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2010

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$$M_p = \begin{matrix} E_1 \\ E_2 \\ \vdots \\ E_S \\ E_{S+1} \\ \vdots \\ E_n \end{matrix} \begin{bmatrix} E_1 & E_2 & \cdots & E_S & E_{S+1} & \cdots & E_n \\ p_{1,1} & p_{1,2} & \cdots & p_{1,S} & p_{1,S+1} & \cdots & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,S} & p_{2,S+1} & \cdots & p_{2,n} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ p_{S,1} & p_{S,2} & \cdots & p_{S,S} & p_{S,S+1} & \cdots & p_{S,n} \\ p_{S+1,1} & p_{S+1,2} & \cdots & p_{S+1,S} & p_{S+1,S+1} & \cdots & p_{S+1,n} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ p_{n,1} & p_{n,2} & \cdots & p_{n,S} & p_{n,S+1} & \cdots & p_{n,n} \end{bmatrix} \quad (1)$$

$$I = \begin{matrix} E_1 \\ E_2 \\ \vdots \\ E_S \end{matrix} \begin{bmatrix} E_1 & E_2 & \cdots & E_S \\ P_{1,1} & P_{1,2} & \cdots & P_{1,S} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,S} \\ \vdots & \vdots & & \vdots \\ P_{S,1} & P_{S,2} & \cdots & P_{S,S} \end{bmatrix} \quad (2)$$

$$O = \begin{matrix} E_1 \\ E_2 \\ \vdots \\ E_S \end{matrix} \begin{bmatrix} E_{S+1} & E_{S+2} & \cdots & E_n \\ P_{1,S+1} & P_{1,S+2} & \cdots & P_{1,n} \\ P_{2,S+1} & P_{2,S+2} & \cdots & P_{2,n} \\ \vdots & \vdots & & \vdots \\ P_{S,S+1} & P_{S,S+2} & \cdots & P_{S,n} \end{bmatrix} \quad (3)$$

:

$$R = \begin{matrix} E_{S+1} \\ E_{S+2} \\ \vdots \\ E_n \end{matrix} \begin{bmatrix} E_1 & E_2 & \cdots & E_S \\ P_{S+1,1} & P_{S+1,2} & \cdots & P_{S+1,S} \\ P_{S+2,1} & P_{S+2,2} & \cdots & P_{S+2,n} \\ \vdots & \vdots & & \vdots \\ P_{n,1} & P_{n,2} & \cdots & P_{n,S} \end{bmatrix} \quad (4)$$

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: M_p

$$M_P = \begin{matrix} & E_1 & E_2 & E_3 & E_4 & E_I & E_{II} \\ E_1 & \left[\begin{array}{cccccc} 0 & P_{1,2} & 0 & 0 & P_{1,I} & 0 \end{array} \right] \\ E_2 & \left[\begin{array}{cccccc} 0 & 0 & P_{2,3} & 0 & P_{2,I} & 0 \end{array} \right] \\ E_3 & \left[\begin{array}{cccccc} 0 & 0 & 0 & P_{3,4} & P_{3,I} & 0 \end{array} \right] \\ E_4 & \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & P_{4,I} & P_{4,II} \end{array} \right] \end{matrix} \quad (5)$$

:

$$\begin{matrix} & E_1 & E_2 & E_3 & E_4 \\ E_1 & \left[\begin{array}{cccc} 0 & P_{1,2} & 0 & 0 \end{array} \right] \\ E_2 & \left[\begin{array}{cccc} 0 & 0 & P_{2,3} & 0 \end{array} \right] \\ E_3 & \left[\begin{array}{cccc} 0 & 0 & 0 & P_{3,4} \end{array} \right] \\ E_4 & \left[\begin{array}{cccc} 0 & 0 & 0 & E_{4,4} \end{array} \right] \end{matrix} \quad (6)$$

