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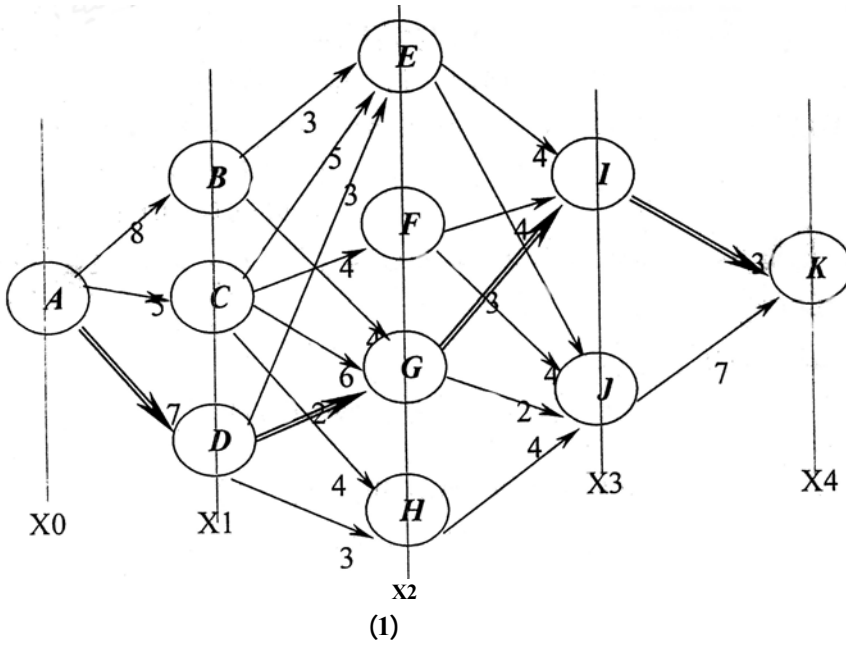
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1601 - 1665	Farmat	
	1944	Pierre Masse
.1952	Richard Bellman	

¹ J.L.LAURIERE:Elements de programmation dynamique Gauthier-villars Paris 1979 P 17 et 27.



$$\begin{array}{c}
 A \\
 V_{k+1}(x_k, x_{k+1}) \\
 x_{k+1} \in X_{k+1}
 \end{array}
 \begin{array}{c}
 V_k^*(x_k) \\
 X_k \quad x_k \\
 \cdot (x_k, x_{k+1}) \\
 \cdot^2
 \end{array}$$

$$V_{k+1}^*(x_{k+1}) = \text{opt}_{x_k \in X_k} [V_{k+1}(x_k, x_{k+1}) + V_k^*(x_k)]$$

² Robert Faure: *Precis de recherche operationnelle* Dunod decision Paris 1979 P 62-63

Min Max opt

:

$$V_1^*(B) = 8, V_1^*(C) = 5, V_1^*(D) = 7$$

:

$$x_2 \in [E, F, G, H]$$

: E

$$V_2^*(x_2)$$

$$V_2^*(E) = \text{opt}_{x_1 \in X_1} [V_2(x_1, E) + V_1^*(x_1)] =$$

$$= \text{opt}_{B,C,D} [V_2(B, E) + V_1^*(B); V_2(C, E) + V_1^*(C); V_2(D, E) + V_1^*(D)] =$$

$$= \text{opt} [3 + 8, 5 + 5, 3 + 7]$$

$$= \text{opt} [11; 10; 10] = 10$$

10

A,D,E A,C,E

: F

$$V_2^*(F) = \text{opt}_{x_1 \in X_1} [V_2(x_1, F) + V_1^*(x_1)] =$$

$$V_1^*(F) = \text{opt}_{B,C,D} [V_2(C, F) + V_1^*(C)] = 4 + 5 = 9$$

F

G

$$V_2^*(G) = \text{opt}_{x_1 \in X_1} [V_2(x_1, G) + V_1^*(x_1)] =$$

$$V_2^*(G) = \text{opt}_{B,C,D} [V_2(B, G) + V_1^*(B); V_2(C, G) + V_1^*(C); V_2(D, G) + V_1^*(D)] =$$

$$= \text{opt} [4 + 8; 6 + 5; 2 + 7] =$$

$$= \text{opt} [12; 11; 9] = 9$$

ADG

ADG

: J

$$V_3^*(I) = \text{opt}_{x_2 \in X_2} [V_3(x_2, I) + V_2^*(x_2)] =$$

$$\begin{aligned}
 &= \text{opt}_{E.F.G} [V_3(E, I) + V^*(E); V_3(F, I) + V_2^*(F); V_2(G, I) + V_2^*(G)] = \\
 &= \text{opt} [4 + 10; 4 + 9; 3 + 9] = \\
 &= \text{opt} [14; 13; 12] = 12
 \end{aligned}$$

