On the inverse limit of finite dimensional lie algebras

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Received 14/08/2005
Accepted 07/03/2006

ABSTRACT

We extend the well Known Levi-Malcev decomposition theorem of finite dimensional Lie algebras to the case of pro-finite dimensional Lie algebras L = $\varprojlim L_n$ (n $\in$ N). We also prove that every finite dimensional homomorphic image of the Cartesian product of finite dimensional nilpotent Lie algebras is also nilpotent.

Key Words: Finite Lie algebras, Inverse limit, Lie groups.

حول النهاية المعكوسة لجبور لـي منتهية البعد

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تاريـخ الإيداع 2005/08/14

قبل للنشـر في 2006/03/07

الملخص

عممت في هذا البحث مبرهنة Levi – Malcev عن جبور لي منتهية البعد، على النهاية المعكوسة لجبور لي منتهية البعد .

كما أثبت أن كل صورة تشاكلية homomorphic image منتهية البعد للجداء الديكارتي لمجموعة من جبور لي العدومة nilpotent والمنتهية البعد هي أيضاً عدومة .

الكلمات المفتاحية: جبور لي المنتهية، النهاية المعكوسة، زمرة لي.

1. INTRODUCTION

Most of the general theory on Lie algebras has been established for finite dimensional Lie algebras. However, little is known about the general theory of infinite dimensional Lie algebras.

Important classes of such Lie algebras are the pro-finite dimensional Lie algebras L = $\varprojlim L_i$ which are inverse limits of finite dimensional Lie algebras. Such Lie algebras appear as the Lie algebras of pro-affine algebraic groups which play an important role in the representation theory of Lie groups ([4], [5]). So it is of interest to extend the basic theory (found for example in ([1], [2]) concerning the finite dimensional Lie algebras to the category of pro-finite dimensional Lie algebras.

In this paper, we generalize the Levi-Malcev decomposition theorem of finite dimensional Lie algebras to the inverse limit of finite dimensional Lie algebras L = $\varprojlim L_n$ (n $\in$ **N**). We prove that if L = $\varprojlim L_n$ (n $\in$ **N**), where the L_n are finite dimensional Lie algebras, then L = R$\oplus$S, where R is a pro-solvable ideal of L and S is a prosemisimple Lie sub-algebra.

We also prove that every finite dimensional homomorphic image of the Cartesian product of finite dimensional nilpotent Lie algebras $L = \prod_{i \in N} N_i$ is also nilpotent.

All Lie algebras in this paper are considered over a fixed algebraically closed field **K** of characteristic 0. [(1)]

2. BASIC DEFINITIONS

Definition 1. Let I be a set with a partial ordering $\leq$. Suppose I is directed upwards, i.e., for every i , j $\in$ I there exists k $\in$ I such that i $\leq$ k and j $\leq$ k.

Let S = {S_i : i $\in$ I} be a family of sets such that for every pair (i, j) $\in$ I $\times$ I with j $\geq$ i, there is a map $\pi_{ji} : S_j \to S_i$ satisfying

[(1)] 1991 Mathematics Subject Classification. 14L, 16W, 17B45. 1

the following two conditions:

(i) π_{ii} is the identity map for every $i \in I$

(ii) if $i \leq j \leq k$, then $\pi_{ki} = \pi_{ji} \circ \pi_{kj}$

Let $\pi = \{ \pi_{ji} : (\pi_{ji} : S_j \to S_i) \,;\, i, j \in I \,,\, j \geq i \}$.

Then (S, π) is called an inverse system and the maps $\pi_{ji} : S_j \to S_i$ are called the transition maps of the inverse system. The inverse limit of this system, denoted by $\varprojlim S_i$, is the subset of the Cartesian product $\prod_{i \in I} S_i$ consisting of all elements $s = (s_i)_{i \in I}$ such that $\pi_{ji}(s_j) = s_i$ for every $j \geq i$.

In other words, $\varprojlim S_i$ is the set of all elements $(s_i)_{i \in I}$ which are compatible with the transition maps $\pi_{ji} : S_j \to S_i$.

