Simulation of Irreversibility line of thin superconducting disk in perpendicular magnetic fields

Maan Salim(1)

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ABSTRACT

Simulation of a theoretical model for the real and imaginary parts of part susceptibility of superconducting thin disk as a function of temperature at different perpendicular applied magnetic field were effectuated. Matlab codes were written to compute this model. During these calculations, the critical current density was assumed independent of the applied magnetic field (that is according to Bean critical state model). The susceptibility parts as a function of temperature were calculated at different disk thickness and at different applied perpendicular magnetic fields. It was found that the temperature corresponding to the peak of the imaginary part of the susceptibility (Irreversibility temperature) nonlinearly increases as the disk thickness increases. From the temperature dependent susceptibility at different applied magnetic field, the Irreversibility line (H_{irr}, T_{irr}) was obtained and found to follow the expression $H_{irr} = 3699(1 - T_{irr}/92)^2$. The results of using the ac susceptibility model showed agreement with experimental works by others.

Keywords: Simulating Irreversibility line, Simulating magnetic susceptibility, superconducting thin disk, perpendicular magnetic field to superconducting specimen, critical current densities, susceptibility geometry dependent.

(1)Assistant Prof., Department of Physics, Faculty of Sciences, Damascus University, Damascus, Syria

محاكاة نموذج نظري للخط اللاعكوس لغشاء ناقل فائق رقيق بشكل قرص في حقول مغناطيسية عمودية على سطحه

معن سليم(1)

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الملخص

تمت محاكاة نموذج نظري للقسمين الحقيقي والتخيلي للطواعية المغناطيسية المتناوبة للنواقل الفائقة على شكل قرص رقيق بدلالة درجة الحرارة وفي حقول مغناطيسية عمودية على سطحه. من أجل إجراء حسابات القسمين الحقيقي والتخيلي للطواعية المغناطيسية المتناوبة للنواقل الفائقة، وفق هذا المبدأ، كُتب عدد من الكودات بلغة الماتلاب. أجريت حسابات القسمين الحقيقي والتخيلي للطواعية المغناطيسية المتناوبة للنواقل الفائقة بفرض أن كثافة التيار مستقلة عن الحقل المغناطيسي المطبق (نموذج بين). بعدها أُجريت حسابات القسمين الحقيقي والتخيلي للطواعية المغناطيسية المتناوبة للنواقل الفائقة بدلالة درجة الحرارة من أجل ثخانات مختلفة للقرص، ومن أجل حقول مغناطيسية مختلفة مطبقة معامدة لسطح العينة.

وجد أن درجة الحرارة الموافقة لقمة القسم التخيلي للطواعية المغناطيسية (الحرارة اللاعكوسة) تزداد لاخطياً بزيادة ثخانة القرص. ومن تابعية الطواعية لدرجة الحرارة في حقول مغناطيسية مختلفة تم الحصول على الخط اللاعكوس، ووجد أنه يتبع العلاقة: $H_{irr} = 3699(1 - T_{irr}/92)^2$. أخيراً وجد أن النتائج التي تم الحصول عليها باستخدام هذا النموذج تتوافق مع النتائج التجريبية المنشورة.

الكلمات المفتاحية: محاكاة الخط اللاعكوس للنواقل الفائقة، محاكاة الطواعية المغناطيسية، غشاء رقيق على شكل قرص ناقل فائق، حقل مغناطيسي معامد لسطح الناقل الفائق، كثافة التيار الحرج، تابعية الطواعية على هندسة العينة.

(1) مدرس، قسم الفيزياء، كلية العلوم، جامعة دمشق، سورية.

Introduction

The ac susceptibility technique has been used widely for characterizing superconducting materials. This can be used to determine the critical temperature, penetration depth, irreversibility line, providing important information about the mechanism of vortex dynamics, flux penetration, flux motion, intergranular as well as intragranular contributions. This technique also is classified as a non-destructive method for measuring critical current density. The measured susceptibility at superconducting state is originated from the circulating persistent shielding currents and to the magnetic properties of the material. Therefore, the real part of the susceptibility is a measure of the screening ability however the imaginary part of the susceptibility is a measure of the power loss due to flux flow.