: J

ADGI

$$\begin{aligned}
 &V_3^*(J) = \text{opt}_{x_2 \in X_2} [V_3(x_2, J + V_2^*(x_2))] = \\
 &= \text{opt}_{E.F.G.H} [V_3(E, J) + V_2^*(E); V_3(F, J) + V_2^*(F); V_3(G, J) + V_2^*(G); V_3(H, J) + V_2^*(H)] = \\
 &= \text{opt} [5 + 10; 4 + 9; 2 + 9; 4 + 9] = \\
 &= \text{opt} [15; 13; 11; 13] = 11
 \end{aligned}$$

.(ADGJ)

: k

x4

$$\begin{aligned}
 &= \text{opt}_{I.J} [V_4(I, K) + V_3^*(I); V_4(J, K) + V_3^*(J)] = \\
 &= \text{opt} [3 + 12; 7 + 11] = \\
 &= \text{opt} [15; 18] = 15
 \end{aligned}$$

.A.D.G.I.K

:

:

- 6

:

2001

144269

14.4269

2000 1963

:

" (1)

0	0	0	0	0	0
0.15	0.18	0.22	0.28	0.25	1
0.20	0.23	0.26	0.37	0.36	2
0.25	0.27	0.32	0.50	0.48	3
0.30	0.32	0.37	0.64	0.60	4
0.36	0.40	0.46	0.73	0.68	5
0.42	0.48	0.50	0.80	0.74	6
0.45	0.54	0.58	0.90	0.82	7
0.52	0.60	0.64	1.01	0.90	8
0.58	0.64	0.68	1.16	1.02	9
0.62	0.69	0.73	1.26	1.2	10

2002

:

14.4269

:

6.1.1.1.1
5.2.1.1.1
4.3.1.1.1
3.3.2.1.1
3.2.2.2.1
2.2.2.2.2

:

2.2.2.2.2

1.1.1.1.6

:

$$\frac{5!}{4!!} = 5$$

:

5.2.1.1.1
4.3.1.1.1
3.1.2.2.2

:

$$\frac{5!}{3!!} = 5.4 = 20$$

60

2.3.1.3.1

:

$$\frac{5!}{2!2!!} = \frac{5.4.3}{2} = 30$$

:

:

$$30 + 60 + 5 + 1 = 96$$

96

x_1, x_2, x_3, x_4, x_5

$f_i(x_i)$

:³

I

$$f_1(x_1), f_2(x_2), f_3(x_3), f_4(x_4), f_5(x_5),$$

$$\begin{aligned}
 &: \\
 x_1 + x_2 + x_3 + x_4 + x_5 &= 10 \\
 &: \tag{1}
 \end{aligned}$$

$$F(x_1, x_2, x_3, x_4, x_5) = f_1(x_1) + f_2(x_2) + f_3(x_3) + f_4(x_4) + f_5(x_5) \dots \tag{1}$$

$$x_1, x_2, x_3, x_4, x_5$$

$$[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

$$\begin{aligned}
 &: \\
 x_1 + x_2 &= u_1 \\
 u_1 + x_3 &= u_2 \\
 u_2 + x_4 &= u_3 \\
 u_3 + x_5 &= 10 = A \\
 &A
 \end{aligned}$$

144.269

2001

$$\begin{aligned}
 &: \\
 u_1 \leq A, u_2 \leq A, u_3 \leq A \\
 &: \tag{1}
 \end{aligned}$$

$$F(x_1, u_1, u_2, u_3, A) = f_1(x_1) + f_2(u_1 - x_1) + f_3(u_2 - u_1) + f_4(u_3 - u_2) + f_5(A - u_3) \dots \tag{2}$$

$$\begin{aligned}
 &: \\
 f_{1,2}(u_1) &= \text{Max}[f_{1,2}(x_1) + f_2(u_1 - x_1)] \dots \tag{3} \\
 x_1 &\in [0, 1, 2, 3, 4, \dots, 10]
 \end{aligned}$$

