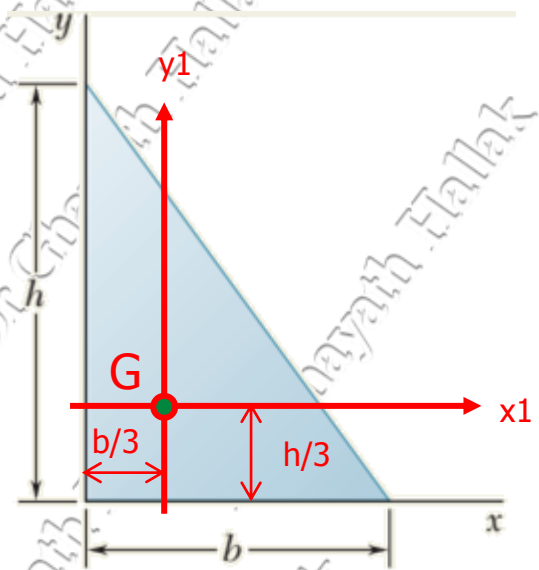
$$I_{xy} = \int \bar{x} \cdot \bar{y} \cdot dA = \int_0^b x \left[\frac{h}{2b} (b-x) \right] \cdot \frac{h}{b} (b-x) \cdot dx$$



MOMENTS OF INERTIA OF COMPOSITE AREAS

$$I_{xy} = \frac{h^2}{2b^2} \left[b^2 \frac{x^2}{2} + \frac{x^4}{4} - 2b \frac{x^3}{3} \right]_0^b$$

$$I_{xy} = \frac{b^2 h^2}{24}$$

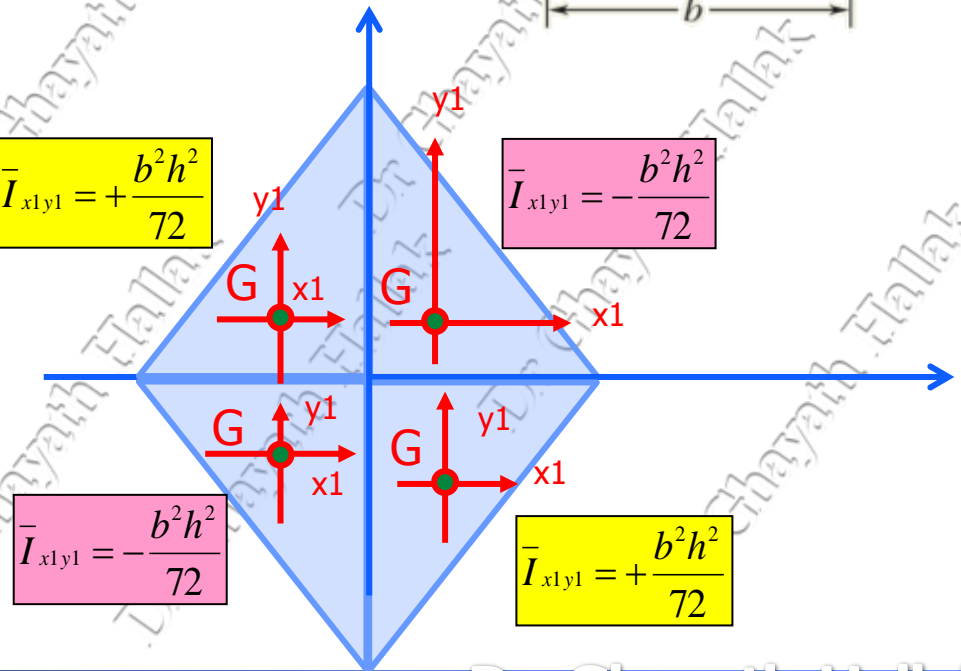


$$I_{XY} = \bar{I}_{x_1 y_1} + (A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x_1 y_1} = I_{XY} - (A_i) \times X_G \times Y_G = \frac{b^2 h^2}{24} - \left(\frac{b \cdot h}{2} \right) \times \left(-\frac{b}{3} \right) \times \left(-\frac{h}{3} \right)$$

