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	Subject: Example 3 - Simply supported beam with lateral restraint at load application points		
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		Checked by: Dr. G. Hallak	Date:

The beam shown in Figure 4.1 is laterally restrained at the ends and at the points of load application only. For the loading shown, design the beam in S275 steel.

References are to BS EN 1993-1-1: 2005, including its National Annex, unless otherwise stated

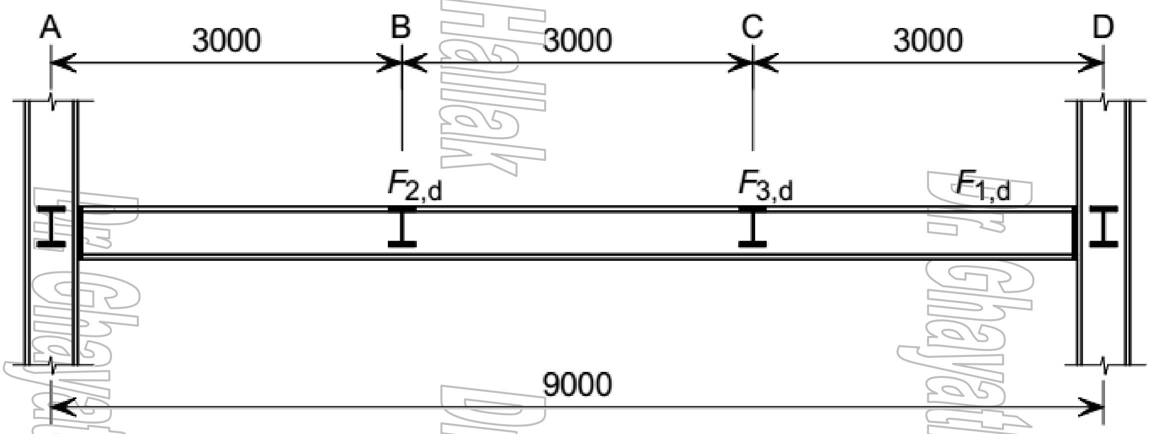

Figure 4.1

1.2 Actions (loading)

1.2.1 Permanent actions

Uniformly distributed load (including self weight) g = 3 kN/m

Concentrated load 1 $G1$ = 40 kN

Concentrated load 2 $G2$ = 20 kN

1.2.2 Variable actions

Concentrated load 1 $Q1$ = 60 kN

Concentrated load 2 $Q2$ = 30 kN

The variable actions are not due to storage and are not independent of each other.

1.2.3 Partial factors for actions

BS EN 1990 A1.3.1(4)

Table NA.A1.2(B)

Partial factor for permanent actions γ_G = 1.35

Partial factor for variable actions γ_Q = 1.50

Reduction factor ξ = 0.925

1.2.4 Design values of combined actions for Ultimate Limit State

Use Expression (6.10) or the less favourable combination from Expression (6.10a) and (6.10b). The UK National Annex to BS EN 1990 allows the designer to choose which of those options to use.

$$\gamma_{Gj,sup} G_{j,sup} + \gamma_{Gj,inf} G_{j,inf} + \gamma_{Q,1} \psi_{0,1} Q_1 + \gamma_{Q,i} \psi_{0,i} Q_i \qquad (6.10a)$$

$$\xi \gamma_{Gj,sup} G_{j,sup} + \gamma_{Gj,inf} G_{j,inf} + \gamma_{Q,1} Q_1 + \gamma_{Q,i} \psi_{0,i} Q_i \qquad (6.10b)$$

BS EN 1990 Table NA.A1.2(B)

Use 6.10b:

UDL (including self weight)

$F_{1,d} = \xi \, \gamma_G \, g = (0.925 \times 1.35 \times 3) = 3.7$ kN/m

Concentrated load 1

$F_{2,d} = \xi \, \gamma_G \, G_1 + \gamma_Q \, Q_1 = (0.925 \times 1.35 \times 40) + (1.5 \times 60) = 140.0$ kN