$$\begin{aligned}
 f_{1,2,3}(u_2) &= \text{Max}[f_{1,2}(u_1) + f_{1,2,3}(u_2 - u_1)] \dots \tag{4} \\
 u_1 &\in [0, 1, 2, 3, 4, \dots, 10]
 \end{aligned}$$

$$\begin{aligned}
 &: \\
 f_{1,2,3,4}(u_3) &= \text{Max}[f_{1,2,3}(u_2) + f_{1,2,3,4}(u_3 - u_2)] \dots \tag{5} \\
 u_2 &\in [0, 1, 2, 3, 4, \dots, 10]
 \end{aligned}$$

$$f_{1.2.3.4.5}(A) = \text{Max}[f_{1.2.3.4}(u_3) + f_{1.2.3.4.5}(A - u_3)] \dots (6)$$

$$u_3 \in [0.1.2.3.4 \dots 10]$$

: (3)

$$f_{1.2}(u_1 = 1) = \text{Max}[f_1(0) + f_2(1), f_1(1) + f_2(0)]$$

$$= \text{Max}[0 + 0.28, 0.25 + 0] = 0.28$$

0.1

$$f_{1.2}(u_1 = 2) = \text{Max}[f_1(0) + f_2(2), f_1(1) + f_2(1), f_1(2) + f_0(0)]$$

$$= \text{Max}[0 + 0.37, 0.36 + 0.37, 0.36 + 0]$$

$$= \text{Max}[0.37, 0.53, 0.36] = 0.53$$

(1.1)

$$f_{1.2}(u_1 = 3) = \text{Max}[f_1(0) + f_2(3), f_1(1) + f_2(2), f_1(2) + f_2(1), f_1(3) + f_2(0)]$$

$$= \text{Max}[0 + 0.5, 0.25 + 0.37, 0.36 + 0.28, 0.48 + 0]$$

$$= \text{Max}[0.5, 0.62, 0.64, 0.48] = 0.64$$

2.1

u₁=10

(2) :

	$f_1(x_1)$	$f_2(x_2)$	$f_{1,2}(u_1)$	
0	0	0	0	0.0
1	0.25	0.28	0.28	0.1
2	0.36	0.37	0.53	1.1
3	0.48	0.50	0.64	2.1
4	0.60	0.64	0.76	3.1
5	0.68	0.73	0.89	1.4
6	0.74	0.80	1	2.4
7	0.82	0.90	1.12	3.4
8	0.90	1.01	1.24	4.4
9	1.02	1.16	1.33	4.5
10	1.2	1.26	1.41	5.5

(5.5)

5

5

1.41

1.41 , 14.4269 = 20.341929

(4)

(3)

	$f_{1,2}(u_1)$	$f_3(x_3)$	$f_{1,2,3}(u_2)$	
0	0	0	0	0.0.0
1	0.28	0.22	0.28	0.1.0
2	0.53	0.26	0.53	1.1.0
3	0.64	0.32	0.75	1.1.1
4	0.76	0.37	0.89	2.1.1
5	0.89	0.46	0.98	3.1.1
6	1	0.50	1.11	1.4.1
7	0.12	0.58	1.22	2.4.1
8	1.24	0.64	1.34	3.4.1
9	1.33	0.68	1.46	4.4.1
10	1.41	0.73	1.55	4.5.1

10

(4,5,1)

5

:

(4)

	$f_{1,2,3}(u_2)$	$f_4(x_4)$	$f_{1,2,3,4}(u_3)$	
0	0	0	0	0.0.0.0
1	0.28	0.18	0.28	0.1.0.0
2	0.53	0.23	0.53	1.1.0.0
3	0.75	0.27	0.75	1.1.1.0
4	0.98	0.32	0.93	1.1.1.1
5	0.98	0.40	1.07	2.1.1.1
6	1.11	0.48	1.16	3.1.1.1
7	1.22	0.54	1.29	1.4.1.1
8	1.34	0.60	1.40	2.4.1.1
9	1.46	0.64	1.52	3.4.1.1
10	1.55	0.69	1.64	4.4.1.1

(4,4,1,1)

:

6

(5)