Irreversibility line

Müller, Tagashige and Bednorz have introduced the "Irreversibility line (IL)" in high temperature superconductors (HTS) that is on a polycrystalline sample of La2BaCuO4-y [1]. They showed that this line exists between $\mathrm{H_{c1}}$ and $\mathrm{H_{c2}}$ in the H-T magnetic phase diagram for HTS as shown in Figure (1). The mixed state region in the H-T plane of the HTS phase is divided into two parts by the IL. These parts are the irreversible region (Abrikosov Lattice phase) below IL and the reversible region above IL. The magnetic flux in the irreversible region is pinned and the reversible region corresponds to the melting of the Abrikosov lattice into a flux line liquid (Vortex liquid). This melting transition can take place more easily in the HTS materials due to the higher temperatures that are available for measurements [2]. Some authors refer to the IL as a depinning line or melting line due to its definition. The IL was also discovered in low temperature superconductors and was interpreted as a melting line [3-4]. The irreversible magnetic field H_{irr} in low temperature superconductor is very close to $\mathrm{H_{c2}}$ thus measurements need to be carried out carefully in order to distinguish H_{irr} and $\mathrm{H_{c2}}$ from each other [3-4].

In order to determine the IL, the AC susceptibility versus temperature is often used [5-6-7]. At the irreversibility temperature, $\mathrm{T_{irr}}$, the out-of-phase susceptibility, χ'', shows a maximum. The maximum of χ''

or (χ'' peak) will occur in the transition regime between almost reversible flux flow above T_{irr} and strong irreversible screening due to flux pinning below T_{irr}. In the range $T_{irr} < T$, flux lines are depinned by thermal activation, a process called thermally assisted flux flow (TAFF) to discriminate it from the flux creep with vortex pinning that occurs at $T_{irr} > T$. Müller, Tagashige and Bednorz [1] found that by increasing the applied magnetic field, the irreversible temperature shifts to the lower temperatures following the scaling law

$$B_{irr}(T) = a(1 - t)^{q} \qquad (1)$$

where *t* is the reduced temperature $t = T/T_c$ and *q*=3/2 for La-Ba-Cu-O, a is constant. Recent experiments found that the IL exists at low temperature T>4.2 K for single crystal of $Bi_2Sr_2CaCu2O8_{+\delta}$ that obeys the exponential function [8], $B_{irr}(T) = B_{iro}\exp(T/T_0)$, Where B_{iro} and T_0 are fitting parameters. Morello *et al* [9] observed the IL of Bi-2201 single crystal in the temperature range 0-4 K and they found out that the IL follows B_{irr}&$\exp(1/T_{irr})$ below *T*=0.5 K. Anderson and Kim [10] have explained the vortex behaviour above and below the irreversibility line in terms of various vortex phases such as flux creep model. Using the concept of the Anderson and Kim model, the giant flux creep model was introduced by Yeshurun and Malozemoff [11] to describe the form of the IL. Matsushita [12] has generalised the form of the IL given by Yeshurun and Malozemoff by taking into account the pinning strength. The collective pinning theory was introduced by Larkin and Ovchinnikov [13] followed by thermally assisted flux flow introduced by Dew-Hughes [14], then vortex glass phase introduced by Fisher *et al* [15] that is all to explain the nature of IL.

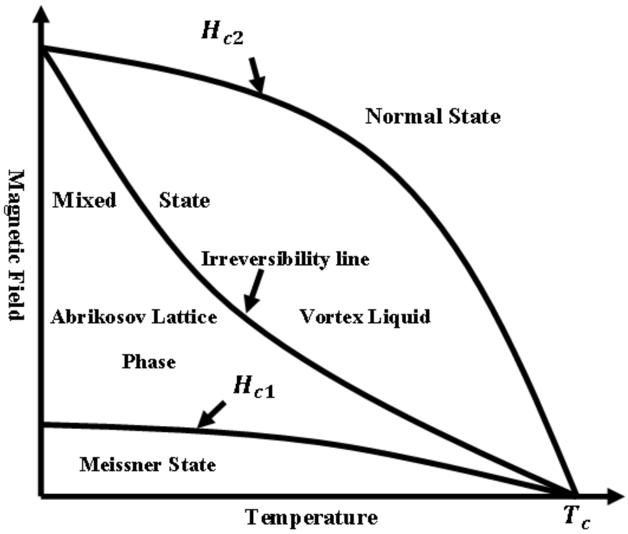

Figure (1): Typical magnetic phase diagram for High Temperature Superconductors.