$$\bar{I}_{x_1 y_1} = -\frac{b^2 h^2}{72}$$

$$\bar{I}_{x_1 y_1} = +\frac{b^2 h^2}{72}$$

$$\bar{I}_{x_1 y_1} = -\frac{b^2 h^2}{72}$$

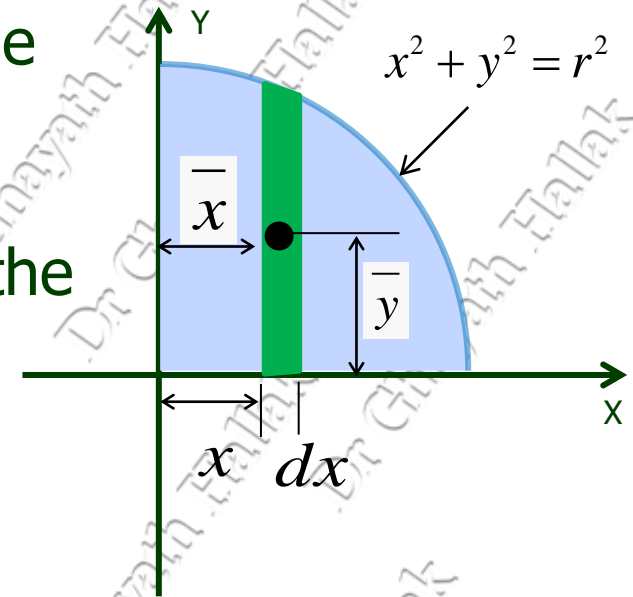


$$\bar{I}_{x_1 y_1} = -\frac{b^2 h^2}{72}$$

$$\bar{I}_{x_1 y_1} = +\frac{b^2 h^2}{72}$$

MOMENTS OF INERTIA OF COMPOSITE AREAS

Determine the product of inertia of the Quarter circle shown (a) with respect to the x and y axes and (b) with respect to centroidal axes parallel to the x and y axes.



$$I_{xy} = \int \bar{x} \cdot \bar{y} \cdot dA$$

$$dA = y \cdot dx$$

$$y^2 = r^2 - x^2 \Rightarrow y = (r^2 - x^2)^{0.5}$$

$$\therefore dA = (r^2 - x^2)^{0.5} \cdot dx$$

$$\bar{x} = x, \quad \bar{y} = \frac{y}{2} = \frac{(r^2 - x^2)^{0.5}}{2}$$

$$I_{xy} = \int_0^r x \frac{(r^2 - x^2)^{0.5}}{2} \cdot (r^2 - x^2)^{0.5} \cdot dx = \frac{1}{2} \left[r^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_0^r$$

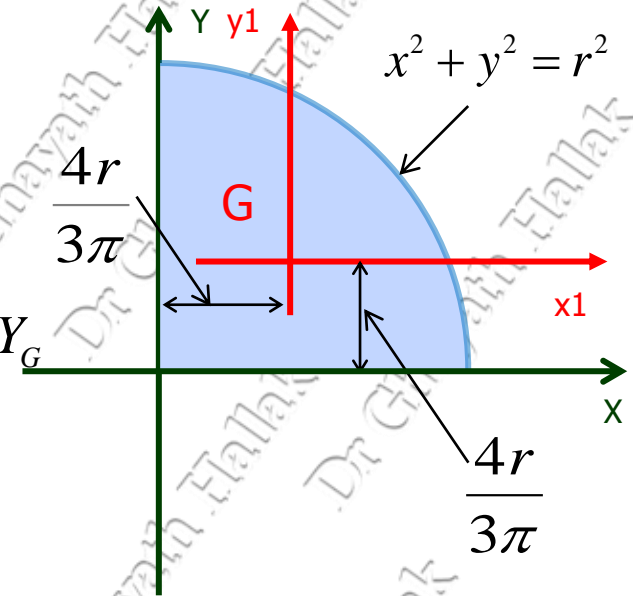
MOMENTS OF INERTIA OF COMPOSITE AREAS

$$I_{xy} = \frac{r^4}{8}$$

$$I_{XY} = \bar{I}_{x_1y_1} + (A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x_1y_1} = I_{XY} - (A_i) \times X_G \times Y_G$$

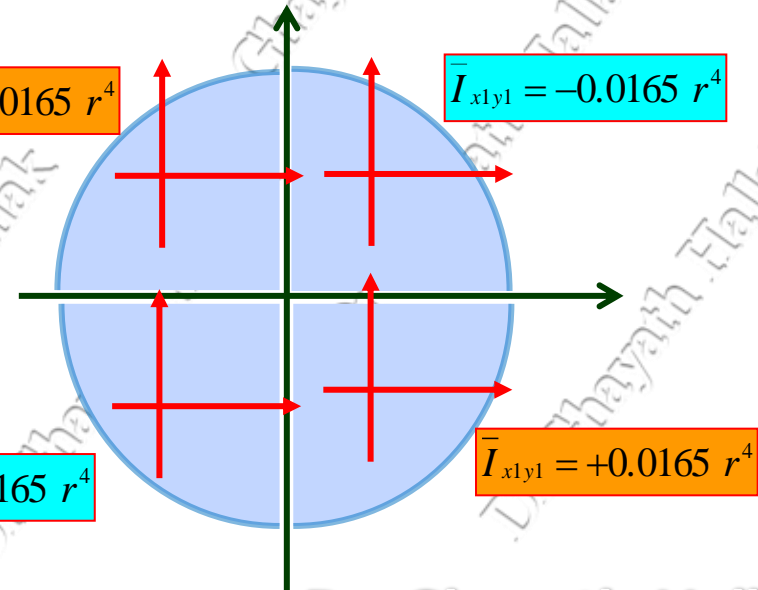
$$= \frac{r^4}{8} - \frac{\pi r^2}{4} \times \left(-\frac{4r}{3\pi}\right) \times \left(-\frac{4r}{3\pi}\right)$$