Concentrated load 2

$F_{3,d} = \xi \gamma_G \, G_2 + \gamma_Q \, Q_2 = (0.925 \times 1.35 \times 20) + (1.5 \times 30) = 70.0$ kN

1.3 Design bending moments and shear forces

The design bending moments and shear forces are shown in Figure 4.2

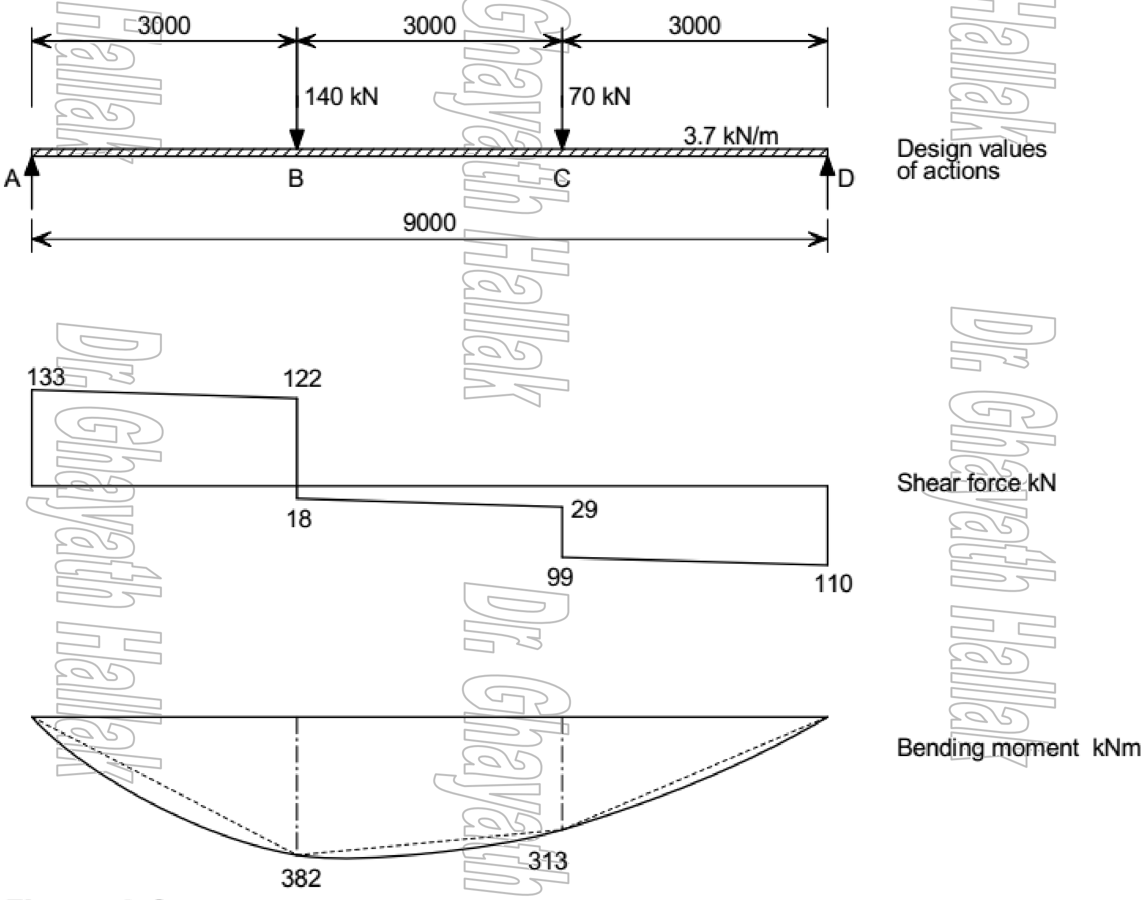

Figure 4.2

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1.4 Buckling length (L_{cr})

Since the beam is restrained at its ends and at the loading points, there are three segments to consider. From the bending moment diagram, it can be seen that the maximum bending moment occurs within segment B to C. Therefore only this segment is considered.