Results and Discussions

The branches of the hysteresis magnetic moment $m_{\uparrow}$ and $m_{\downarrow}$in increasing and decreasing magnetic filed are given as [16],[17], [18], [19],[20]:

$$m_{\uparrow} = -m(H_0) + 2m\left(\frac{H_0 + H_a}{2}\right) \qquad (2)$$

$$m_{\downarrow} = m(H_0) - 2m\left(\frac{H_0 - H_a}{2}\right) \qquad (3)$$

Where H_a is the ac magnetic field and is given by $H_a = H_0 \exp(i\emptyset)$, which it can be written in terms of complex components as $H_a(t) = H_0 \cos(\emptyset)$ for the real part and $H_a(t) = H_0 \sin(\emptyset)$ for the imaginary part, H_0. is the field magnitude and $\emptyset$ is the phase. The magnetic moment of a circular disk of radius a and thickness d $(d \ll a)$ in transverse magnetic filed is given by [21]:

$$m_{disk}(h) = -J_c d a^3 \frac{2}{3}\left(cos^{-1}\frac{1}{\cosh(h)} + \frac{sinh|h|}{cosh^2(h)}\right) \qquad (4)$$

With $h = H_a/H_c$, where $H_c = J_c d/2$ is the critical magnetic field for the superconducting disk, and J_c is the critical current density of the disk. From equation (3) and for the type II superconducting samples, the maximum real part magnetic moment corresponds to ($\emptyset = 0$) hence $H_a = H_0 \cos(\emptyset) = H_0$, therefore the real part of equation (3) can be written as:

$$m'_{\downarrow} = m(H_0) \qquad (5)$$

On the other hand, since there is a 90 degree phase shift between the in- and out-of-phase, the phase for the imaginary part of the magnetic moment is $\emptyset = 90$ hence $H_a = H_0 \sin(\emptyset) = 0$, therefore using equation (3) the imaginary part of the magnetic moment can be written as:

$$m''_{\downarrow} = m(H_0) - 2m\left(\frac{H_0}{2}\right) \qquad (6)$$

Substituting equation (4) into equations (5) and (6) and using the expression $\chi = \chi' + \chi'' = m/(H_0 V)$; one can calculate the real and the imaginary parts susceptibility respectively as follows $\chi' = m'/VH_0$ and $\chi'' = m''/VH_0$. Where $V = \pi a^2 d$ is the disk volume. Hence the real and imaginary part susceptibility is given re spectively by:

$$\chi' = -\frac{\chi_0}{2h}\left[\left(cos^{-1}\frac{1}{cosh(h)} + \frac{sinh|h|}{cosh^2(h)}\right)\right] \qquad (7)$$

$$\chi'' = \frac{\chi_0}{2h}\left[-\left(cos^{-1}\frac{1}{\cosh(h)} + \frac{sinh|h|}{cosh^2(h)}\right) + 2\left(cos^{-1}\frac{1}{\cosh\left(\frac{h}{2}\right)} + \frac{sinh\left|\frac{h}{2}\right|}{cosh^2\left(\frac{h}{2}\right)}\right)\right] \qquad (8)$$