$$\bar{I}_{x_1y_1} = r^4 \left(\frac{1}{8} - \frac{4}{9\pi} \right) = -0.0165 r^4$$



$$\bar{I}_{x_1y_1} = +0.0165 r^4$$

$$\bar{I}_{x_1y_1} = -0.0165 r^4$$



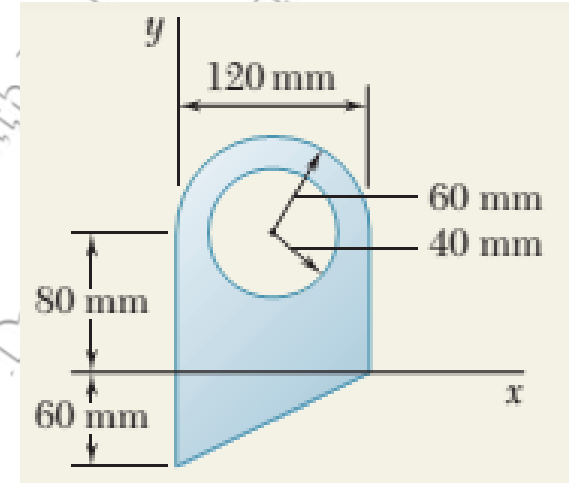
$$\bar{I}_{x_1y_1} = -0.0165 r^4$$

$$\bar{I}_{x_1y_1} = +0.0165 r^4$$

MOMENTS OF INERTIA OF COMPOSITE AREAS

For the plane area shown, determine

- (a) the location of the centroid.
- (b) The moment of inertia about x & y axis.
- (c) The moment of inertia about the centroidal axis.



$$\bar{I}_x = \frac{80^3 \times 120}{12}$$

$$\bar{I}_x = 512 \times 10^4 \text{ mm}^4$$

$$\bar{I}_y = \frac{120^3 \times 80}{12}$$

$$\bar{I}_y = 1152 \times 10^4 \text{ mm}^4$$

$$\bar{I}_x = \frac{60^3 \times 120}{36} = 72 \times 10^4 \text{ mm}^4$$

$$\bar{I}_y = \frac{120^3 \times 60}{36} = 288 \times 10^4 \text{ mm}^4$$

$$\bar{I}_{xy} = \frac{120^2 \times 60^2}{72} = 72 \times 10^4 \text{ mm}^4$$

$$\frac{4r_1}{3\pi} = 25.46 \text{ mm}$$

$$\bar{I}_x = r^4 \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) \approx 0.11 r^4$$

$$= 142.56 \times 10^4 \text{ mm}^4$$

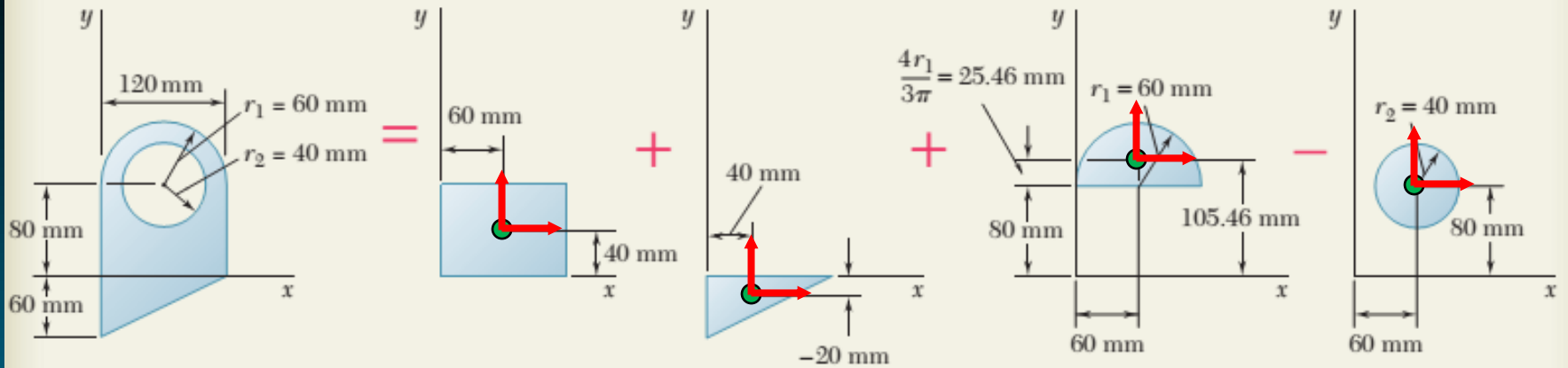
$$\bar{I}_y = \frac{\pi \times r^4}{8} = 508.68 \times 10^4 \text{ mm}^4$$

