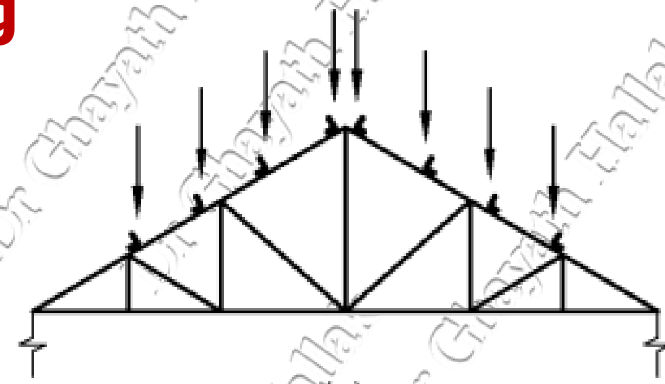
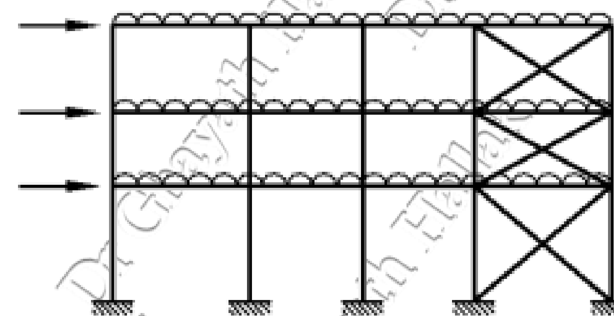
BEAM COLUMNS ((Combined moment and axial load))

Occurrence of combined loading

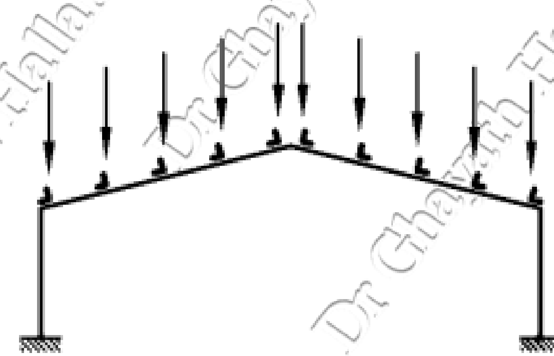
Roof truss-top chord members subject to bending from purlin loads and compression due to overall bending.



Simple framing - columns subject to bending from eccentric beam reactions and compression due to gravity loading.



Portal frame - rafters and columns subject to bending and compression due to frame action

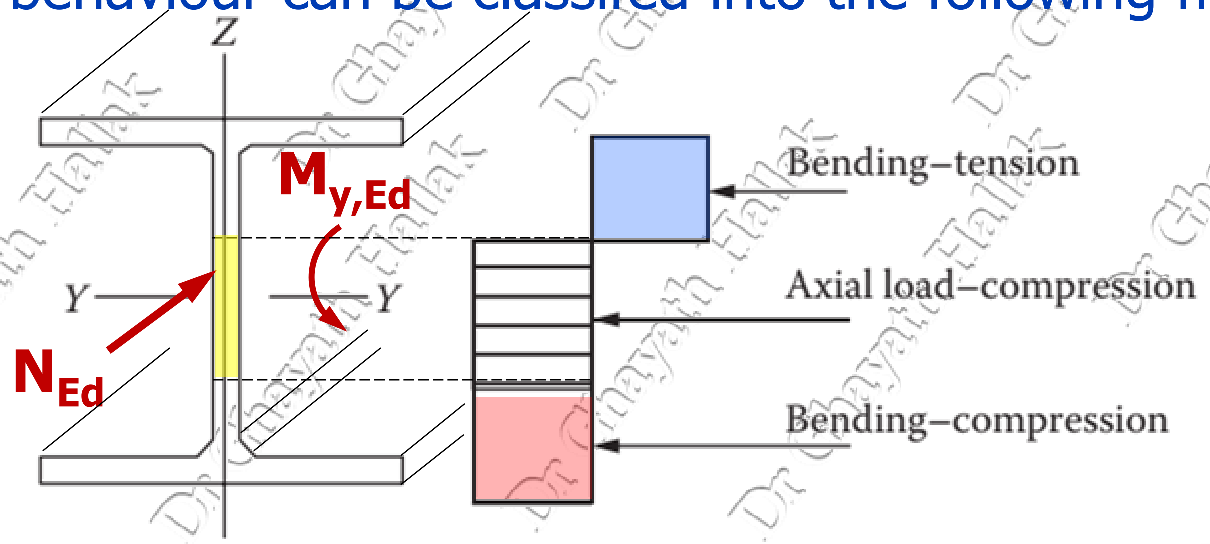


BEAM COLUMNS ((Combined moment and axial load))

Types of response – interaction:

The behaviour of a member under bending and axial force results from the ***interaction between instability and plasticity*** and is influenced by geometrical and material imperfections.

Consider a class 1 or class 2 H section column as shown in the Figure. The behaviour depends on the **column length**, how the **moments are applied** and the **lateral** support, if any, provided. The behaviour can be classifed into the following five cases:



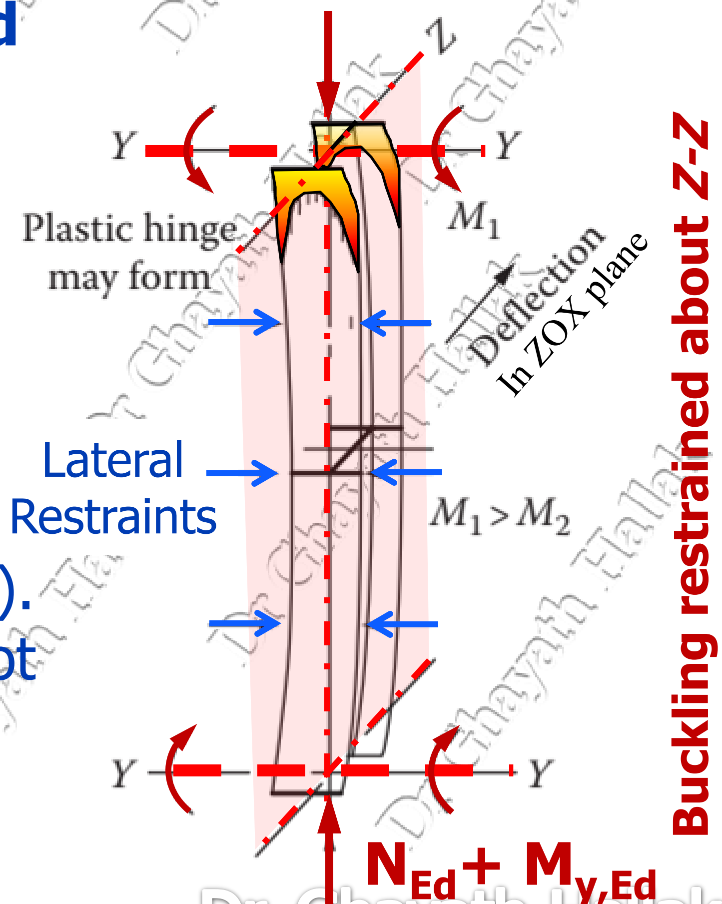
BEAM COLUMNS ((Combined moment and axial load))

Types of response – interaction:

Case 1: A short column subjected to axial load and uniaxial bending about either axis or biaxial bending. Failure generally occurs when the plastic capacity of the section is reached.

Case 2: A **slender column subjected to axial load and uniaxial bending about the major axis *y–y*.** If the column is supported laterally against buckling about the minor axis *z–z* out of the plane of bending, the column **fails by buckling about the *y–y* axis**.

This is not a common case (see Figure). At low axial loads or if the column is not very slender, a plastic hinge forms at the end or point of maximum moment.



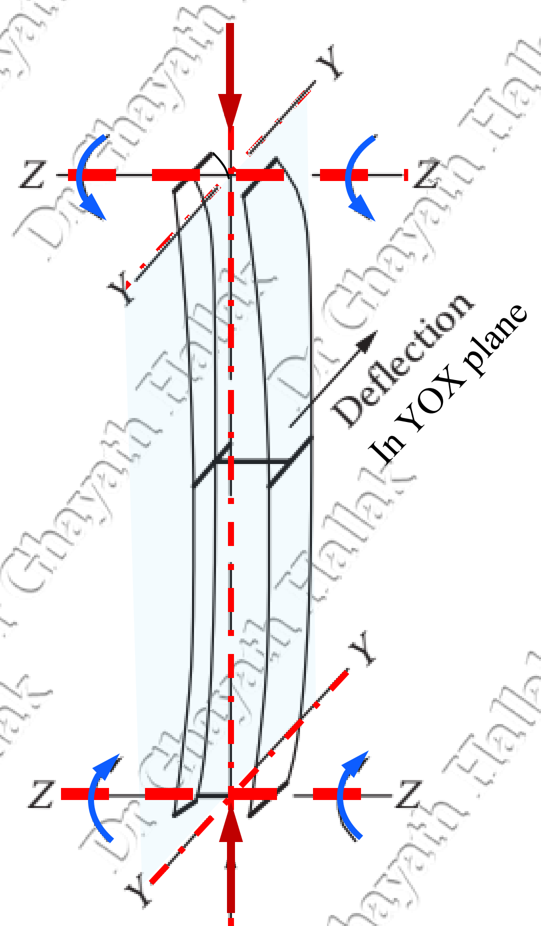
BEAM COLUMNS ((Combined moment and axial load))

Types of response – interaction:

Case 3 : **A slender column subjected to axial load and uniaxial bending about the minor axis *z–z*.**

The column does not require lateral support and there is no buckling out of the plane of bending.