Where $h = H_0/H_c$, and $H_c = J_c d/2$, and $\chi_0 = 8a/(3\pi d)$. Equations (7) and (8) give the theoretical model for the real and imaginary parts of the susceptibility of superconducting thin disk as a function of perpendicular applied magnetic field. Taking into account the temperature dependent critical current density one can derive equations (7) and (8) to be temperature dependent. The temperature dependent critical current density $J_c(T)$ is often given by the following expression:

$$J_c(T) = J_0(1 - T/T_c)^n \qquad (9)$$

Where J_0 is the critical current density at zero temperature, T_c is the critical temperature, and n is order number. Substituting equation (9) into $H_c = J_c d/2$ and H_c into $h = H_0/H_c$ hence substituting h into equations (7) and (8) gives temperature dependent the real and the imaginary parts of the susceptibility of superconducting thin disk in perpendicular magnetic field. Two matlab codes were written in order to calculate the susceptibility parts as a function of temperature. First code used to calculate the susceptibility as a function of temperature at different disk thickness and the second code used to do the calculation at different applied magnetic field. The value of J_0 was given the value of $J_0 = 2 \times 10^{10}\ A/m^2$ and the value of n was given the value of $n = 2$. These values were used in running both codes.

Using equations (7), (8), and (9), figure (2) shows the calculated real and imaginary part of the susceptibility as a function of temperature and at different disc thickness. The values of d from left to the right are 100nm- 200nm- 300nm- 400nm- 500nm- 600nm- 700nm- 800nm- 900nm- 1000nm. The critical temperature was taken $T_c = 92\ K$, and the applied magnetic field was $H_0 = 10\ A/m$. From this figure, the imaginary part of the susceptibility peaks shift to higher temperature as the film thickness increases. As mentioned above the temperature corresponding to the peak in the imaginary part of the susceptibility represents the depinning temperature or irreversible temperature T_{irr}. Where below this temperature the flux creeps (vortex pinned) and above this temperature the flux flow (vortex unpinned). Hence plotting the values of disk thickness as a function of the depinning temperature T_{irr}. (figure (3) and figure (4)) gives nonlinear relationship between d and T_{irr}. The difference between figure (3) and figure (4) is that figure (3) presents values of disk thickness up to 500 nm but figure (4) display values of d up to 70000 nm. Note figure (2) shows only few results, and the rest of results were used to take the temperature corresponding to the position of the imaginary part peaks T_{irr} as a function of disk thickness (see figure (3) and figure (4)). From theses figures it can be concluded that the depinning temperature increases as the disk thickness increases or in the other words the vortices pinning decreases as the disk thickness increases.

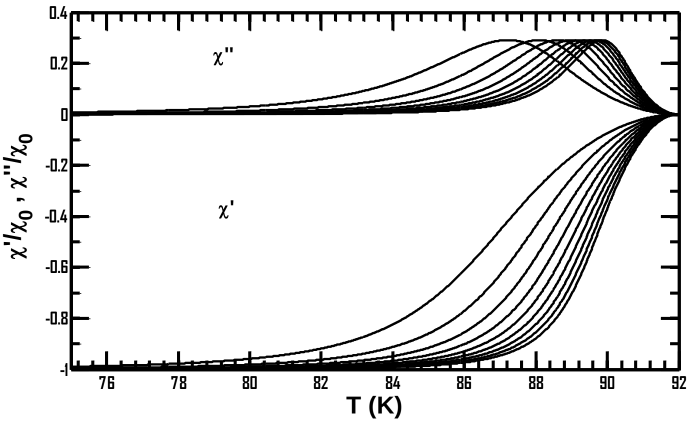

Figure (2): The real and the imaginary part of the susceptibility as a function of temperature and at different film thickness (Bean Model). The values of d from left to the right are 200nm - 300nm - 400nm - 500nm - 600nm - 700nm - 800nm- 900nm- 1000nm, $J_0 = 2 \times 10^{10} A/m^2$,$H_0 = 10\ A/m, T_c = 92\ K$