BS EN 1993-1-1 does not give guidance for determining buckling lengths. Therefore take the buckling length (L_{cr}) equal the span length between lateral restraints,

L_{cr} = 3000 mm

1.5 Section properties

Trial section can be calculated as follows:

Mb,R= Ω Wpl,y fy/γM0=MEd ⇒ Wpl,y= MEd/(Ω fy/γM0) ⇒

Wpl,y=382x106/(0.80x275)=1736cm3 where Ω =0.65 to 0.8

Chose from the UKB section tables a section has Wpl,y >1736cm3

Try section 457 × 191 × 82 UKB in S275 with W pl,y = 1800 cm3

From section property tables:

Depth	h = 460.0 mm
Width	b = 191.3 mm
Web thickness	t_w = 9.9 mm
Flange thickness	t_f = 16.0 mm
Root radius	r = 10.2 mm
Depth between flange fillets	d = 407.6 mm
Second moment of area, y -y axis	Iy = 37 100 cm4
Second moment of area, z -z axis	Iz = 1 870 cm4
Radius of gyration y-y axis	i_y=18.8 cm
Radius of gyration z-z axis	i_z = 4.23 cm
Plastic modulus, y -y axis	W pl,y = 1 830 cm3
Plastic modulus, z -z axis	W pl,z = 304 cm3
Area	A = 104 cm2
Buckling parameter	U=0.879
Torsional constant	I_T = 96.2 cm^4
Warping constant	I_W = 0.922x10^6 cm^6
Modulus of elasticity	E = 210 000 N/mm^2

Use Ω =0.65 if there are no lateral restrains between supports

And Ω =0.8 if lateral restrains between supports were provided

For S275 steel and $t \leq 16$ mm

Yield strength $f_y = R_{eH} = 275\ \text{N/mm}^2$

BS EN 10025-2 Table 7

1.6 Cross section classification

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

Table 5.2

Outstand of compression flange

$$c = \frac{b - t_w - 2r}{2} = \frac{191.3 - 9.9 - (2 \times 10.2)}{2} = 80.50\ \text{mm}$$

$$\frac{c}{t_f} = \frac{80.5}{16.0} = 5.03$$

The limiting value for Class 1 is $\frac{c}{t_f} \leq 9\varepsilon = 9 \times 0.92 = 8.28$

5.03 < 8.28

Therefore the flange is Class 1 under compression.

Web subject to bending

$c = d = 407.6$ mm

$$\frac{C}{t_w} = \frac{407.6}{9.9} = 41.17$$

The limiting value for Class 1 is $\frac{C}{t_w} \leq 72\varepsilon = 72 \times 0.92 = 66.24$

41.17 < 66.24

Therefore the web is Class 1 under bending.

Therefore the section is Class 1 under bending.

1.7 Partial factors for resistance

$\gamma_{M0} = 1.0$

$\gamma_{M1} = 1.0$

NA.2.15

1.8 Cross-sectional resistance

1.8.1 Shear buckling

The shear buckling resistance for webs should be verified according to Section 5 of BS EN 1993-1-5 if: 6.2.6(6)

$$\frac{h_w}{t_w} > \frac{72\varepsilon}{\eta}$$

$\eta = 1.0$ BS EN 1993-1-5 NA.2.4

$h_w = h - 2t_f = 460.0 - (2 \times 16.0) = 428.0$ mm

$$\frac{h_w}{t_w} = \frac{428.0}{9.9} = 43.23$$

$$\frac{72\varepsilon}{\eta} = \frac{72 \times 0.92}{1.0} = 66.2$$

43.23 < 66.2

Therefore the shear buckling resistance of the web does not need to be verified.