The column **fails by buckling about the *z–z* axis**. At very low axial loads, it will reach the bending capacity for *z–z* axis (see Figure).

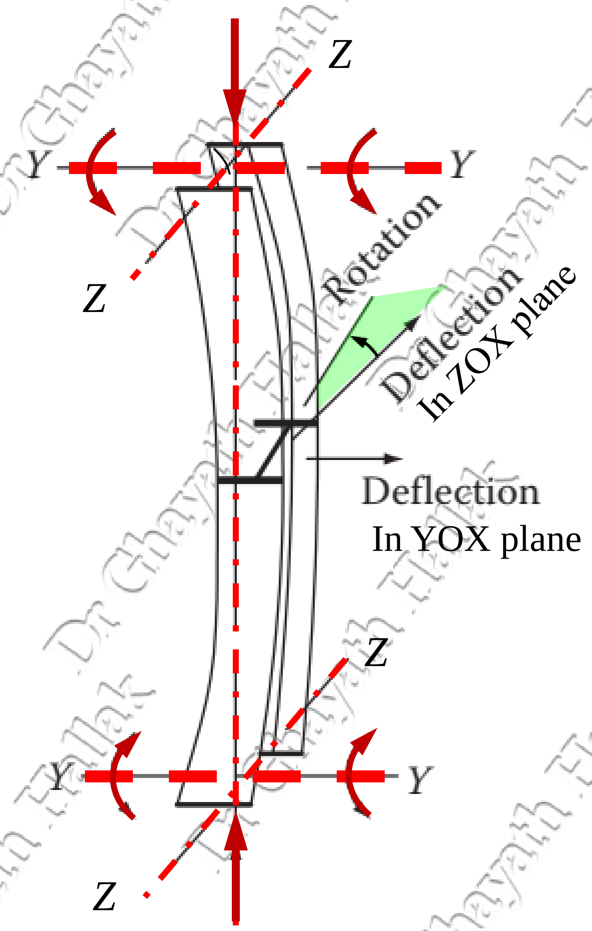


N_{Ed}+ $M_{z,Ed}$-(no restraint)

BEAM COLUMNS ((Combined moment and axial load))

Types of response – interaction:

Case 4: **A slender column subjected to axial load and uniaxial bending about the major axis *y–y*.** This time the column has no lateral support. The column **fails due to a combination of column buckling about the *y–y* axis and lateral torsional buckling** where the column section twists as well as deflecting in the *y–y* and *z–z* planes (see Figure).



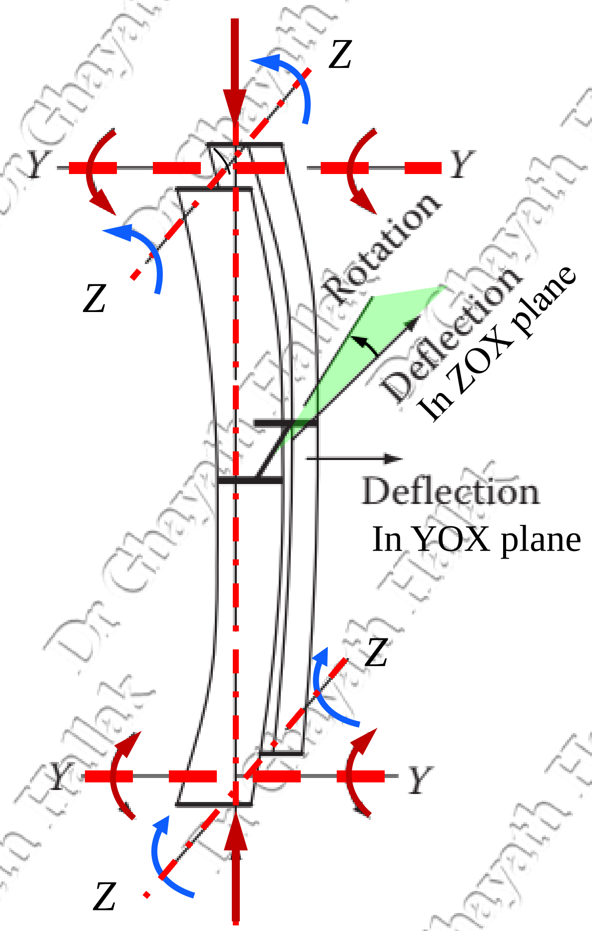
$N_{Ed} + M_{y,Ed}$

NO restraint about *Z-Z*

BEAM COLUMNS ((Combined moment and axial load))

Types of response – interaction:

Case 5 : **A slender column subject to axial load and biaxial bending**. The column has no lateral support. The failure is the same as in Case 4 earlier, but minor axis buckling will usually have the greatest effect.



General loading $N_{Ed} + M_{y,Ed} + M_{z,Ed}$

NO restraint about *Z-Z*

DESIGN PROCEDURE

The verification of the safety of members subject to bending and axial force is made in two steps:

- ❑- verification of the resistance of cross sections;
- ❑- verification of the member buckling resistance (in general governed by flexural or lateral-torsional buckling).

❑-Cross-section resistance:

For Class 1 , 2 & 3 cross-sections -Clause 6.2 of BS EN 1993-1-1

$$\boxed{\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1}$$

conservative approach

N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the applied loads and moments,
N_{Rd} , $M_{y,Rd}$ and $M_{z,Rd}$ are the axial and bending resistances.

❑- Cross-section resistance:

A more economic solution is to use Clause 6.2.9.1(2) of BS EN 1993-1-1, which states:

$$M_{Ed} \leq M_{N,Rd}$$

$M_{N,Rd}$ is the design plastic moment of resistance allowing for the presence of the axial force N_{Ed} .

For doubly symmetric I- and H-sections ((SMALL N_{Ed}))

The effect of axial load on reducing the moment capacity can be **IGNORD** IF

Moment about the y-y axis
$$\begin{cases} N_{Ed} \leqslant 0.25 N_{pl,Rd} \\ N_{Ed} \leq \dfrac{0.5 h_w t_w f_y}{\gamma_{M0}} \end{cases}$$

Moment about the z-z axis
$$\begin{cases} N_{Ed} \leq \dfrac{h_w t_w f_y}{\gamma_{M0}} \end{cases}$$

❑- Cross-section resistance:

For Class 1 & 2 cross-sections (Large N_{Ed})

$$M_{Ed} \leq M_{N,Rd}$$

rolled or welded I or H sections

$M_{N,y,Rd} = M_{pl,y,Rd}\,(1\text{-}n)/(1\text{-}0.5a) \leq M_{pl,y,Rd}$

$n = N_{Ed} / N_{pl.Rd}$

$a = (A\text{-}2bt_f\,)/A \leq 0.5$

$n \leq a \Longrightarrow M_{N,z,Rd} = M_{pl,z,Rd}$

$n > a \Longrightarrow M_{N,z,Rd} = M_{pl,z,Rd}\{1\text{-}[(n\text{-}a)/(1\text{-}a)]^2\}$

Rectangular Hollow sections

$a_w = (A - 2bt)/A \leq 0.5$

$a_f = (A - 2ht)/A \leq 0.5$

Welded Box sections

$a_w = (A\text{-}2bt_f)/A \leq 0.5$

$a_f = (A\text{-}2ht_w\,)/A \leq 0.5$

$M_{N,y,Rd} = M_{pl,y,Rd}\,(1 - n)/(1 - 0.5a_w) \leq M_{pl,y.R}$

$M_{N,z,Rd} = M_{pl,z,Rd}\,(1 - n)/(1 - 0.5a_f\,) \leq M_{pl,z,Rd}$

❑- Cross-section resistance:

For Class 1 & 2 cross-sections (Large N_{Ed})

Circular Hollow Sections

$$M_{Ed} \le M_{N,Rd}$$

$n = N_{Ed} / N_{pl.Rd}$ $M_{N,Rd} = M_{pl,Rd}\,(1 - n^{1.7}) \le M_{pl,Rd}$

Rectangular Solid sections

$n = N_{Ed} / N_{pl.Rd}$ $M_{N,Rd} = M_{pl,Rd}\,(1 - n^{2}) \le M_{pl,Rd}$

Bi-axial bending and axial force, the $N + M_y + M_z$ interaction

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \le 1$$

Sections	α	β
I or H sections	2	$5n \ge 1$
circular hollow	2	2
rectangular hollow	$1.66/(1-1.13n^2) \leqslant 6$	$1.66/(1-1.13n^2) \leqslant 6$

$M_{N,y,Rd}$ and $M_{N,z,Rd}$ are the reduced plastic moments resistances evaluated as previously described.