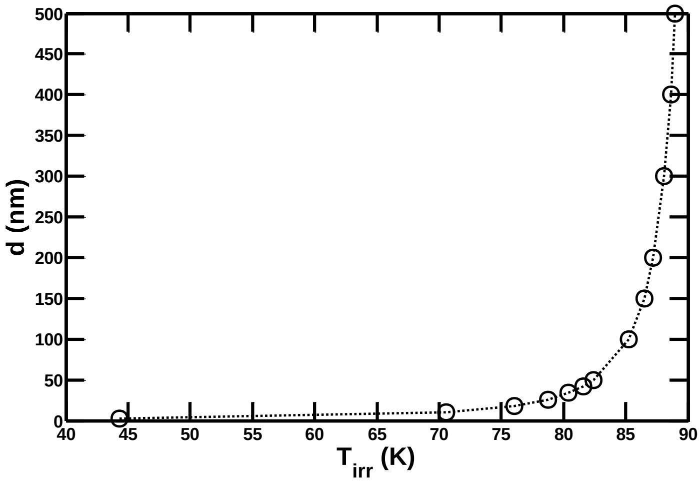

Figure (3):The disk thickness as a function of temperature corresponding to the imaginary part of the susceptibility peaks, the disk thickness was in the range from 10 nm to 500 nm, $J_0 = 2 \times 10^{10} A/m^2$, $H_0 = 10\ A/m, T_c = 92\ K$

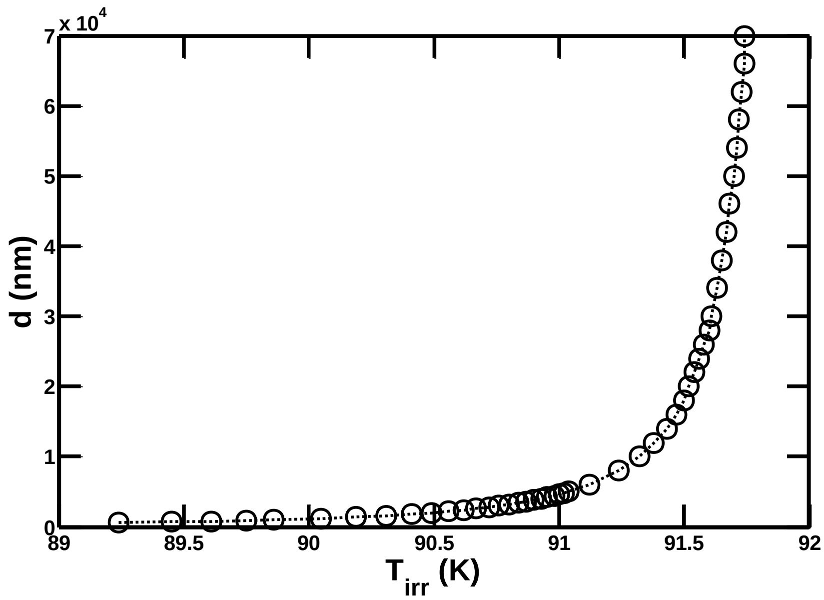

Figure (4): The disk thickness as a function of temperature corresponding to the imaginary part of the susceptibility peaks, the disk thickness was in the range from 500 nm to 70000 nm, $\mathrm{J}_0 = 2 \times 10^{10} \mathrm{A/m^2}$, $H_0 = 10\ A/m$, $T_c = 92\ K$

Using equations (7), (8), and (9), figures (5), (6), and (8) show the calculated real and imaginary parts of the susceptibility as a function of temperature and at different applied magnetic fields H_0. The range of the applied magnetic field was from 0.04 A/m to 1000 A/m, the critical current density at zero temperature was given the value of $\mathrm{J}_0 = 2 \times 10^{10} \mathrm{A/m^2}$, the disk thickness was $d = 200nm$, and the critical temperature was $T_c = 92\ K$. Figure (7) magnify the imaginary peaks and was taken from figure (6). From these figures it is obvious that the peak in the imaginary part of the susceptibility shift to lower temperature as the applied magnetic field increases. This is consistent with published work by others [5].