:

$$Q = \begin{matrix} & E_{S+1} & E_{S+2} & \cdots & E_n \\ E_{S+1} & \left[\begin{array}{cccc} P_{S+1,S+1} & P_{S+1,S+2} & \cdots & P_{S+1,n} \end{array} \right] \\ E_{S+2} & \left[\begin{array}{cccc} P_{S+2,S+1} & P_{S+2,S+2} & \cdots & P_{S+2,n} \end{array} \right] \\ \vdots & \left[\begin{array}{cccc} \vdots & \vdots & \cdots & \vdots \end{array} \right] \\ E_n & \left[\begin{array}{cccc} P_{n,S+1} & P_{n,S+2} & \cdots & P_{n,n} \end{array} \right] \end{matrix} \quad (7)$$

:

$$(I-Q) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} P_{S+1,S+1} & P_{S+1,S+2} & \cdots & P_{S+1,n} \\ P_{S+2,S+1} & P_{S+2,S+2} & \cdots & P_{S+2,n} \\ \vdots & \vdots & \cdots & \vdots \\ P_{n,S+1} & P_{n,S+2} & \cdots & P_{n,n} \end{bmatrix} \quad (8)$$

:

$$(I - Q) = \begin{bmatrix} 0 & -P_{1,2} & 0 & 1 \\ 0 & 1 & -P_{2,3} & 0 \\ 0 & 0 & 0 & -P_{3,4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

:

$$(I - Q)^{-1} = \begin{bmatrix} 1 & -P_{1,2} & 0 & 0 \\ 0 & 1 & -P_{2,3} & 0 \\ 0 & 0 & 1 & -P_{3,4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

:

: 2006 1996

.2006 -1996 : (1)

13924	4859	3265	2599	3201	1997-1996
13020	4294	2813	2646	3267	1998-1997
12838	3743	3304	2451	3340	1999-1998
12621	3781	2214	2922	3704	2000-1999
12014	3310	3064	2720	2920	2001-2000
12674	3638	2912	2997	3127	2002-2001
15667	5201	3536	3497	3433	2003-2002
15623	5121	3950	3066	3486	2004-2003
13923	4160	3514	3085	3164	2005-2004
15971	5427	4145	3111	3288	2006-2005
138275	43534	32717	29094	32930	

. 4353

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.2006-1996

:(2)

1203	611	339	253	1997-1996
1817	911	622	284	1998-1997
1375	609	522	244	1999-1998
1268	758	297	213	2000-1999
1023	537	237	249	2001-2000
1071	421	377	273	2002-2001
1298	631	346	321	2003-2002
1355	506	454	395	2004-2003
1570	718	512	340	2005-2004
2053	785	839	429	2006-2005
14033	6487	4545	3001	

1403

(2)

:

$$100 \times \frac{\text{متوسط عدد الخريجين في السنة الواحدة}}{\text{متوسط عدد الطلاب السنة الرابعة}} = \quad (11)$$

$$100 \times \frac{1403}{4353} =$$

$$= 32\%$$

(3)

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.2006-1996 : (3)

2234	553	689	505	487	1997-1996
1408	368	389	257	394	1998-1997
2872	1022	828	554	468	1999-1998
890	200	211	217	262	2000-1999
3658	1379	979	734	566	2001-2000
2836	1239	691	512	394	2002-2001
352	57	127	109	59	2003-2002
2960	1196	738	592	434	2004-2003
2257	1513	289	230	225	2005-2004
2618	1135	748	410	325	2006-2005
22085	8662	5689	4120	3614	
2208	866	569	412	361	