❑- Cross-section resistance:

For Class 3 cross-sections

maximum longitudinal stress due to moment and axial force

$\sigma_{x,Ed} \leq f_y/\gamma_{M0}$ taking account of fastener holes where relevant

For Class 4 cross-sections

maximum longitudinal stress due to moment and axial force calculated using the effective cross sections

$\sigma_{x,Ed} \leq f_y/\gamma_{M0}$ taking account of fastener holes where relevant

$$\frac{N_{Ed}}{A_{eff}\; f_y/\gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed}\, e_{Ny}}{W_{eff,y,min}\; f_y/\gamma_{M0}} + \frac{M_{z,Ed} + N_{Ed}\, e_{Nz}}{W_{eff,z,min}\; f_y/\gamma_{M0}} \leq 1$$

A_{eff} is the effective area of the cross-section when subjected to uniform compression

$W_{eff,min}$ is the effective section modulus (corresponding to the fibre with the maximum elastic stress) of the cross-section when subjected only to moment about the relevant axis

e_N is the shift of the relevant centroidal axis when the cross-section is subjected to compression

❑- Member resistance

The instability of a member of **doubly symmetric** cross section, **not susceptible to distortional deformations**, and subject to bending and axial compression, can be due to **flexural buckling** or to **lateral torsional buckling**. Therefore, clause 6.3.3(1) considers two distinct situations:

✓- members not susceptible to torsional deformation, such as **members of circular hollow section or other sections restrained from torsion**. Here, flexural buckling is the relevant instability mode

✓- members that are susceptible to torsional deformations, such as members of **open section (I or H sections)** that are not restrained from torsion. Here, lateral torsional buckling tends to be the relevant instability mode.

❑- Member resistance

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + K_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} + K_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1 \quad \Lambda\ (6-61)$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + K_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} + K_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1 \quad \Lambda\ (6-62)$$

N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the design values of the axial compression force and the maximum bending moments along the member about *y* and *z*, respectively;

$\Delta M_{y,Ed}$ and $\Delta M_{z,Ed}$ are the moments due to the shift of the centroidal axis on a reduced effective class 4 cross section;

χ_y and χ_z are the reduction factors due to flexural buckling about *y* and *z*, respectively, evaluated according to clause 6.3.1 of EN 1993-1-1: 2005

χ_{LT} is the reduction factor due to lateral-torsional buckling, evaluated according to clause 6.3.2 of EN 1993-1-1: 2005 (χ_{LT} = 1.0 for members that are not susceptible to torsional deformation);

k_{yy}, k_{yz}, k_{zy} and k_{zz} are interaction factors that depend on the relevant instability and plasticity phenomena, obtained through Annex A (Method 1) or Annex B(Method 2);

❑- Member resistance

Table 6.7: Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y}\,N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z}\,N_{Ed}$

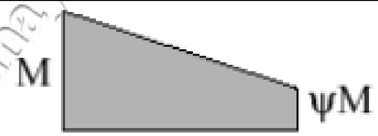
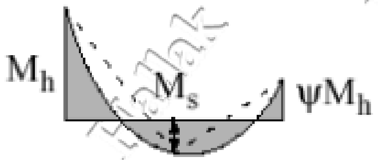
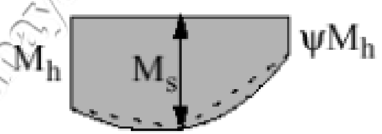
Safe (maximum) values for interaction factors

Interaction factor	Class 1 and 2	Class 3
k_{yy}	$1.8C_{my}$	$1.6C_{my}$
k_{yz}	$0.6k_{zz}$	k_{zz}
k_{zy}	1.0	1.0
k_{zz}	$2.4C_{mz}$	$1.6C_{mz}$

Conservatively and for simplicity interaction factors $K_{ij}=1$

❑- Member resistance

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

Moment diagram	range		C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated load
	$-1 \le \psi \le 1$		$0{,}6 + 0{,}4\psi \ge 0{,}4$	
 $\alpha_s = M_s / M_h$	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0{,}2 + 0{,}8\alpha_s \ge 0{,}4$	$0{,}2 + 0{,}8\alpha_s \ge 0{,}4$
	$-1 \le \alpha_s < 0$	$0 \le \psi \le 1$	$0{,}1 - 0{,}8\alpha_s \ge 0{,}4$	$-0{,}8\alpha_s \ge 0{,}4$
		$-1 \le \psi < 0$	$0{,}1(1-\psi) - 0{,}8\alpha_s \ge 0{,}4$	$0{,}2(-\psi) - 0{,}8\alpha_s \ge 0{,}4$
 $\alpha_h = M_h / M_s$	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0{,}95 + 0{,}05\alpha_h$	$0{,}90 + 0{,}10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \psi \le 1$	$0{,}95 + 0{,}05\alpha_h$	$0{,}90 + 0{,}10\alpha_h$
		$-1 \le \psi < 0$	$0{,}95 + 0{,}05\alpha_h(1+2\psi)$	$0{,}90 - 0{,}10\alpha_h(1+2\psi)$

For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0{,}9$ or $C_{Mz} = 0{,}9$ respectively.

C_{my}, C_{mz} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:

moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

hogging moment ---- negative
sagging moment +++ positive.

❑- Member resistance

According to **The Institution of Structural Engineers** "Manual for the design of steelwork building structures to Eurocode 3".2010

If the column is subject to moments other than from beam eccentricity the following expressions can be used:

For class I and H sections (susceptible to lateral torsional buckling):

$$\frac{N_{Ed}}{\chi_{min}(Af_{yd})} + \frac{M_{y,Ed}}{\chi_{LT}(W_y f_{yd})} + C_{mz}\frac{M_{z,Ed}}{W_z f_{yd}} \leqslant 0.78 \text{ for class 1 and 2, and 0.85 for class 3 and 4}$$

For RHS sections (not susceptible to lateral torsional buckling):

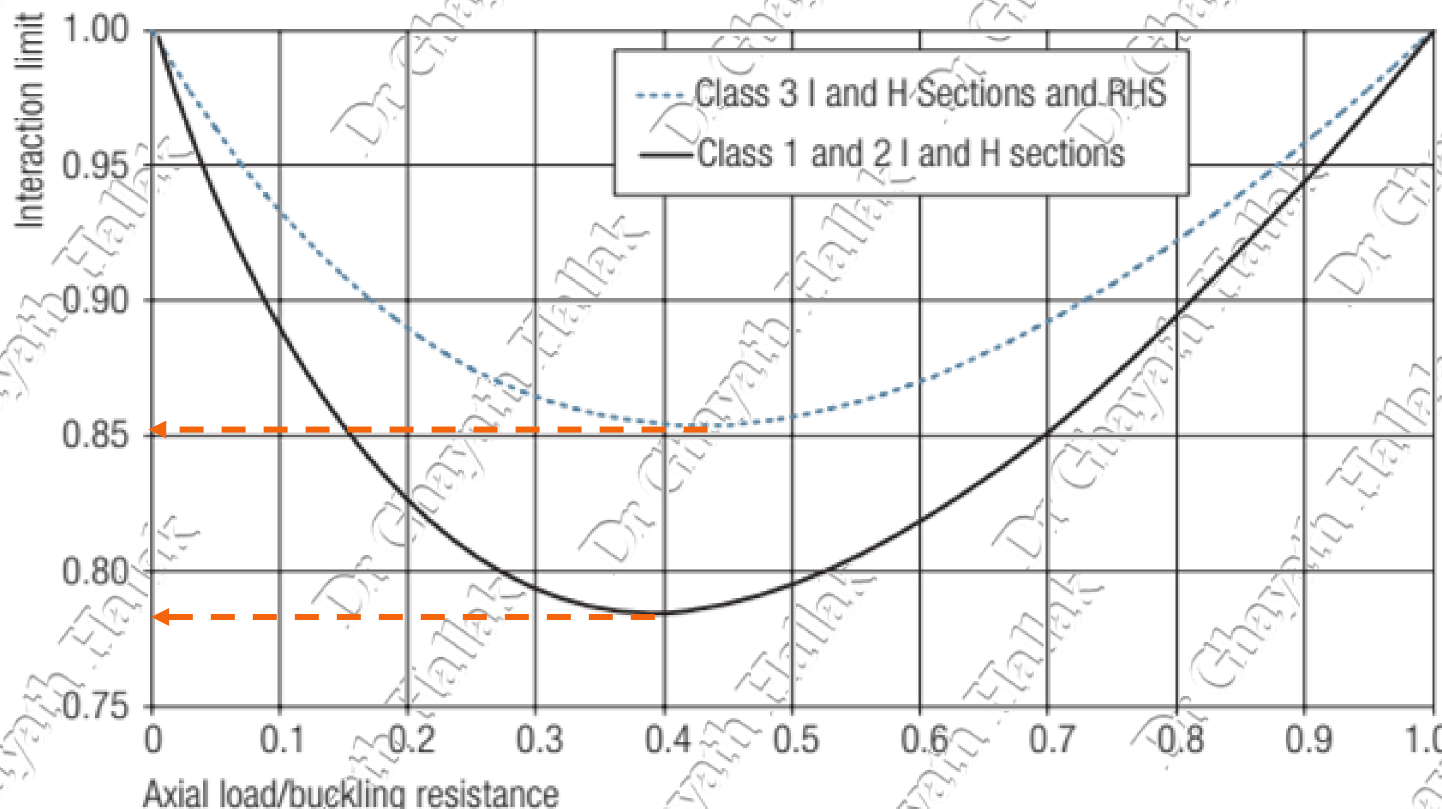
$$\frac{N_{Ed}}{\chi_{min} A f_{yd}} + C_{my}\frac{M_{y,Ed}}{W_y f_{yd}} + C_{mz}\frac{M_{z,Ed}}{W_z f_{yd}} \leqslant 0.85$$

The interaction limits of 0.85 and 0.78 are minimum values and apply at a particular axial load. The variation of the limits with applied load can be seen in the following Figure.