$$\bar{I}_x = \bar{I}_y$$

$$= \frac{\pi \times r^4}{4}$$

$$= 200.96 \times 10^4 \text{ mm}^4$$

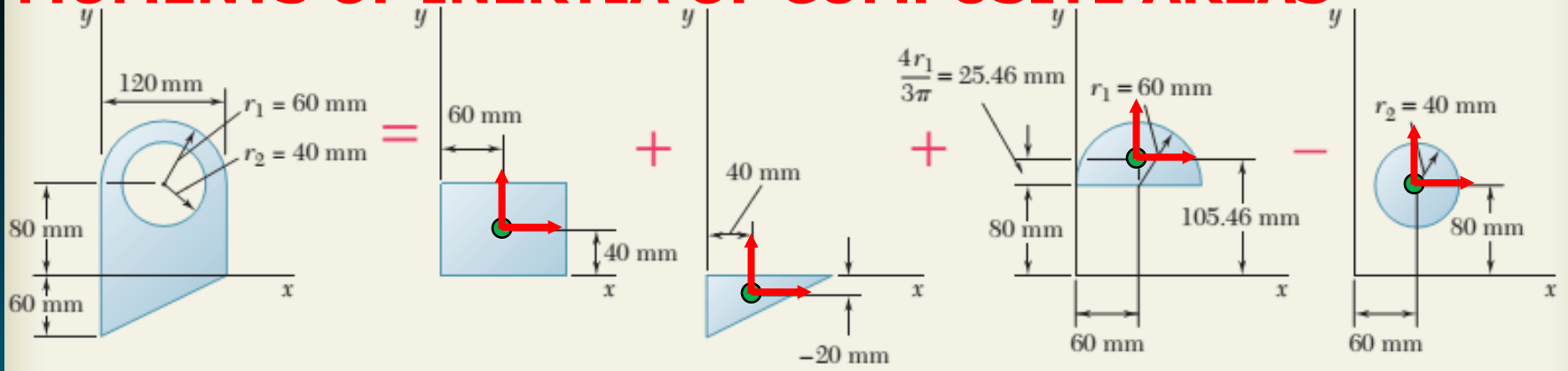
MOMENTS OF INERTIA OF COMPOSITE AREAS



Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	\bar{I}_x $\times 10^4$ mm ⁴	\bar{I}_y $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴	\bar{I}_{xy} $\times 10^4$ mm ⁴	$A_i \cdot x_i \cdot y_i$ $\times 10^4$ mm ⁴
Rect	9600	60	40	576	384	512	1152	3456	1536	0	2304
triang	3600	40	-20	144	-72	72	288	576	144	72	-288
Semi-cir	5652	60	105.5	339.1	596.3	142.6	508.7	2034.7	6290.8	0	3577.7
circle	- 5024	60	80	- 301.4	- 401.9	- 200.9	- 200.9	- 1808.6	- 3215.4	0	- 2411.5
Σ	13828			757.7	506.4	525.7	1747.8	4258.1	4755.4	72	3182.2

$$X_G = \frac{\sum A_i \times x_i}{\sum A_i} = \frac{757.7 \times 10^3}{13828} = 54.8 \text{ mm}, \quad Y_G = \frac{\sum A_i \times y_i}{\sum A_i} = \frac{506.4 \times 10^3}{13828} = 36.6 \text{ mm}$$

MOMENTS OF INERTIA OF COMPOSITE AREAS



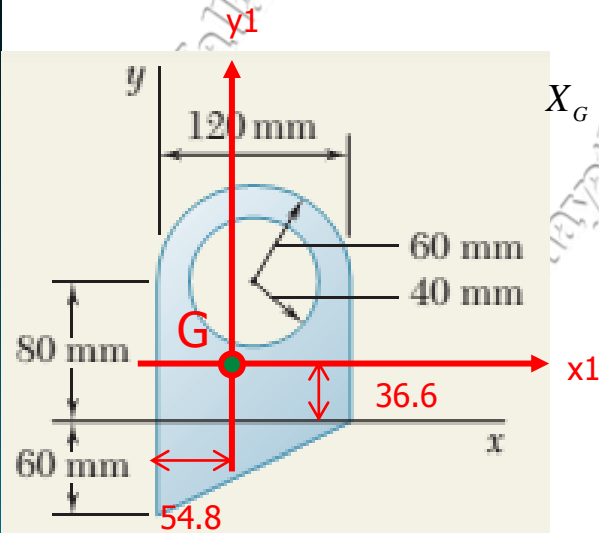
Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	\bar{I}_x $\times 10^4$ mm ⁴	\bar{I}_y $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴	\bar{I}_{xy} $\times 10^4$ mm ⁴	$A_i \cdot x_i \cdot y_i$ $\times 10^4$ mm ⁴
Rect	9600	60	40	576	384	512	1152	3456	1536	0	2304
triang	3600	40	-20	144	-72	72	288	576	144	72	-288
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circle	- 5024	60	80	- 301.4	- 401.9	- 200.9	- 200.9	- 1808.6	- 3215.4	0	- 2411.5
Σ	13828			757.7	506.4	525.7	1747.8	4258.1	4755.4	72	3182.2