If (S, π) is an inverse system, let $\pi_i : \lim S_i \to S_i$ be the canonical projection sending $(s_i)_{i \in I}$ to s_i. Then $\pi_i = \pi_{ji} \circ \pi_j$ for every $i \leq j$.

A surjective inverse system is an inverse system (S, π) whose transition maps $\pi_{ji} : S_j \to S_i$ are surjective.

Definition 2. Let I be a directed poset (partially ordered set). A subset J of I is said to be cofinal in I if for each $i \in I$ there exists $j \in J$ such that $i \leq j$.

Define a sequence of ideals of a Lie algebra L, called the derived series,by $L^{(0)} = L$, $L^{(1)} = [L, L]$, $L^{(2)} = [L^{(1)}, L^{(1)}]$, …, $L^{(i)} = [L^{(i-1)}, L^{(i-1)}]$.

Definition 3. A Lie algebra L is called solvable if $L^{(n)} = 0$ for some n in **N**.

Let S be a maximal solvable ideal of a Lie algebra L. If I is any other solvable ideal of L, then $I + S = S$, or $I \subset S$. Thus L has a unique maximal solvable ideal which is called the radical of L and it is denoted by Rad L.

Definition4.A Lie algebra L is called semi-simple if its radical is 0.

Define a sequence of ideals of a Lie algebra L, called the descending central series, by

$$L^0 = L, L^1 = [L, L], L^2 = [L, L^1], \dots, L^i = [L, L^{i-1}].$$

Definition 5. A Lie algebra L is called nilpotent if $L^n = 0$ for some n in **N**.

Definition 6. A Lie algebra L is abelian if and only if [L, L] = 0.

Definition 7. A Lie algebra is called prosemisimple if it is the inverse limit of finite dimensional semi-simple Lie algebras.

Definition 8. A Lie algebra is called pro-solvable if it is the inverse limit of finite dimensional solvable Lie algebras.

3. PRELIMINARIES

Proposition 1. *If $X = \varprojlim X_i$, $i \in I$ is an inverse limit of sets over a directed poset I and J is a cofinal subset of I, then a compatible family $\{x_j\}_{j\in J} \in \varprojlim X_j$ can be completed to a compatible family $\{x_i\}_{i\in I} \in \varprojlim X_i$ in a unique way.*

Proof. Given a compatible family $\{x_j\}_{j\in J} \in \varprojlim X_j$, then because J is cofinal in I, for every i in I there exists j in J such that $j \geq i$, let $x_i = \pi_{ji}(x_j)$. This is well defined because, if j_1 and j_2 are both greater than i, and x_{j1}, x_{j2} are components of a compatible family $\{x_j\}_{j\in J}$, then there exists j_3 in J such that $j_3 \geq j_1$ and $j_3 \geq j_2$. So $\pi_{j3,j1}$, $\pi_{j3,j2}$ exist.

Moreover, $\pi_{j1,i}(x_{j1}) = \pi_{j2,i}(x_{j2}) = x_i$ because

$x_i = \pi_{j3,i}(x_{j3}) = \pi_{j1,i}(\pi_{j3,j1}(x_{j3})) = \pi_{j1,i}(x_{j1})$ and
$x_i = \pi_{j3,i}(x_{j3}) = \pi_{j2,i}(\pi_{j3,j2}(x_{j3})) = \pi_{j2,i}(x_{j2})$.

Thus $\{x_i\}_{i\in I} \in \varprojlim X_i$.

This implies that $\varprojlim X_i$ (i ∈ I) is bijective to $\varprojlim X_j$ (j ∈ J).

Theorem 1. [1, p.65] *Let L be a finite dimensional Lie algebra. Then:*

(i) $L = R \oplus S$, where R is the radical of L and S is a Levi subalgebra of L.

(ii) For a subalgebra S of L to be a Levi subalgebra, it is necessary and sufficient that S is a maximal semisimple subalgebra of L.

(iii) Every ideal A of L can be written as $A = (A \cap R) \oplus (A \cap S)$.

Theorem 2. [3, p.12] *Let L be a finite dimensional Lie algebra. Then*

(i) If L is nilpotent, then so are all subalgebras and homomorphic images of L.