C_{my} and C_{mz} are uniform moment factors, taken from table B.3

❑- Member resistance

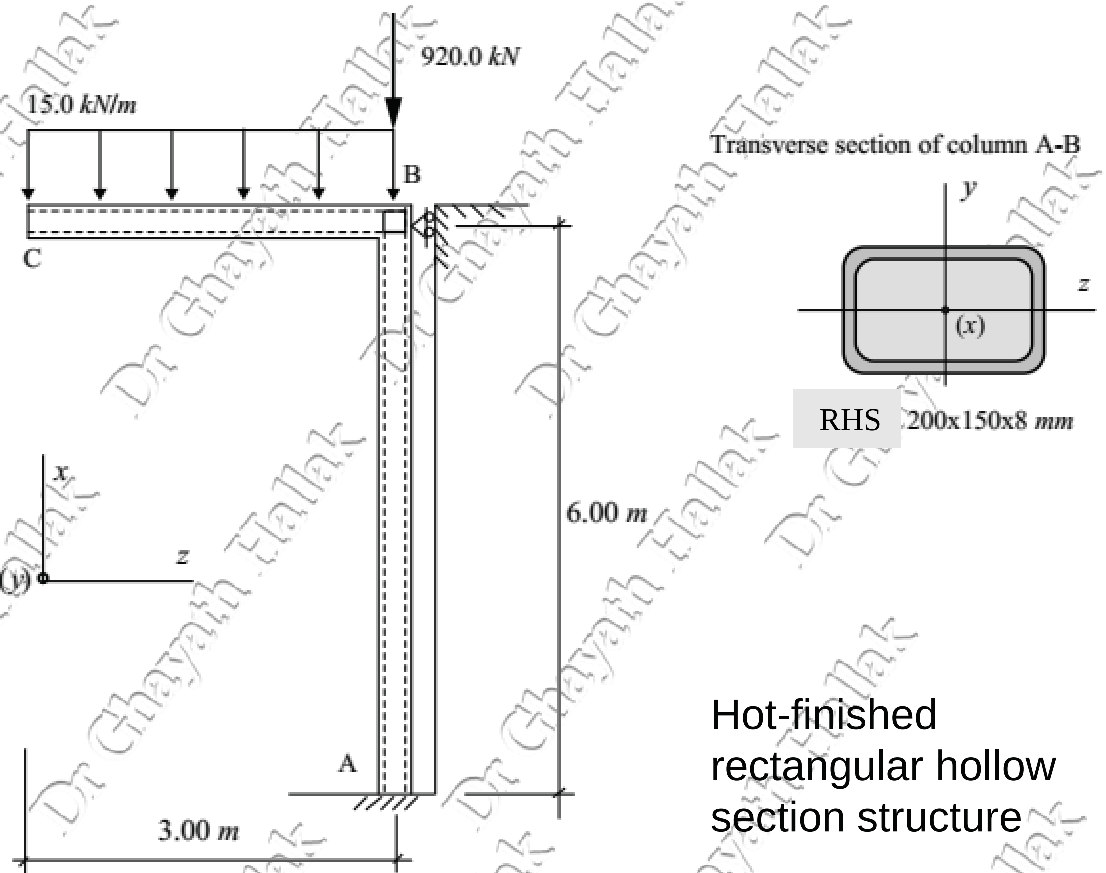
According to **The Institution of Structural Engineers** "Manual for the design of steelwork building structures to Eurocode 3".2010



Variation of interaction limit with axial load

➢- Example 1

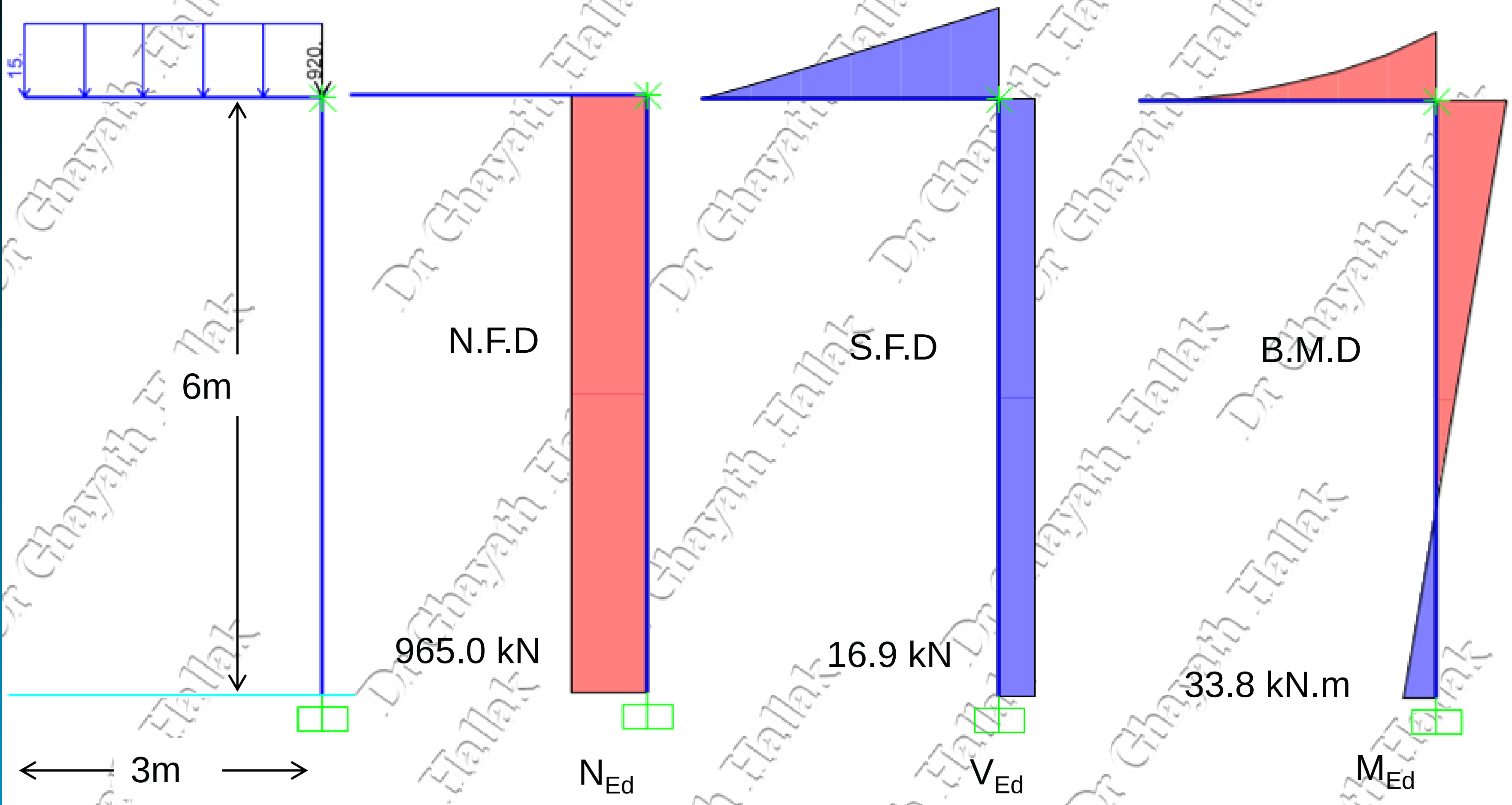
Consider column A-B that supports a steel cantilever B-C, represented in Figure. The column is fixed at section A, while the top section (B) is free to rotate, but restrained from horizontal displacements in both directions. The column has a rectangular



Hot-finished rectangular hollow section structure

hollow section RHS 200x150x8 *mm* in S 355 steel (E = 210 *GPa* and G = 81 *GPa*). Assuming that the indicated loading is already factored for ULS, verify the column according to EN 1993-1-1: 2005.

➢- Example 1



i) *Internal force diagrams*

➢- Example 1

ii) *Verification of the cross section resistance*

Classification of the cross section resistance

RHS 200x150x8 *mm*

$A = 52.8\ cm^2$, $W_{pl,y} = 359\ cm^3$, $W_{el,y} = 297\ cm^3$, $I_y = 2970\ cm^4$, $i_y = 7.5 cm$,
$W_{pl,z} = 294\ cm^3$, $W_{el,z} = 253\ cm^3$, $I_z = 1890\ cm^4$, $i_z = 5.99\ cm$,
$I_T = 3640 cm^4$, Iw=398cm3, $C_w/t=22$, $C_f/t=15.8$. $\varepsilon=(235/f_y)^{0.5}=(235/355)^{0.5}=0.81$

For a member subjected to varying bending and compression, the class of the cross section may vary along the member.

In this example, a simplified approach is adopted, whereby the class of the cross section is verified for the most unfavourable situation (compressed section only).

Table 5.2 in EN 1993-1-1:2005.

$c/t \approx$ (b-3t)/t $\approx$ (200-3x8)/8=22 < 33 ε (33 × 0.81) = 26.73 . (Class 1)

The cross section is class 1 in compression and can be treated as a class 1 cross section for any other combination of stresses.