$$I_x = \Sigma \bar{I}_x + \Sigma A_i \times y_i^2 = 525.7 \times 10^4 + 4755.4 \times 10^4 = 5281.1 \times 10^4 \text{ mm}^4$$

$$I_y = \Sigma \bar{I}_y + \Sigma A_i \times x_i^2 = 1747.8 \times 10^4 + 4258.1 \times 10^4 = 6005.9 \times 10^4 \text{ mm}^4 ,$$

$$I_{xy} = \Sigma \bar{I}_{xy} + \Sigma A_i \times x_i \times y_i = 72 \times 10^4 + 3182.2 \times 10^4 = 3254.2 \times 10^4 \text{ mm}^4$$

MOMENTS OF INERTIA OF COMPOSITE AREAS



$$X_G = \frac{\sum A_i \times x_i}{\sum A_i} = \frac{757.7 \times 10^3}{13828} = 54.8 \text{ mm} \quad , \quad Y_G = \frac{\sum A_i \times y_i}{\sum A_i} = \frac{506.4 \times 10^3}{13828} = 36.6 \text{ mm}$$

$$I_X = \sum \bar{I}_x + \sum A_i \times y_i^2 = 525.7 \times 10^4 + 4755.4 \times 10^4 = 5281.1 \times 10^4 \text{ mm}^4$$

$$I_Y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 1747.8 \times 10^4 + 4258.1 \times 10^4 = 6005.9 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \sum \bar{I}_{xy} + \sum A_i \times x_i \times y_i = 72 \times 10^4 + 3182.2 \times 10^4 = 3254.2 \times 10^4 \text{ mm}^4$$

$$I_X = \bar{I}_{x1} + (\sum A_i) \times Y_G^2 \Rightarrow \bar{I}_{x1} = I_X - (\sum A_i) \times Y_G^2 = 5281.1 \times 10^4 - (13828) \times (-36.6)^2 = 3428.75 \times 10^4 \text{ mm}^4$$

$$I_Y = \bar{I}_{y1} + (\sum A_i) \times X_G^2 \Rightarrow \bar{I}_{y1} = I_Y - (\sum A_i) \times X_G^2 = 6005.9 \times 10^4 - (13828) \times (-54.8)^2 = 1853.3 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \bar{I}_{x1y1} + (\sum A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x1y1} = I_{XY} - (\sum A_i) \times X_G \times Y_G = 3254.2 \times 10^4 - (13828) \times (-36.6) \times (-54.8)$$

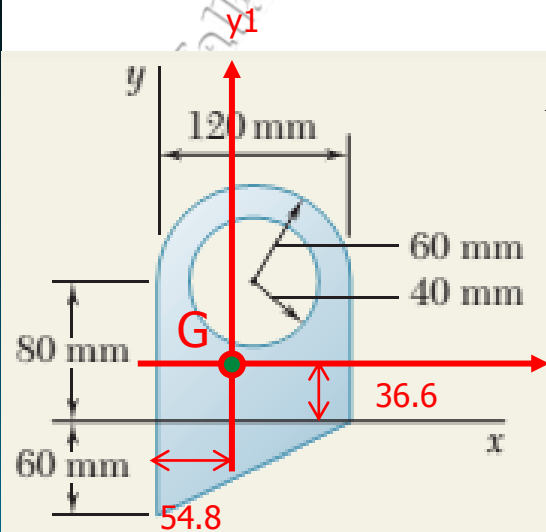
$$\bar{I}_{x1y1} = 480.75 \times 10^4 \text{ mm}^4$$

البعد بين المحاور المتوازية
منسوبة للمحاور القديمة
المركزية

المحاور الجديدة

المحاور القديمة المركزية

MOMENTS OF INERTIA OF COMPOSITE AREAS



$$X_G = \frac{\sum A_i \times x_i}{\sum A_i} = \frac{757.7 \times 10^3}{13828} = 54.8 \text{ mm} \quad , \quad Y_G = \frac{\sum A_i \times y_i}{\sum A_i} = \frac{506.4 \times 10^3}{13828} = 36.6 \text{ mm}$$

$$I_X = \sum \bar{I}_x + \sum A_i \times y_i^2 = 525.7 \times 10^4 + 4755.4 \times 10^4 = 5281.1 \times 10^4 \text{ mm}^4$$

$$I_Y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 1747.8 \times 10^4 + 4258.1 \times 10^4 = 6005.9 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \sum \bar{I}_{xy} + \sum A_i \times x_i \times y_i = 72 \times 10^4 + 3182.2 \times 10^4 = 3254.2 \times 10^4 \text{ mm}^4$$

$$I_X = \bar{I}_{x1} + (\sum A_i) \times Y_G^2 \Rightarrow \bar{I}_{x1} = I_X - (\sum A_i) \times Y_G^2 = 5281.1 \times 10^4 - (13828) \times (-36.6)^2 = 3428.75 \times 10^4 \text{ mm}^4$$