(ii) L is nilpotent if and only if all elements of L are ad-nilpotent.

(iii) If A is a nilpotent ideal of L such that L/A is nilpotent, then L itself is nilpotent.

Proposition 2. *Let I be a solvable ideal of a finite dimensional Lie algebra L such that L/I is semisimple. Then I is the radical of L.*

Proof. Because I is solvable, I + Rad L is a solvable ideal of L. Thus (I + Rad L)/I is a solvable ideal of L/I, but L/I is semisimple, therefore (I + Rad L)/I is 0, i.e. I + Rad L = I and I = Rad L.

Proposition 3. *Let $f: L \to L'$ be a surjective homomorphism of finite dimensional Lie algebras. Then f (Rad L) = Rad L'.*

Proof. Let $f': L/\text{Rad } L \to L'/f(\text{Rad } L)$ be given by

$f'(l + \text{Rad } L) = f(l) + f(\text{Rad } L)$, then f' is a well defined surjective Lie algebra homomorphism. $(L/\text{Rad } L)/\text{Ker } f' \cong L'/f(\text{Rad } L)$, but $(L/\text{Rad } L)/\text{Ker } f'$ is semisimple because $L/\text{Rad } L$ is semisimple, thus $L'/f(\text{Rad } L)$ is semisimple, and by Proposition 2, $f(\text{Rad } L) = \text{Rad } L'$.

Proposition 4. *Let $f: L \to L'$ be a surjective homomorphism of finite dimensional Lie algebras, and let S be a maximal semisimple Lie subalgebra of L. Then $f(S)$ is a maximal semisimple Lie subalgebra of L'.*

Proof. $L = \text{Rad } L \oplus S$, $L' = f(L) = f(\text{Rad } L) + f(S) = \text{Rad } L' + f(S)$.

But $\text{Rad } L'$ is a solvable ideal of L' and $f(S)$ is a semisimple subalgebra of L', thus $\text{Rad } L' \cap f(S)$ is $\{0\}$.

Thus $L' = \text{Rad } L' \oplus f(S)$ and by Theorem 1 $f(S)$ is a maximal semisimple Lie subalgebra of L'.

Lemma 1. *Let $f: L \to L'$ be a surjective homomorphism of finite dimensional Lie algebras. If S' is a maximal semisimple Lie subalgebra of L', then there exists a maximal semisimple Lie subalgebra S of L such that $f(S) = S'$.*

Proof. According to Theorem 1, $f^{-1}(S')$ can be written as $f^{-1}(S') = \text{Rad } [f^{-1}(S')] \oplus \mathbf{S}$, where $\mathbf{S}$ is a maximal semisimple Lie subalgebra of $f^{-1}(S')$. Let M be a maximal semisimple Lie subalgebra of L containing S, i.e. $\mathbf{S} \subset M$, then $f(\mathbf{S}) \subset f(M)$, but $f(\mathbf{S}) = S'$, because $f(\text{Rad } [f^{-1}(S')]) = 0$. Thus $S' \subset f(M)$, but S' is a maximal semisimple Lie subalgebra of L', hence $f(M) = S'$, i.e. $M \subset f^{-1}(S')$. But $\mathbf{S}$ is a maximal semisimple Lie algebra of $f^{-1}(S')$ therefore $M = \mathbf{S}$.

4. levi-malcev decomposition of the inverse limit of finite dimensional lie algebras

Theorem 3. *Let* $L = \varprojlim L_n, (n \in \mathbf{N})$*, be the inverse limit of a surjective inverse system of finite dimensional Lie algebras over* **K**.

Then $L = R \oplus S$*, where* $R = \varprojlim R_n$ *and* $S = \varprojlim S_n$ *for a compatible family of Levi subalgebras* S_n *of* L_n.

Proof. Let $L_0 = R_0 \oplus S_0$ be a Levi decomposition of L_0. By Lemma 1, there exists a maximal semisimple Lie algebra S_1 of L_1 such that $\pi_{1,0}(S_1) = S_0$. Similarly, there exists a maximal semisimple Lie algebra S_2 of L_2 such that $\pi_{2,1}(S_2) = S_1$, and so on we construct a compatible family of S_n. Also, the R_n form a compatible subinverse system of L since the radical is unique.