	$f_{1,2,3,4}(u_3)$	$f_5(x_5)$	$f_{1,2,3,4,5}(A)$	
0	0	0	0	0.0.0.0.0
1	0.28	0.15	0.28	0.1.0.0.0
2	0.53	0.20	0.53	1.1.0.0.0
3	0.75	0.25	0.75	1.1.1.0.0
4	0.93	0.30	0.93	1.1.1.1.0
5	1.07	0.36	1.08	1.1.1.1.1
6	1.16	0.42	1.22	2.1.1.1.1
7	1.29	0.48	1.31	3.1.1.1.1
8	1.40	0.52	1.44	1.4.1.1.1
9 (1)	1.52	0.85	1.67	2.4.1.1.1
10	1.64	0.62	1.67	3.4.1.1.1

3.4.1.1.1

:

144.269

:

14.4269

$$3 \times 14.4269 = 43.2807$$

14.4269

$$4 \times 14.4269 = 57.7076$$

14.4269

14.4269

.4

$$1.67 \times 14.4269 = 23.992.923$$

23992.923

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-1

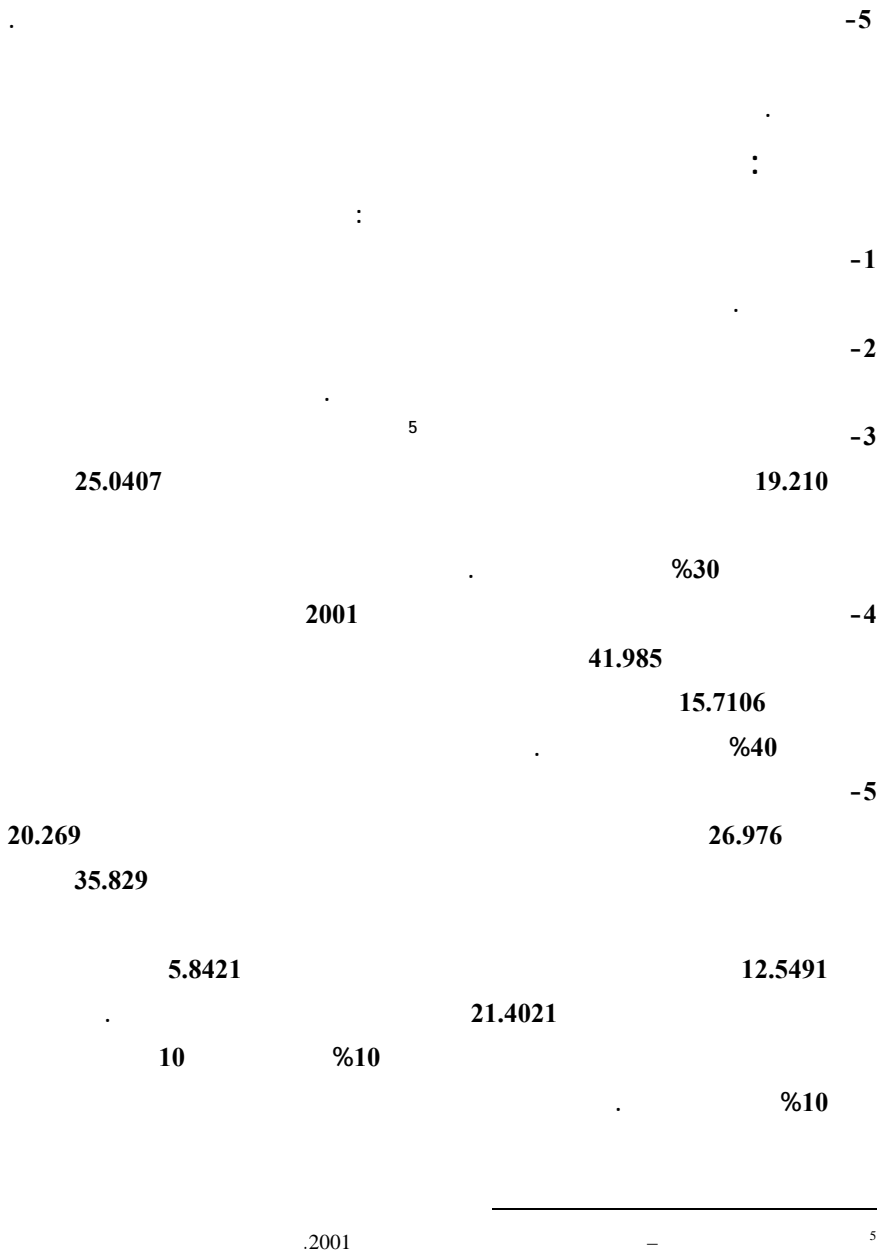
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3,4,1,1,1



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