$$I_Y = \bar{I}_{y1} + (\sum A_i) \times X_G^2 \Rightarrow \bar{I}_{y1} = I_Y - (\sum A_i) \times X_G^2 = 6005.9 \times 10^4 - (13828) \times (-54.8)^2 = 1853.3 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \bar{I}_{x1y1} + (\sum A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x1y1} = I_{XY} - (\sum A_i) \times X_G \times Y_G = 3254.2 \times 10^4 - (13828) \times (-36.6) \times (-54.8)$$

$$\bar{I}_{x1y1} = 480.75 \times 10^4 \text{ mm}^4$$

$$\tan 2\phi_p = \frac{2\bar{I}_{x1y1}}{\bar{I}_{y1} - \bar{I}_{x1}} = \frac{2(480.75 \times 10^4)}{(1853.3 - 3428.75) \times 10^4} = -0.61 \Rightarrow \phi_p = -15.7 \text{ degree}$$

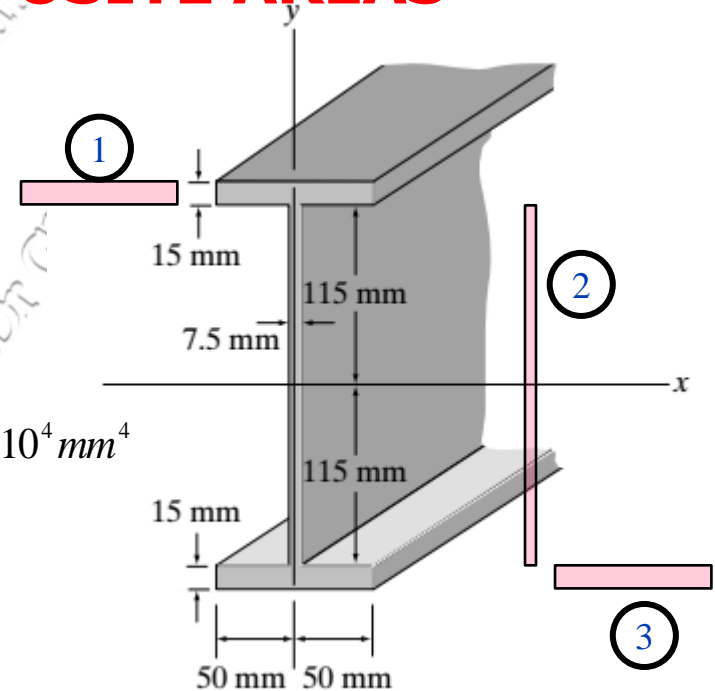
$$I_{xp} = \bar{I}_{x1} \cos^2 \phi_p + \bar{I}_{y1} \sin^2 \phi_p - \bar{I}_{x1y1} \sin 2\phi_p = 3563.8 \times 10^4 \text{ mm}^4$$

$$I_{yp} = \bar{I}_{y1} \cos^2 \phi_p + \bar{I}_{x1} \sin^2 \phi_p + \bar{I}_{x1y1} \sin 2\phi_p = 1718.2 \times 10^4 \text{ mm}^4$$

$$I_{max,min} = \frac{\bar{I}_{x1} + \bar{I}_{y1}}{2} \pm \frac{1}{2} \sqrt{(\bar{I}_{x1} - \bar{I}_{y1})^2 + 4\bar{I}_{x1y1}^2} = \{3563.8 \times 10^4, 1718.0 \times 10^4\} \text{ mm}^4$$

MOMENTS OF INERTIA OF COMPOSITE AREAS

Determine the moment of inertia of the beam's cross-sectional area about the x & y axis.



$$I_x = \sum \bar{I}_x + \sum A_i \times y_i^2 = 766.07 \times 10^4 + 4501.8 \times 10^4 = 5267.87 \times 10^4 \text{ mm}^4$$

$$I_y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 250.809 \times 10^4 + 0 = 250.809 \times 10^4 \text{ mm}^4$$

Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	I_{xi} $\times 10^4$ mm ⁴	I_{yi} $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴
1	1500	0	122.5	0	183.75	2.8125	125	0	2250.9
2	1725	0	0	0	0	760.44	0.809	0	0
3	1500	0	-122.5	0	-183.75	2.8125	125	0	2250.9
Σ	4725			0	0	766.07	250.809	0	4501.8