Let $R = \varprojlim R_n$ and $S = \varprojlim S_n$. Then, $L = R \oplus S$: for an arbitrary element l in L, $l = (l_n)_{n \in N} = (r_n + s_n)$, where $r_n \in R_n$, $s_n \in S_n$. If $l_{n+1} = r_{n+1} + s_{n+1}$, $l_n = r_n + s_n$ and $l_{n-1} = r_{n-1} + s_{n-1}$, where s_{n-1}, s_n and s_{n+1} belong respectively to the elements S_{n-1}, S_n and S_{n+1} of the compatible family $\{S_n\}$. Then, because the l_n are compatible,

$\pi_{n+1,n-1}(l_{n+1}) = l_{n-1}$, thus $\pi_{n+1,n-1}(r_{n+1} + s_{n+1}) = r_{n-1} + s_{n-1}$, i.e.,

$\pi_{n+1,n-1}(r_{n+1}) - r_{n-1} = s_{n-1} - \pi_{n+1,n-1}(s_{n+1}) = 0$, since $R_{n-1} \cap S_{n-1} = \{0\}$.

Thus, $\pi_{n+1,n-1}(r_{n+1}) = r_{n-1}$ and $\pi_{n+1,n-1}(s_{n+1}) = s_{n-1}$.

Thus the r_n and the s_n are compatible.

Hence $l = (r_n) + (s_n) \in R + S$. Also $R \cap S = \{0\}$: Suppose there exists an element $a = (a_n)$ in $R \cap S$. Then, for every n in **N**, $a_n \in R_n \cap S_n$, but $R_n \cap S_n = \{0\}$, thus $a_n = 0$ for every n in **N**, and consequently $a = (a_n) = (0)$.

Hence $R \cap S = \{0\}$ and $L = R \oplus S$.

We will call S a prolevi subalgebra of L and R the proradical of L.

From the above theorem the following results follow directly:

Proposition 5. *Let* $L = R \oplus S$ *be the inverse limit of a surjective inverse system of finite dimensional Lie algebras over* **K***, where R is the proradical of L and S is a prolevi subalgebra of L. Then:*

(i)R is the largest prosolvable ideal of L.

(ii) S is a maximal prosemisimple subalgebra of L.

Corollary 1. *Let A be a closed ideal of a pro-finite dimensional Lie algebra* $L = R \oplus S$. *Then* $A = (A \cap R) \oplus (A \cap S)$.

Proof. Let S_n, R_n and A_n be the n[th] projection of S, R and A respectively. Then, from Theorem 1, we know that

$A_n = (A_n \cap R_n) \oplus (A_n \cap S_n.)$

Also because A is closed, $A = \varprojlim A_n$. Thus,

$A = \varprojlim [(A_n \cap R_n) \oplus (A_n \cap S_n)]$. For an arbitrary element a in A,

$a = (a_n)_{n \in N} = (r_n + s_n)$, where $r_n \in A_n \cap R_n$ and $s_n \in A_n \cap S_n$. If $a_{n+1} = r_{n+1} + s_{n+1}$, $a_n = r_n + s_n$ and $a_{n-1} = r_{n-1} + s_{n-1}$, where s_{n+1}, s_n and s_{n-1} belong respectively to the elements $A_{n-1} \cap S_{n-1}$, $A_n \cap S_n$ and $A_{n+1} \cap S_{n+1}$ of the compatible family $\{A_n \cap S_n\}$. Then, because the a_n are compatible,

$\pi_{n+1,n-1}(a_{n+1}) = a_{n-1}$, thus $\pi_{n+1,n-1}(r_{n+1} + s_{n+1}) = r_{n-1} + s_{n-1}$, i.e.,

$\pi_{n+1,n-1}(r_{n+1}) + \pi_{n+1,n-1}(s_{n+1}) = r_{n-1} + s_{n-1}$,

$\pi_{n+1,n-1}(r_{n+1}) - r_{n-1} = s_{n-1} - \pi_{n+1,n-1}(s_{n+1}) = 0$.