➢- Example 1

ii) *Verification of the cross section resistance*

Major axis bending resistance

$M_{pl,y,Rd} = W_{pl,y}\, f_y/\gamma_{M0} = 359x10^3x355/1 = 127.45kN.m$

Bending resistance about the y axis, combined with the axial force:

the critical cross section (top of the column),

N_{Ed} = 965.0 kN and $M_{y,Ed}$ = 67.5 kN.m

$$M_{N,y,Rd} = M_{pl,y,Rd}\,(1 - n)/(1 - 0.5a_w) \leq M_{pl,y.Rd}$$

$a_w = (A - 2bt)/A = (5280-2x150x8)/5280 = 0.55 \geq 0.5 \Longrightarrow a_w = 0.50$

$n = N_{Ed} / N_{pl.Rd} = N_{Ed} / (A\, f_y/\gamma_{M1}) = 965x10^3/(5280x355/1) = 0.51$

$M_{N,y,Rd} = 127.45\,(1 - 0.51)/(1 - 0.5x0.5) = 83.27kN.m \leq M_{pl,y.Rd}$

$M_{N,y,Rd} = 83.27kN.m \geq 67.5kN.m = M_{y,Ed}$ O.K.

Shear verification: From clause 6.2.6(3):

$A_v = A\, h/(b+h) = 5280x200/(150+200) = 3017.1mm2$

$V_{pl,Rd} = A_v\, f_y/(\gamma_{M0} x\sqrt{3}) = 3017.1x355/(1x\sqrt{3}) = 618.4kN > V_{Ed} = 16.9kN$ OK

➢- Example 1

ii) *Verification of the cross section resistance*

Shear verification:

V_{Ed}=16.9kN<0.5$V_{pl,Rd}$=309.2kN low shear, No reduction to bending resistance.

Shear buckling of the web, according to clause 6.2.6(6), with η = 1

h_w/t_w = (200 -3×8)/8=22.0 <72 ε/ η= 58.32 No shear buckling verfication is required

iii) *Verification of the stability of the member*

class 1 section, χ_{LT} = 1.0 for members that are not susceptible to torsional deformation:

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + K_{yy}\frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} \le 1 \Rightarrow \frac{N_{Ed}}{N_{b,y,Rd}} + K_{yy}\frac{M_{y,Ed}}{M_{pl,y,Rd}} \le 1 ...(6-61)$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + K_{zy}\frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} \le 1 \Rightarrow \frac{N_{Ed}}{N_{b,z,Rd}} + K_{zy}\frac{M_{y,Ed}}{M_{pl,y,Rd}} \le 1 ...(6-62)$$

➢- Example 1

iii) *Verification of the stability of the member*

Major and minor axis column buckling resistances

Effective lengths:

Plane *xz* (buckling about *y*): in plane

$L_{E,y}$ = 0.7×6=4.2m

$$\overline{\lambda}_y = \frac{\lambda_y}{\lambda_1} = \frac{L_{E,y}/i_y}{93.9\,\varepsilon} = \frac{4.2}{7.5\times10^{-2}}\times\frac{1}{93.9\times0.81} = 0.74$$

Plane *xy* (buckling about *z*):out of plane

$L_{E,z}$ = 0.85 × 6.0 = 5.1m ;

$$\overline{\lambda}_z = \frac{\lambda_z}{\lambda_1} = \frac{L_{E,z}/i_z}{93.9\,\varepsilon} = 1.12$$

From Table 6.2 of EN 1993-1-1: For a hot-rolled CHS, use buckling curve **a**

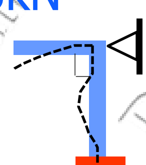
α=0.21

$$\phi_y = 0.5\left[1+\alpha\left(\overline{\lambda}_y - 0.2\right)+\overline{\lambda}_y^{\,2}\right]$$

$$\phi_y = 0.5\left[1+0.21\times(0.74-0.2)+0.74^2\right] = 0.83$$

$$\chi_y = \frac{1}{\left[0.83+\sqrt{0.83^2-0.74^2}\right]} = 0.83$$

$N_{b,y,Rd}$= χ_y A f_y/γ_{M1}=0.83x5280x355/1.0 =1556x10³N=1556kN> N_{Ed} =965kN



α=0.21

$$\phi_z = 0.5\left[1+\alpha\left(\overline{\lambda}_z - 0.2\right)+\overline{\lambda}_z^{\,2}\right]$$

$$\phi_z = 0.5\left[1+0.21\times(1.12-0.2)+1.12^2\right] = 1.22$$

$$\chi_z = \frac{1}{\left[1.22+\sqrt{1.22^2-1.12^2}\right]} = 0.59$$

$N_{b,z,Rd}$= χ_z A f_y/γ_{M1}=0.59x5280x355/1.0 =1106x10³N=1106kN > N_{Ed} =965kN

➢- Example 1

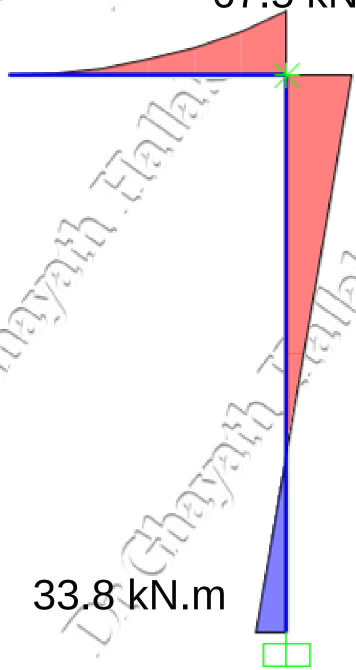
iii) *Verification of the stability of the member*

$$\frac{N_{Ed}}{N_{b,y,Rd}} + K_{yy}\frac{M_{y,Ed}}{M_{pl,y,Rd}} \le 1 \quad \Lambda\ (6-61)$$

$$\frac{N_{Ed}}{N_{b,z,Rd}} + K_{zy}\frac{M_{y,Ed}}{M_{pl,y,Rd}} \le 1 \quad \Lambda\ (6-62)$$

The interaction factors k_{yy} and k_{zy} can be obtained using:

1- Conservative method $k_{yy} = k_{zy} = 1.0$:



$$\frac{965}{1556} + 1.0\frac{67.5}{127.45} \le 1.15 \rangle 1.0 \quad NOT\ OK$$

$$\frac{965}{1106} + 1.0\frac{67.5}{127.45} \le 1.4 \rangle 1.0 \quad NOT\ OK$$

2- Safe (maximum) method:

Table on slide 14 $k_{yy} = 1.8\ C_{my}$, $k_{zy} = 1.0$:

Table B.3 $\psi = 33.8/-67.5 = -0.5 \rightarrow C_{my} = 0.6 + 0.4(-0.5) = 0.4$

$k_{yy} = 1.8\ C_{my} = 1.8 \times 0.4 = 0.72$, $k_{zy} = 1.0$

➢- Example 1

iii) *Verification of the stability of the member*

2- Safe (maximum) method:

$$\frac{965}{1556}+0.72\frac{67.5}{127.45}\leq 1.01\rangle 1.0 \quad NOT\ OK$$

$$\frac{965}{1106}+1.0\frac{67.5}{127.45}\leq 1.4\rangle 1.0 \quad NOT\ OK$$

3- Method 1 in the code Annex A:

I_T = 3643 cm^4 > I_y = 2971 cm^4, the member is not susceptible to torsional deformation

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}+k_{yy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}}=$$

$$=\frac{965.0}{0.83\times 1872.6/1.0}+0.69\times\frac{67.5}{1.0\times 127.4/1.0}=0.986<1.0 \quad \text{OK}$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}+k_{zy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}}=$$

$$=\frac{965.0}{0.59\times 1872.6/1.0}+0.40\times\frac{67.5}{1.0\times 127.4/1.0}=1.09>1.0 \quad \text{Not OK}$$

➢- Example 1

iii) *Verification of the stability of the member*

4- Method 2 in the code Annex B:

$$k_{yy} = 0.53 . k_{zy} \;=\; 0.$$

$$\frac{965.0}{0.83 \times 1872.6/1.0} + 0.53 \times \frac{67.5}{1.0 \times 127.4/1.0} = 0.9 < 1.0 \text{ OK}$$

$$\frac{965}{0.59 \times 1872.6/1.0} = 0.87 < 1.0 \text{ OK}$$

iv) Verification of the stability of the member using The Institution of Structural Engineers Manual