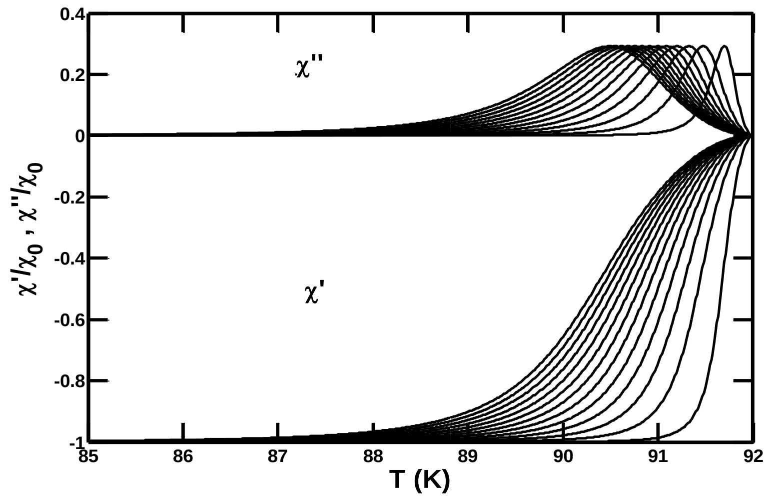

Figure (5): The real and the imaginary part of the susceptibility as a function of temperature and at different applied magnetic fields H_0. The values of H_0. from right to the left are (0.04, 0.12, 0.2, 0.28, 0.36, 0.44, 0.52, 0.6, 0.68, 0.76, 0.84, 0.92, 1) A/m, $J_0 = 2 \times 10^{10} A/m^2$, $d = 200nm, T_c = 92\ K$

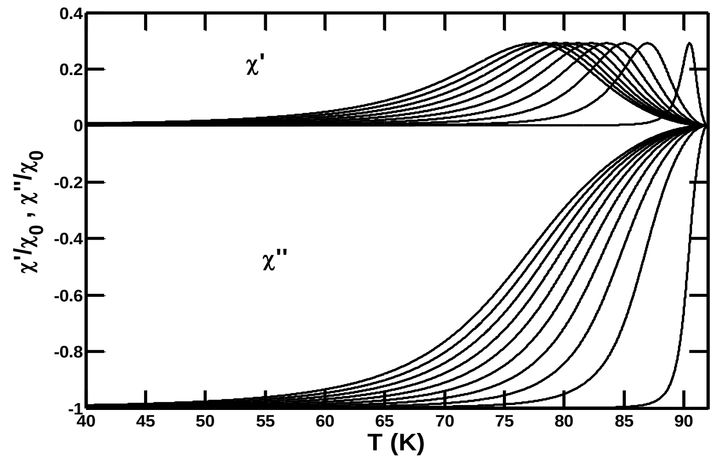

Figure (6): The real and the imaginary part of the susceptibility as a function of temperature and at different applied magnetic fields H_0. The values of H_0. from right to the left are (1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100) A/m, $J_0 = 2 \times 10^{10} A/m^2$, $d = 200nm, T_c = 92\ K$

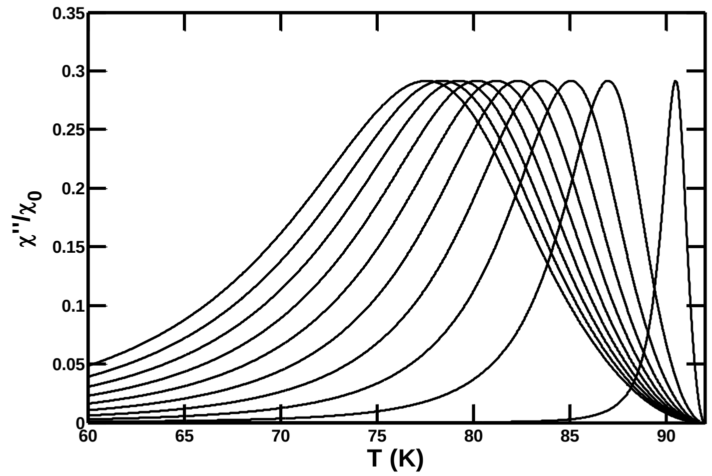

Figure (7): The imaginary part of the susceptibility as a function of temperature and at different applied magnetic fields H_0. The values of H_0. from right to the left are (1, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100) A/m, $\mathrm{J_0} = 2 \times 10^{10}\mathrm{A/m^2}$, $d = 200nm, T_c = 92\ K$