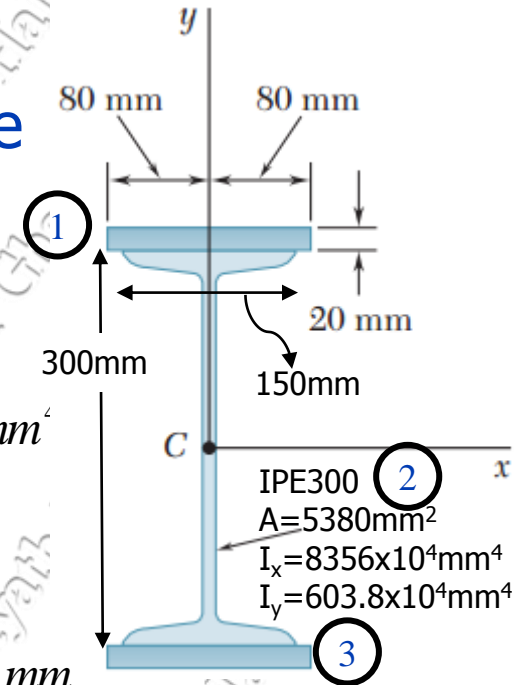
MOMENTS OF INERTIA OF COMPOSITE AREAS

Two 20-mm steel plates are welded to a rolled IPE 300 section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

$$I_X = \sum \bar{I}_x + \sum A_i \times y_i^2 = 8377.3 \times 10^4 + 16384 \times 10^4 = 9761.3 \times 10^4 \text{ mm}^4$$

$$I_Y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 1969.14 \times 10^4 + 0 = 1969.14 \times 10^4 \text{ mm}^4$$

$$r_x = \sqrt{\frac{I_X}{A}} = \sqrt{\frac{9761.3 \times 10^4}{11780}} = 91 \text{ mm}, r_y = \sqrt{\frac{I_Y}{A}} = \sqrt{\frac{1969.1 \times 10^4}{11780}} = 41 \text{ mm}$$



Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	I_{xi} $\times 10^4$ mm ⁴	I_{yi} $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴
1	3200	0	160	0	512	10.67	682.67	0	8192
2	5380	0	0	0	0	8356	603.8	0	0
3	3200	0	-160	0	-512	10.67	682.67	0	8192
Σ	11780			0	0	8377.3	1969.14	0	16384

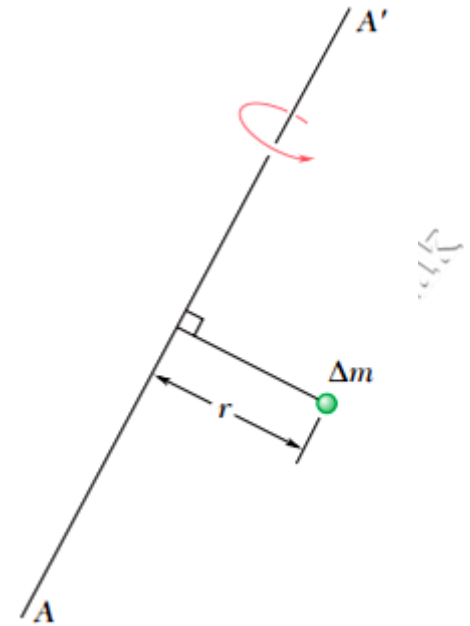
MOMENT OF INERTIA OF A MASS

Consider a small mass Δm mounted on a rod of negligible mass which can rotate freely about an axis AA' . If a couple is applied to the system, the rod and mass, assumed to be initially at rest, will start rotating about AA' .

The time required for the system to reach a given speed of rotation is proportional to the mass Δm and to the square of the distance r .

The product $r^2 \Delta m$ provides, therefore, a measure of the *inertia* of the system, i.e., a measure of the resistance the system offers when we try to set it in motion.

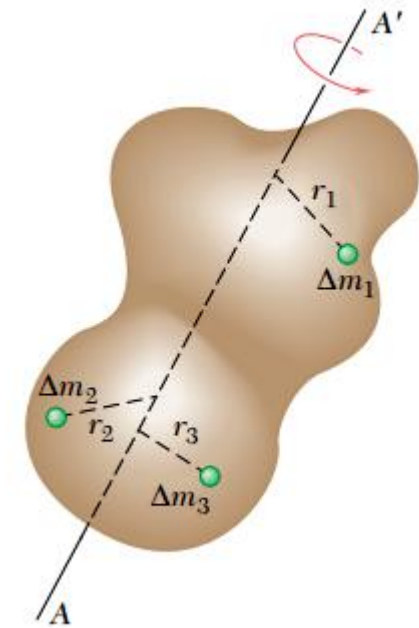
For this reason, the product $r^2 \Delta m$ is called the *moment of inertia* of the mass Δm with respect to the axis AA' .