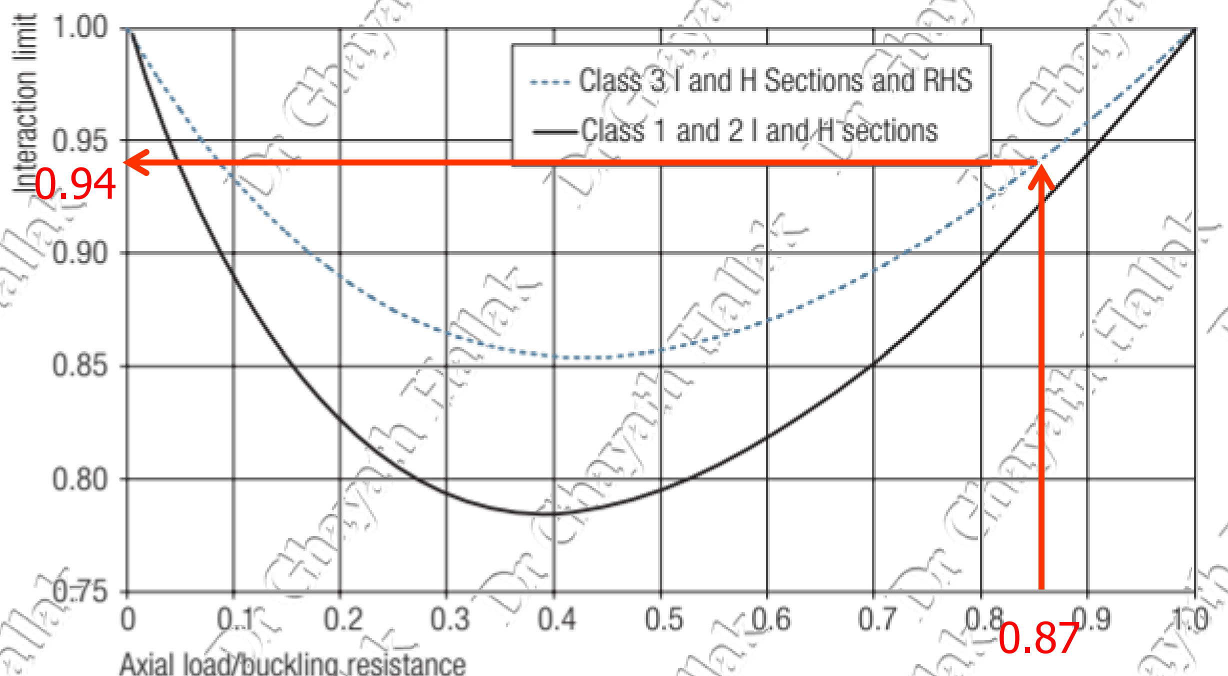
$$\frac{N_{Ed}}{\chi_{min} A f_{yd}} + C_{my} \frac{M_{y,Ed}}{W_y f_{yd}} + C_{mz} \frac{M_{z,Ed}}{W_z f_{yd}} \leqslant 0.85$$

C_{my}=0.6+0.4(-0.5)=0.4

$N_{b,z,Rd}$= 1106kN , N_{Ed} =965kN , N_{Ed}/ $N_{b,z,Rd}$=0.87

➢- Example 1

iv) Verification of the stability of the member using The Institution of Structural Engineers Manual

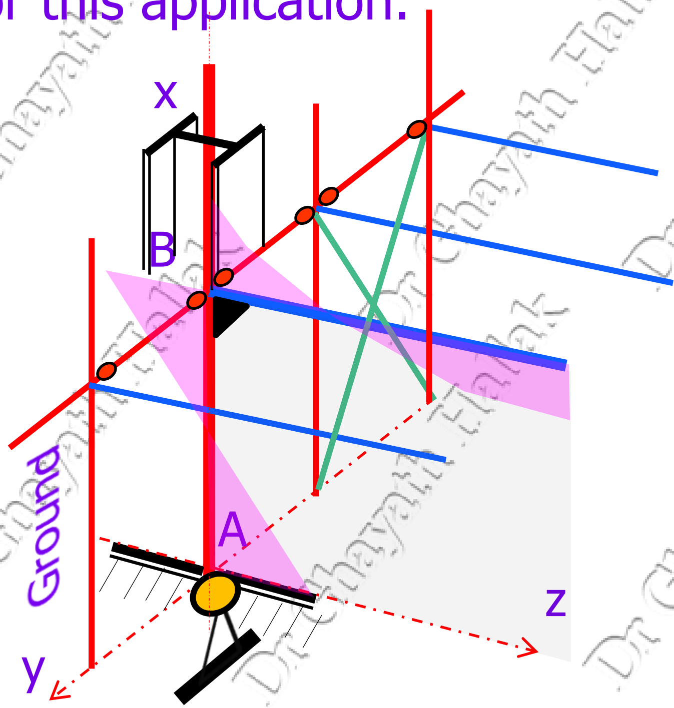


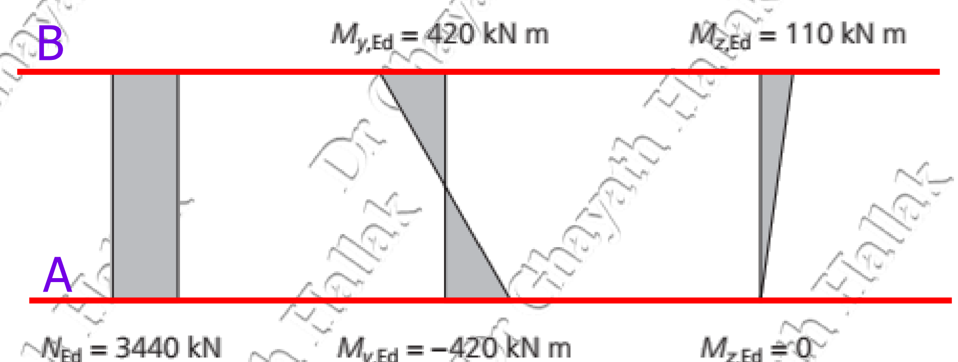
$$\frac{N_{Ed}}{\chi_{min} N_{Rk} / \gamma_{M1}} + C_{my} \frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk} / \gamma_{M1}} \leq 0.94$$

$$\frac{965}{1106} + 0.4 \frac{67.5}{127.45} = 1.08 \rangle 0.94 \quad NOT\ OK$$

➢- Example 2

An H-section member of length 4.2 m is to be designed as a ground-floor column in a multi-storey building. The frame is moment resisting in-plane and braced out-of-plane. The column is subjected to major axis bending due to horizontal forces and minor axis bending due to eccentric loading from the floor beams. From the structural analysis, the design action effects arise in the column are shown in the Figure below. Assess the suitability of a hot-rolled 305 x 305 x 240 H-section in grade S275 steel for this application.





➢- Example 2

Section properties

305 x 305 x 240 H-section

h = 352.5 mm, b = 318.4 mm, t_w= 23.0 mm, t_f= 37.7 mm, r = 15.2 mm
A = 30 600 mm^2, I_y= 642.0 × 10^6 mm^4, I_z= 203.1 × 10^6 mm^4, i_y=145mm,
i_z=81.5mm, I_T= 12.71 × 10^6 mm^4, I_w= 5.03 × 10^{12} mm^6, U=0.854
$W_{el,y}$ = 3 643 000 mm^3, $W_{el,z}$ = 1 276 000 mm^3, $W_{pl,y}$ = 4247000 mm^3,
$W_{el,z}$ = 1 951 000 mm^3

From EN 10025-2 for 16mm$<t_f\leqslant$40mm → f_y= 265 N/mm^2

Cross-section classification (clause 5.5.2)

$\varepsilon=\sqrt{(235/f_y)}=\sqrt{(235/265)}=0.94$

Outstand flanges (Table 5.2, sheet 2):
c_f = (b - t_w - 2r)/2 = 132.5 mm
c_f/t_f = 132.5/37.7 = 3.51
Limit for Class 1 flange = 9 ε = 8.46
8.46 > 3.51 ; flanges are Class 1
Web – internal compression part (Table 5.2, sheet 1):
c_w = h - 2t_f - 2r = 246.7 mm
c_w/t_w = 246.7/23.0 = 10.73

➢- Example 2

Cross-section classification (clause 5.5.2)

Limit for Class 1 web = 33 ε = 31.02

31.02 > 10.73 ; web is Class 1

The overall cross-section classification is therefore Class 1.

Compression resistance of cross-section (clause 6.2.4)

$N_{c,Rd} = A\, f_y/\gamma_{M0}$ = [(30600x265)/1.0]x10^{-3}=8109kN>3440kN

∴ Cross section resistance is OK

Bending resistance of cross-section (clause 6.2.5)

Major (y–y) axis

Maximum bending moment $M_{y,Ed}$ = 420.0 kN m

$M_{pl,y,Rd} = W_{pl,y}\, f_y/\gamma_{M0}$= 4247x10^{3}x265/1=1125.46kN.m>420kN.m OK

Minor (z–z) axis

Maximum bending moment $M_{z,Ed}$ = 110.0 kN m

$M_{pl,z,Rd} = W_{pl,z}\, f_y/\gamma_{M0}$= 1951 x10^{3}x265/1=517.02kN.m>110kN.m OK

➢- Example 2

Shear resistance of cross-section (clause 6.2.6)

Load parallel to web

Maximum shear force

V_{Ed} =(M_{top}-M_{bottom})/L= 840/4.2 = 200 kN

For a rolled H-section, loaded parallel to the web,

$A_v = A - 2bt_f + (t_w+2r)t_f$ (but not less than $\eta\, h_w\, t_w$), $\eta = 1$

$h_w = (h - 2t_f) = 352.5 - (2x37.7) = 277.1$ mm

A_v = 30 600 – (2 x 318.4 x 37.7) + (23.0 + [2 x 15.2]) x 37.7

= 8606 mm2 (but not less than 1.0 x 277.1 x 23.0 = 6373 mm2)

$V_{pl,Rd} = A_v\, f_y/(\gamma_{M0} x \sqrt{3}) = 8606x265/(1x\sqrt{3}) = 1316.7kN > V_{Ed} = 200kN$ OK

$V_{Ed} = 200kN < 0.5V_{pl,Rd} = 658.35kN$ low shear, No reduction to bending resistance.