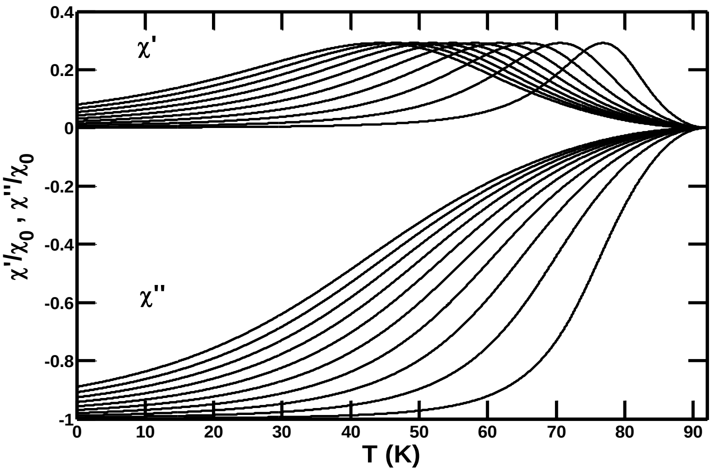

Figure (8): The real and the imaginary part of the susceptibility as a function of temperature and at different applied magnetic fields H_0. The values of H_0 from right to the left are (100, 200, 300, 400, 500, 600, 700, 800, 900, 1000) A/m, $\mathrm{J_0} = 2 \times 10^{10}\mathrm{A/m^2}, d = 200nm, T_c = 92\ K$

Figure (9) shows the irreversibility line of superconducting thin disk in a perpendicular applied magnetic field, where T_{irr} is the depinning temperature that corresponded to the peak in the imaginary part of the susceptibility at the applied magnetic field H_{irr}. The data of this line was taken from figures (5), (6), (7), and (8). Fitting this curve with equation(1) ($B_{irr}(T) = a(1-t)^q$) gives $H_{irr} = 3699(1 - T_{irr}/92)^2$. From figure (9) it can be concluded that the depinning temperature decreases as the applied magnetic field increases. This is consistent with published work by others [5], [7].

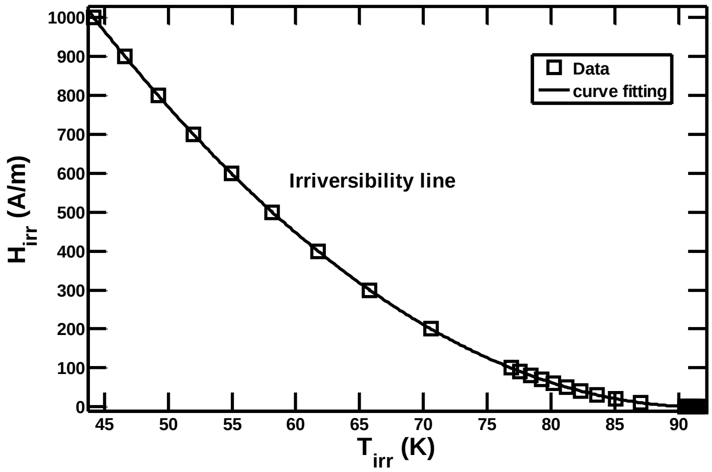

Figure (9): The magnetic field of full penetration as a function of temperature and at different applied magnetic fields H_0. The values of H_0 was in the range from 0.04 A/m to 1000 A/m. $J_0 = 2 \times 10^{10}$ A/m^2, $d = 200nm, T_c = 92\ K$

Conclusion

Simulations of a theoretical model for the real and imaginary parts of the susceptibility of thin superconducting thin disk as a function of temperature at different perpendicular applied magnetic field were effectuated. Matlab codes were written to compute this model. The irreversibility temperature (d, T_{irr}) was found nonlinearly increases as the disk thickness increases. The Irreversibility line (H_{irr}, T_{irr}) was obtained and found to follow the expression $H_{irr} = 3699(1 - T_{irr}/92)^2$. The results of using the ac susceptibility model showed agreement with experimental works by others.

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