1- MOMENT OF INERTIA OF A MASS

Consider now a body of mass m which is to be rotated about an axis AA' . Dividing the body into elements of mass $\Delta m_1, \Delta m_2$, etc., we find that the body's resistance to being rotated is measured by the sum $(r_1)^2 \Delta m_1 + (r_2)^2 \Delta m_2 + \dots$

$$I_{AA'} = \int r^2 dm$$



MOMENTS OF INERTIA OF THIN PLATES

Consider a thin plate of uniform thickness t , which is made of a homogeneous material of density ρ (density = mass per unit volume). The mass moment of inertia of the plate with respect to an axis AA' contained in the plane of the plate

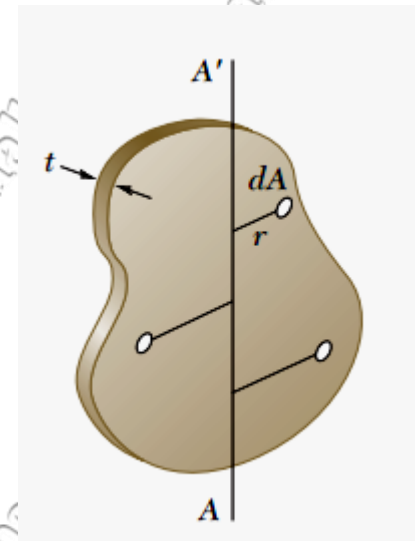
$$I_{AA',\text{mass}} = \int r^2 dm$$

Since $dm = \rho t dA$, we write

$$I_{AA',\text{mass}} = \rho t \int r^2 dA$$

But r represents the distance of the element of area dA to the axis AA' ; the integral is therefore equal to the moment of inertia of the area of the plate with respect to AA' . We have

$$I_{AA',\text{mass}} = \rho t I_{AA',\text{area}}$$



MOMENTS OF INERTIA OF THIN PLATES

Similarly, for an axis BB' which is contained in the plane of the plate and is perpendicular to AA' , we have

$$I_{BB',\text{mass}} = \rho t I_{BB',\text{area}}$$

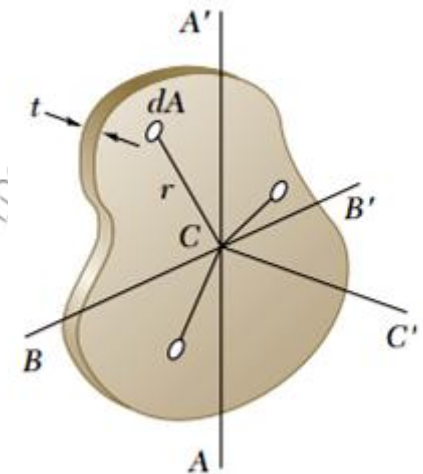
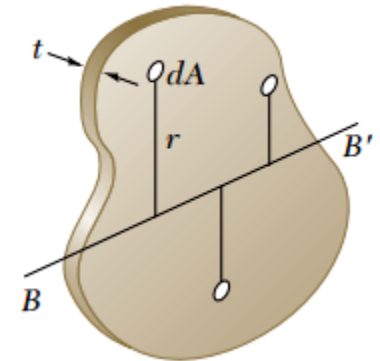
Considering now the axis CC' which is *perpendicular* to the plate and passes through the point of intersection C of AA' and BB' , we write

$$I_{CC',\text{mass}} = \rho t J_{C,\text{area}}$$

where J_C is the *polar* moment of inertia of the area of the plate with respect to point C .

$$J_C = I_{AA'} + I_{BB'}$$

$$I_{CC',\text{mass}} = I_{AA',\text{mass}} + I_{BB',\text{mass}}$$



MOMENTS OF INERTIA OF THIN PLATES

Rectangular Plate.

The mass moments of inertia with respect to axes through the center of gravity of the plate:

$$I_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho t (a^3 b/12)$$

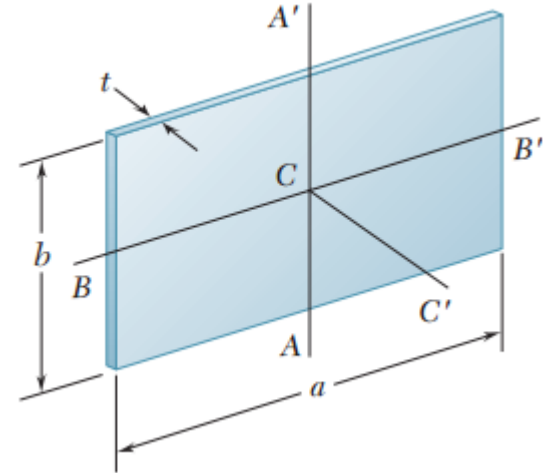
$$I_{BB',\text{mass}} = \rho t I_{BB',\text{area}} = \rho t (b^3 a/12)$$

$$\rho abt = m$$

$$I_{AA',\text{mass}} = m a^2 /12$$

$$I_{BB',\text{mass}} = m b^2 /12$$

$$I_{CC',\text{mass}} = I_{AA',\text{mass}} + I_{BB',\text{mass}} = m [(a^2 + b^2) /12]$$



MOMENTS OF INERTIA OF THIN PLATES

Circular Plate.

The mass moments of inertia with respect to axes through the center of gravity of the plate:

$$I_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho t (\pi r^4 / 4)$$

$$\rho \pi r^2 t = m$$

$$I_{AA',\text{mass}} = I_{BB',\text{mass}} = m r^2 / 4$$

$$I_{CC',\text{mass}} = I_{AA',\text{mass}} + I_{BB',\text{mass}} = m r^2 / 2$$