Load parallel to flanges

Maximum shear force V_{Ed} = 110/4.2 = 26.2 kN

No guidance on the determination of the shear area for a rolled I- or H-section loaded parallel to the flanges is presented in EN 1993-1-1. adopting the recommendations provided for a welded I- or H-section would be acceptable.

➢- Example 2

Shear resistance of cross-section (clause 6.2.6)

Load parallel to flanges

$A_w = A - \sum (h_w\ t_w) = 30\ 600 - (277.1 \times 23.0) = 24\ 227\ mm^2$

$V_{pl,Rd}=A_w\ f_y/(\gamma_{M0} x \sqrt{3})=24277x265/(1x\ \sqrt{3})=3707kN>V_{Ed}=26.2kN$ OK

$V_{Ed}=26.2kN<0.5V_{pl,Rd}=1853.5kN$ low shear, No reduction to bending resistance.

Shear buckling

$h_w/t_w < 72\ \varepsilon/\ \eta$ for unstiffened webs

$\eta= 1.0$, Actual $h_w/t_w=277.1/23.0=12.0 <72\ \varepsilon/\ \eta= 67.7$; no shear buckling check required

Cross-section resistance under bending, shear and axial force (clause 6.2.10)

Since, $V_{Ed} < 0.5V_{pl,Rd}$ for both axes, and shear buckling is not a concern (see above). Therefore, the cross-section need only be checked for bending and axial force.

➢- Example 2

Cross-section resistance under bending, shear and axial force (clause 6.2.10)

No reduction to the major axis plastic resistance moment due to the effect of axial force is required when both of the following criteria are satisfied:

$N_{Ed} \leqslant 0.25 N_{pl,Rd}$ $\rightarrow 0.25N_{pl,Rd} = 0.25 \times 8415 = 2104\ kN < 3440kN$
Not satisfied

$$N_{Ed} \leq \frac{0.5 h_w t_w f_y}{\gamma_{M0}} = \frac{0.5 \times 277.1 \times 23.0 \times 265}{1.0} = 844.46 kN$$

844.46kN<3440kN Not satisfied

Therefore, allowance for the effect of axial force on the major axis plastic moment resistance of the cross-section must be made.

➢- Example 2

Cross-section resistance under bending, shear and axial force (clause 6.2.10)

No reduction to the minor axis plastic resistance moment due to the effect of axial force is required when the following criterion is satisfied:

$$N_{Ed} \leq \frac{h_w t_w f_y}{\gamma_{M0}} \rightarrow \frac{277.1 \times 23.0 \times 265}{1.0} = 1689\,kN \langle 3440\,kN \text{ not satisfied}$$

Therefore, allowance for the effect of axial force on the minor axis plastic moment resistance of the cross-section must be made.

Reduced plastic moment resistances (clause 6.2.9.1(5))

Major (y–y) axis:

$n = N_{Ed} / N_{pl.Rd} = 3440/ 8109 = 0.42$

$a = (A-2bt_f)/A \leq 0.5$, $a = [30600-(2x318.4x37.7)]/30600 = 0.22$

$M_{N,y,Rd} = M_{pl,y,Rd} (1-n)/(1-0.5a) \leq M_{pl,y,Rd}$

$M_{N,y,Rd} = 1125x (1-0.42)/(1-0.5x0.22) = 733kN.m \geq 420kN.m = M_{y,Ed}$ OK

- Example 2

Reduced plastic moment resistances (clause 6.2.9.1(5))

Minor (z–z) axis:

$n = N_{Ed} / N_{pl.Rd} = 3440/8109 = 0.42$

$a = (A-2bt_f)/A \leq 0.5$, $a = [30600-(2x318.4x37.7)]/30600 = 0.22$

$n > a \Longrightarrow M_{N,z,Rd} = M_{pl,z,Rd}\{1-[(n-a)/(1-a)]^2\}$

$M_{N,z,Rd} = 517.0x\{1-[(0.42-0.22)/(1-0.22)]^2\} = 483kN.m > 110kN.m$ OK

Cross-section check for bi-axial bending (with reduced moment resistances)

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha} + \left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta} \leq 1$$

For I- and H-sections: $\alpha = 2$ and $\beta = 5n$ (but $\beta \geq 1$) = (5 x 0.42) = 2.12

$$\left[\frac{420}{733}\right]^{2} + \left[\frac{110}{483}\right]^{2.12} = 0.37 \leq 1 \quad OK$$

➢- Example 2

Member buckling resistance in compression (clause 6.3.1)

For buckling about the major (y–y) axis:

Plane *xz* (buckling about *y*):

$L_{E,y}$ = 0.7×4.2=2.94m

$$\overline{\lambda}_y = \frac{\lambda_y}{\lambda_1} = \frac{L_{E,y}/i_y}{93.9\,\varepsilon} = \frac{2.94}{14.5\times10^{-2}} \times \frac{1}{93.9\times0.94} = 0.23$$

For buckling about the minor (z–z) axis:

Plane *xy* (buckling about *z*):

$L_{E,z}$ = 1.0× 4.2 = 4.2m ;

$$\overline{\lambda}_z = \frac{\lambda_z}{\lambda_1} = \frac{L_{E,z}/i_z}{93.9\,\varepsilon} = 0.58$$

From Table 6.2 of EN 1993-1-1: For a hot-rolled H-section (with h/b ≤ 1.2, S275 steel), use buckling curve

b, α=0.34

$$\phi_y = 0.5\left[1+\alpha\left(\overline{\lambda}_y - 0.2\right)+\overline{\lambda}_y^{\,2}\right]$$

$$\phi_y = 0.5\left[1+0.34\times\left(0.23-0.2\right)+0.23^2\right] = 0.53$$

$$\chi_y = \frac{1}{\left[0.53+\sqrt{0.53^2-0.23^2}\right]} = 0.99$$

$N_{b,y,Rd}$= χ_y A f_y/γ_{M1}=0.99x30600x265/1.0 =8028x10³N=8028 kN> N_{Ed} =3440 kN

OK

c, α=0.49

$$\phi_z = 0.5\left[1+\alpha\left(\overline{\lambda}_z - 0.2\right)+\overline{\lambda}_z^{\,2}\right]$$

$$\phi_z = 0.5\left[1+0.49\times\left(0.58-0.2\right)+0.58^2\right] = 0.76$$

$$\chi_z = \frac{1}{\left[0.76+\sqrt{0.76^2-0.58^2}\right]} = 0.80$$

$N_{b,z,Rd}$= χ_z A f_y/γ_{M1}=0.80x30600x265/1.0 =6487x10³N=6487 kN > N_{Ed} =3440 kN

OK

➢- Example 2

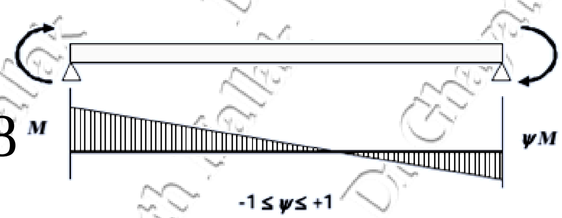
Member buckling resistance in bending (clause 6.3.2)

The 4.2 m column is unsupported along its length with no torsional or lateral restraints. Equal and opposite design end moments of 420 kN.m are applied about the major axis. The full length of the column will therefore be checked for lateral torsional buckling.

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} U V D \bar{\lambda}_z \sqrt{\beta_w}$$

Ψ =-420/420=-1.0 $\Rightarrow \frac{1}{\sqrt{C_1}} = 0.60$, U=0.854 , i_z=81.5mm , β_w =1

, D =1.0 (Normal)

End Moment Loading	ψ	$\frac{1}{\sqrt{C_1}}$	C_1
	+1.00	1.00	1.00
	+0.75	0.92	1.17
	+0.50	0.86	1.36
	+0.25	0.80	1.56
	0.00	0.75	1.77
	-0.25	0.71	2.00
	-0.50	0.67	2.24
	-0.75	0.63	2.49
	-1.00	0.60	2.76

$$\lambda_z = kL/i_z = 1 \times 4200/81.5 = 51.5$$

$$\bar{\lambda}_z = \lambda_z/\lambda_1 = 51.5/(93.9 \times 0.94) = 0.58$$

$$V = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{\lambda_z}{h/t_f}\right)^2}} = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{51.5}{352.5/37.7}\right)^2}} = 0.79$$

➢- Example 2

Member buckling resistance in bending (clause 6.3.2)

$$\bar{\lambda}_{LT} = 0.60 \times 0.854 \times 0.79 \times 1.0 \times 0.58 \times 1.0 = 0.24$$

$$\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,O} = 0.4$$

Hence, lateral torsional buckling effects may be ignored (clause 6.3.2.2(4)).

$$\chi_{LT} = 1.0$$

Lateral torsional buckling resistance

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} = 1.0 \times 4247 \times 10^3 \times \frac{265}{1.0} = 1125 \times 10^6 \, N.mm$$

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{420}{1125} = 0.37 \leq 1.0 \therefore \; OK.$$

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + K_{yy}\frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} + K_{yz}\frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1 \quad \Lambda\ (6-61)$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + K_{zy}\frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} + K_{zz}\frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \le 1 \quad \Lambda\ (6-62)$$