Thus, $\pi_{n+1,n-1}(r_{n+1}) = r_{n-1}$ and $\pi_{n+1,n-1}(s_{n+1}) = s_{n-1}$. Thus the r_n and the s_n are compatible. Hence $a = (r_n) + (s_n)$.

Thus, $A = \varprojlim [(A_n \cap R_n) \oplus (A_n \cap S_n)] = \varprojlim (A_n \cap R_n) \oplus \varprojlim (A_n \cap S_n)$.

But $(A_n \cap R_n) \subset A_n$ and $(A_n \cap R_n) \subset R_n$, for all n, thus

$\varprojlim (A_n \cap R_n) \subset \varprojlim A_n = A$ and $\varprojlim (A_n \cap R_n) \subset \varprojlim R_n = R$, hence $\varprojlim (A_n \cap R_n) \subset (A \cap R)$.

Similarly, $\varprojlim (A_n \cap S_n) \subset (A \cap S)$.

Thus, $A = \varprojlim (A_n \cap R_n) \oplus \varprojlim (A_n \cap S_n) \subset (A \cap R) \oplus (A \cap S)$,

and consequently $A = (A \cap R) \oplus (A \cap S)$.

Remark 1. Let $L = \varprojlim L_i$, $i \in I$, be the inverse limit of a surjective inverse system of finite dimensional Lie algebras over **K**, where I is a countable set with directed upwards partial ordering. Then, if I has a maximum element M, we get $L \cong L_M$. Otherwise, we can construct a cofinal subset J of I such that $J \cong$ **N**, i.e. the bijection between J and **N**

preserves the order. This is done as follows: Suppose I = (a_1, a_2,...). Let $\alpha_1 = a_1$. Choose α_2 to be Max(a_2, α_1), .., α_n to be Max(a_n, α_{n-1}).

Where:

$$Max\ (a_i, a_j) = \begin{cases} a_i & ; if\ \ a_i \geq a_j \\ a_j & ; if\ \ a_j \geq a_i \\ a_k\ \ ; if\ \ a_i\ \ and\ \ a_j\ \ are\ \ not\ \ comparable\ \ . \\ Where\ \ \ a_k \geq a_i\ \ and\ \ \ a_k \geq a_j \end{cases}$$

a_k exists since I is directed upwards.

Let J = {α_1, α_2, …, $\alpha_{n,..}$..}.

Then J is a cofinal subset of I, and J $\cong$ **N**.

Thus, L = $\varprojlim L_i$, (i $\in$ I) $\cong$ $\varprojlim L_n$, (n $\in$ **N**).

From the above Remark and using Proposition 1 and Theorem 3 we get the following result:

Theorem 4. *Let L = $\varprojlim L_i$, $(i \in I)$, be the inverse limit of a surjective inverse system of finite dimensional Lie algebras over* **K**, *where I is a countable set with directed upwards partial ordering, then L= R $\oplus$ S, where R=$\varprojlim R_i$, and S=$\varprojlim S_i$ for a compatible family of Levi subalgebras S_i of L_i.*

5. on the cartesian product of finite dimensional nilpotent lie algebras

Lemma 2. *Let* $L=\prod_{i\in N} N_i$ *,where the* N_i *are finite dimensional nilpotent Lie algebras. Suppose that* J *is an ideal of* L *such that* L/J *is finite dimensional and* J *contains every* N_i *, and let* $l = (b_i)_{i\in \mathbf{N}}$ *be an arbitrary element in* $L \setminus (\bigoplus_{i\in N} N_i)$ *. Then there exists a sequence of non-zero elements* $(\theta_i)_{i\in \mathbf{N}}$ *of* $\mathbf{K}$ *such that* $(\theta_i b_i)_{i\in \mathbf{N}} \in J$.

Proof. Let S be the subspace of L spanned by the vectors $(x_k)_{k\in \mathbf{N}}$ with $x_k = ((1+i)^k b_i)_{i\in \mathbf{N}}$. Then :

1. the family $(x_k)_{k\in \mathbf{N}}$ is linearly independent.