: (1)

$$\bar{N}_1 = \frac{32930}{10} = 3293 :$$

$$\bar{N}_2 = \frac{29094}{10} = 2909 :$$

$$\bar{N}_3 = \frac{32717}{10} = 3272 :$$

$$\bar{N}_4 = \frac{43537}{10} = 4353 :$$

: (3)

$$\bar{R}_1 = \frac{3614}{10} = 361 :$$

$$\bar{R}_2 = \frac{4120}{10} = 412 :$$

$$\bar{R}_3 = \frac{5689}{10} = 568.9 :$$

$$\bar{R}_4 = \frac{8662}{10} = 866.2 :$$

:

$$P_{1,1} = \frac{361}{3293} = 0.11 :$$

$$P_{2,1} = \frac{412}{2909} = 0.14 :$$

$$P_{3,1} = \frac{569}{3272} = 0.17 :$$

$$P_{4,1} = \frac{866}{4353} = 0.2 :$$

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$$P_{1,2} = 1 - P_{1,1} = 1 - 0.11 = 0.89$$

:

$$P_{1,3} = 1 - P_{2,1} = 1 - 0.14 = 0.86$$

:

$$P_{1,3} = 1 - P_{2,1} = 1 - 0.17 = 0.83$$

:

$$P_{4,II} = 1 - P_{4,1} = 1 - 0.2 = 0.8$$

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$$M_p = \begin{matrix} & E_1 & E_2 & E_3 & E_4 & E_I & E_{II} \\ \begin{matrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_I \\ E_{II} \end{matrix} & \begin{bmatrix} 0 & 0.89 & 0 & 0 & 0.11 & 0 \\ 0 & 0 & 0.86 & 0 & 0.14 & 0 \\ 0 & 0 & 0 & 0.83 & 0.17 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \quad (12)$$

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$$\begin{matrix} & E_1 & E_2 & E_3 & E_4 \\ \begin{matrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{matrix} & \begin{bmatrix} 0 & 0.89 & 0 & 0 \\ 0 & 0 & 0.86 & 0 \\ 0 & 0 & 0 & 0.83 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad (13)$$

:

$$Q^0 + Q^1 + Q^2 + \dots \quad (14)$$

$$Q \quad Q^{0=1} :$$

:

$$Q^0 + Q^1 + Q^2 + \dots + \frac{1}{(1-Q)} = (1-Q)^{-1} \quad (15)$$

:

$$(I-Q) = \begin{matrix} & & I & & Q \\ \begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.89 & 0 & 0 \\ 0 & 0 & 0.86 & 0 \\ 0 & 0 & 0 & 0.83 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad (16)$$

$$I - Q = \begin{bmatrix} 1 & -0.89 & 0 & 0 \\ 0 & 1 & -0.86 & 0 \\ 0 & 0 & 1 & -0.83 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

$$N = (I - Q)^{-1} = \begin{bmatrix} 1 & 0.89 & 0.77 & 0.64 \\ 0 & 1 & 0.86 & 0.71 \\ 0 & 0 & 1 & 0.83 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

$$I = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (19)$$

$$(I - Q)^{-1} \cdot I = \begin{bmatrix} 1 & 0.89 & 0.77 & 0.64 \\ 0 & 1 & 0.86 & 0.71 \\ 0 & 0 & 1 & 0.83 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.3 \\ 2.57 \\ 1.83 \\ 1 \end{bmatrix} \quad (20)$$

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P N

$$B = \begin{bmatrix} 1 & 0.89 & 0.77 & 0.64 \\ 0 & 1 & 0.86 & 0.71 \\ 0 & 0 & 1 & 0.83 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.11 & 0 \\ 0.14 & 0 \\ 0.17 & 0 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.49 & 0.51 \\ 0.43 & 0.57 \\ 0.34 & 0.66 \\ 0.2 & 0.8 \end{bmatrix} \quad (23)$$

%57

%51

%80

%66

2007 - 2006

2453

2865

3685

3897

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$$[2865 \ 2453 \ 3897 \ 3685] \quad (21)$$

:

B

$$[2865 \ 2453 \ 3897 \ 3685] \begin{bmatrix} 0.49 & 0.51 \\ 0.43 & 0.57 \\ 0.34 & 0.66 \\ 0.2 & 0.8 \end{bmatrix} \quad (22)$$

2948

1398 **2008**

2572 **2007**

1461 **2009**

. **2010**

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