χ_y =0.99, χ_z =0.80 , N_{Rk}= A f_y=30600x265=8109x10^3 N=8109kN, γ_{M1}=1.0

$M_{y,Rk}$ =$W_{pl,y}$ f_y= 4247x10^3x265=1125.46kN.m

$M_{z,Rk}$ =$W_{pl,z}$ f_y= 1951 x10^3x265=517.02kN.m

1- Conservative method k_{yy} =k_{zy} =k_{zz} =k_{yz} =1.0:

$$\frac{3440}{(0.99\times 8109)/1.0} + 1.0\frac{420}{(1.0\times 1125)/1.0} + 1.0\frac{110}{517/1.0} = 1.01 > 1 \quad NOT \quad OK$$

$$\frac{3440}{(0.8\times 8109)/1.0} + 1.0\frac{420}{(1.0\times 1125)/1.0} + 1.0\frac{110}{517/1.0} = 1.12 > 1 \quad NOT \quad OK$$

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

2- Safe (maximum) method:

Considering y–y bending and in-plane supports:

Table B.3 ψ=420/-420=-1.0 →
C_{my}=0.6+0.4(-1.0) =0.2≥0.4 →
C_{my}=0.4

Safe (maximum) values for interaction factors

Interaction factor	Class 1 and 2
k_{yy}	$1.8C_{my}$
k_{yz}	$0.6k_{zz}$
k_{zy}	1.0
k_{zz}	$2.4C_{mz}$

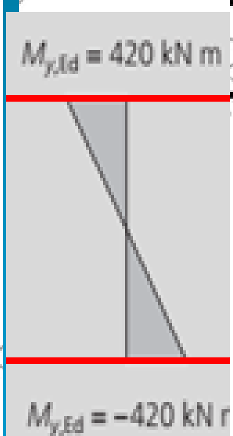


Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

Moment diagram	range	C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated lo:
	$-1 \le \psi \le 1$	$0,6 + 0,4\psi \ge 0,4$	

moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

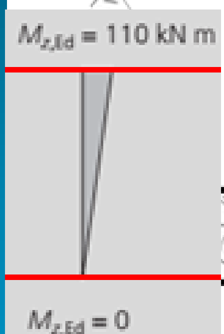
➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

2- Safe (maximum) method:

Considering z–z bending and in-plane supports:

Table B.3 $\psi = 0/110 = 0 \rightarrow$
$C_{mz} = 0.6 + 0.4(0) = 0.6 \geq 0.4 \rightarrow$
$C_{mz} = 0.6$



Safe (maximum) values for interaction factors

Interaction factor	Class 1 and 2
k_{yy}	$1.8C_{my}$
k_{yz}	$0.6k_{zz}$
k_{zy}	1.0
k_{zz}	$2.4C_{mz}$

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

Moment diagram	range	C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated load
	$-1 \leq \psi \leq 1$	$0{,}6 + 0{,}4\psi \geq 0{,}4$	

moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

2- Safe (maximum) method:

k_{yy} =1.8 C_{my} =1.8x0.4=0.72, k_{zy} =1.0 , k_{zz} =2.4 C_{mz}=2.4x0.6=1.44, k_{yz} =0.6 k_{zz} =0.6x1.44=0.86

$$\frac{3440}{(0.99\times 8109)/1.0}+0.72\frac{420}{(1.0\times 1125)/1.0}+0.86\frac{110}{517/1.0}=0.88<1 \quad OK$$

$$\frac{3440}{(0.8\times 8109)/1.0}+1.0\frac{420}{(1.0\times 1125)/1.0}+1.44\frac{110}{517/1.0}=1.21>1 \quad NOT \quad OK$$

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

3- Method 1 Annex A:

$$k_{yy} = 0.74 \qquad k_{yz} = 0.49 \qquad k_{zy} = 0.43 \qquad k_{zz} = 1.33$$

which gives, for *equation* (*6.61*),

$$0.43 + 0.28 + 0.10 = 0.81 \qquad (0.81 \leq 1.0 \;\therefore\; \text{acceptable})$$

and, for *equation* (*6.62*),

$$0.53 + 0.16 + 0.15 = 0.85 \qquad (0.85 \leq 1.0 \;\therefore\; \text{acceptable})$$

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

4- Method 2 Annex B:

k_{yy} =0.41, k_{zy} =0.79 , k_{zz} =0.78, k_{yz} =0.47

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{yz}\frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1 \qquad (6.61)$$

$$\Rightarrow \frac{3440}{(0.99 \times 8109)/1.0} + 0.41 \times \frac{420.0}{(1.0 \times 1125)/1.0} + 0.47 \times \frac{110.0}{517.0/1.0} = 0.43 + 0.15 + 0.10$$

$$= 0.68$$

$0.68 \leq 1.0 \qquad \therefore$ *equation (6.61)* is satisfied

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{zz}\frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1 \qquad (6.62)$$

$$\Rightarrow \frac{3440}{(0.80 \times 8109)/1.0} + 0.79 \times \frac{420.0}{(1.0 \times 1125)/1.0} + 0.78 \times \frac{110.0}{517.0/1.0} = 0.53 + 0.30 + 0.17$$

$$= 1.0$$

$1.0 \leq 1.0 \qquad \therefore$ *equation (6.62)* is satisfied

➢- Example 2

Member buckling resistance in combined bending and axial compression (clause 6.3.3)

Verification of the stability of the member using The Institution of Structural Engineers Manual

$$\frac{N_{Ed}}{\chi_{min}(Af_{yd})} + \frac{M_{y,Ed}}{\chi_{LT}(W_y f_{yd})} + C_{mz}\frac{M_{z,Ed}}{W_z f_{yd}} \leqslant 0.78 \text{ for class 1 and 2,}$$

Table B.3 $\psi=0/110=0 \rightarrow C_{mz}=0.6+0.4(0)=0.6\geq 0.4 \rightarrow C_{mz}=0.6$

$N_{b,z,Rd} = \chi_z\, A\, f_y/\gamma_{M1} = 0.80 \text{x} 30600 \text{x} 265/1.0 = 6487 \text{ kN}$

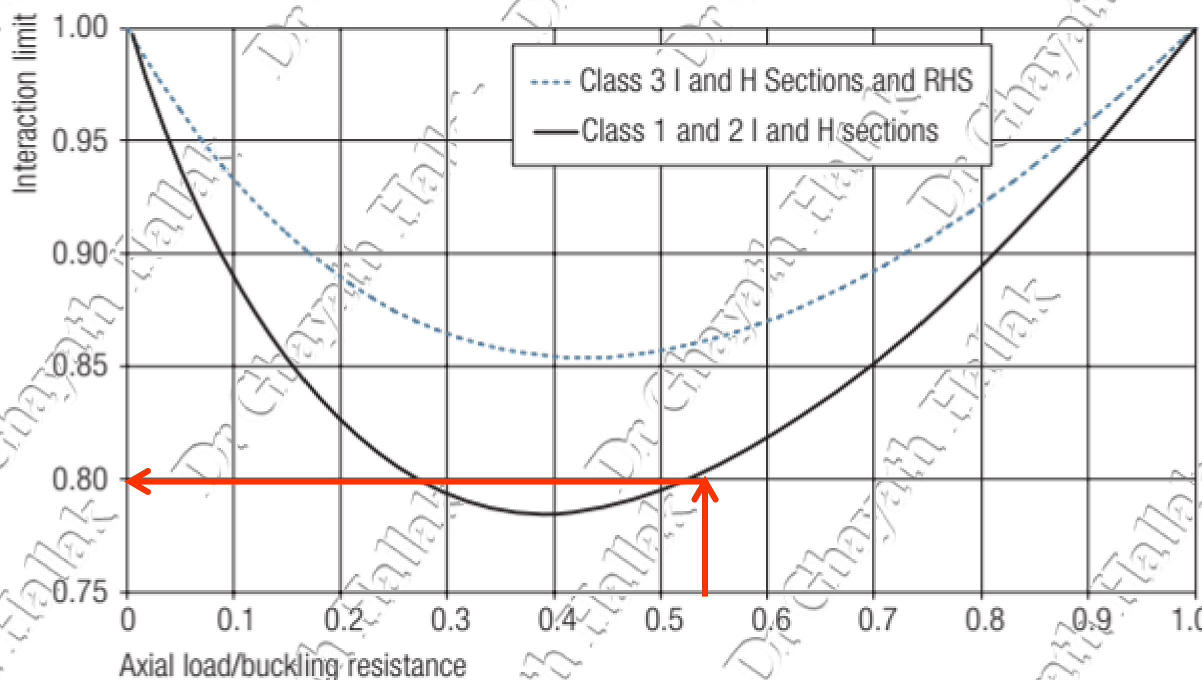
$$\frac{N_{Ed}}{N_{b,z,Rd}} = \frac{3440}{6487} = 0.53 \quad \textit{from the Graph} \Rightarrow \textit{Interaction Limit} = 0.80$$

$$\frac{N_{Ed}}{\chi_{min} N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{\chi_{Lt} M_{y,Rk}/\gamma_{M1}} + C_{mz}\frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 0.80$$

➢- Example 2

<u>Member buckling resistance in combined bending and axial compression (clause 6.3.3)</u>

Verification of the stability of the member using The Institution of Structural Engineers Manual



$$\frac{3440}{(0.8\times 8109)/1.0}+\frac{420}{(1.0\times 1125)/1.0}+0.6\frac{110}{517/1.0}=1.03>0.8 \quad NOT \quad OK$$