For if $\sum_{k\in T} \lambda_k x_k = 0$ and T is a finite subset of N then we have $(P(i)b_i)_{i\in \mathbf{N}} = 0$ with $P(j) = \sum_{k\in T} \lambda_k (j+1)^k$ and from the fact that $\{i : b_i \neq 0\}$ is infinite ($l \notin \bigoplus N_i$) we conclude that P has infinitely many zeros, that is P = 0 i.e. for every k, $\lambda_k = 0$.

2. $J \cap S \neq \{0\}$:

By 1 above, we have $\dim S = +\infty$ so the assumptions that $(\dim L/J) < +\infty$ implies that $J \cap S \neq \{0\}$.

3. Conclusion:

Let a be any element of $(J \cap S) \setminus \{0\}$. Then $a = \sum_{k\in V} \mu_k x_k$ and V is a finite subset of $\mathbf{N}$ and if we put $Q(x) = \sum_{k\in V} \mu_k (x+1)^k$, we see that $a = (Q(i)b_i)_{i\in \mathbf{N}}$.

Since Q is a non-zero polynomial it has a finite number of zeros;

Let $F = \{i \in \mathbf{N} : Q(i) = 0\}$ and let us define the sequence $(\theta_i)_{i\in \mathbf{N}}$ as follows: $\theta_i= Q(i)$ if $Q(i) \neq 0$ and $\theta_i= 1$ if $Q(i) = 0$.

In this case $[(\theta_i b_i)_{i\in \mathbf{N}} - a] \in \bigoplus_{i\in F} N_i \subset J$ and because $a \in J$ we conclude that $(\theta_i b_i)_{i\in \mathbf{N}} \in J$.

Lemma 3. *Let* $L = \prod_{i\in N} N_i$, *where the N_i are finite dimensional nilpotent Lie algebras. If J is an ideal of L such that L/J is finite dimensional and J contains every N_i, then L/J is abelian.*

Proof. We shall prove that $[L, L] \subset J$. For an arbitrary element $[l, l']$ in $[L, L]$ we have $[l, l'] = [(b_i),(b_i')]$, where

$(b_i) = (b_0, b_1, b_2, \ldots)$; $b_i \in N_i$; and $(b'_i) = (b'_0, b'_1, b'_2, \ldots)$; $b_i' \in N_i$. If only finitely many b_i are non-zero, then $(b_i) \in \oplus N_i \subset J$. But J is an ideal of L, thus $[l, l'] = [(b_i), (b_i')] \in J$.

Otherwise, by Lemma 2 there exists a sequence of non-zero elements $(\theta_i)_{i\in \mathbf{N}}$ of **K** such that $(\theta_i b_i)_{i\in \mathbf{N}} \in J$.

Thus $[(\theta_i b_i), (\frac{b_i'}{\theta_i})] \in J$. But $[(\theta_i b_i), (\frac{b_i'}{\theta_i})] = [(b_i), (b'_i)] = [l, l']$.

Thus $[l,l'] \in J$. Hence for an arbitrary element x in $[L, L]$, x which is a finite combination of $[l, l']$ belongs to J. This being true for every x in $[L, L]$.

Therefore $[L, L] \subset J$, and hence L/J is abelian.

Theorem 5. *Let* $L = \prod_{i\in N} N_i$ *,where the N_i are finite dimensional nilpotent Lie algebras. If A is an ideal of L such that L/A is finite dimensional, then L/A is nilpotent.*

Proof. Suppose first that A contains every N_i, then by the above Lemma, L/A is abelian and thus nilpotent. Otherwise, let $V = \oplus N_i$. Then A+V is an ideal of L that contains every N_i, and by Lemma 3, $[L, L] \subset A + V$.

Thus $L/(A+V)$ is abelian. But $(L/A)/[(A + V)/A] \approx L/(A + V)$ and $(A+V)/A$ is nilpotent since $(A + V)/A \approx V/(A \cap V)$ which is nilpotent since it is of finite dimension and every element of V is ad-nilpotent. Thus, by Theorem 2, L/A is nilpotent as desired.

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