1.8.2 Shear resistance

Verify that: 6.2.6(1)

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1.0$$ Eq (6.17)

$$V_{c,Rd} = V_{pl,Rd} = \frac{A_V \times f_y}{\gamma_{M0} \times \sqrt{3}}$$

$A_V = A - 2t_f b + (t_w + 2r)t_f \geq \eta h_w t_w$ 6.2.6(3)

$A_V = 104 \times 10^2 - (2 \times 191.3 \times 16.0) + 16.0 \times [9.9 + (2 \times 10.2)] = 4763.2 mm^2$

$\eta h_w t_w = 1.0 \times 428 \times 9.9 = 4237.2 mm^2$

Therefore,

$A_v = 4763.2$ mm^2

$$V_{c,Rd} = V_{pl,Rd} = \frac{4763.2 \times 275}{1.0 \times \sqrt{3}} = 756kN$$

Maximum design shear occurs at A, therefore the design shear is

$V_{A,Ed} = 133$ kN

$$\frac{V_{Ed}}{V_{c,Rd}} = \frac{133}{756} = 0.18 \leq 1.0$$

Therefore the shear resistance of the section is adequate.

1.8.3 Resistance to bending

Verify that: — 6.2.5(1) Eq (6.12)

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0$$

At the point of maximum bending moment (at B), verify whether the shear force will reduce the bending resistance of the cross section. — 6.2.8(2)

$$\frac{V_{c,Rd}}{2} = \frac{756}{2} = 378\ kN$$

Shear force at maximum bending moment $V_{B,Ed}$ = 122 kN

122 kN < 378 kN

Therefore **no reduction** in bending resistance due to shear is required.(Low shear)

The design resistance for bending for Class 1 and 2 cross sections is: — 6.2.5(2)

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl,y} \times f_y}{\gamma_{M0}} = \frac{1830 \times 10^3 \ \times 275}{1.0} \times 10^{-6} = 503 kN.m$$ — Eq (6.13)

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{382}{503} = 0.76 < 1.0$$ — 6.2.5(1) Eq (6.12)

Therefore the bending moment resistance is adequate.

1.9 Buckling resistance of member in bending

If the lateral torsional buckling slenderness ($\bar{\lambda}_{LT}$) is less than or equal to $\bar{\lambda}_{LT,0}$ the effects of lateral torsional buckling may be neglected, and only cross-sectional verifications apply — 6.3.2.2(4)

$\bar{\lambda}_{LT,0}$ = 0.4 for rolled sections — NA.2.17

BS EN 1993-1-1 does not give a method for determining the elastic critical moment for lateral-torsional buckling (M_{cr}). Here a method presented in Access Steel document SN002 is used to determine a value for $\bar{\lambda}_{LT}$ without having to calculate M_{cr}. — Access Steel document SN002

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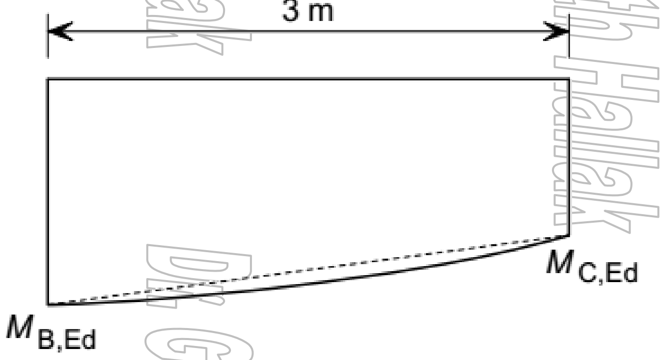
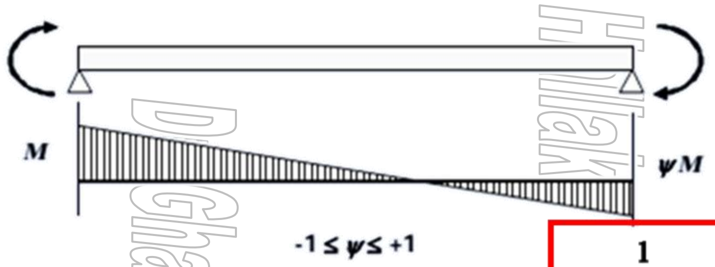
Consider section B – C of the beam.



$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{c1}} UVD\bar{\lambda}_z\sqrt{\beta_w}$$

Calculate 1/√c1

End Moment Loading	ψ	$\frac{1}{\sqrt{C_1}}$	C_1
	+1.00	1.00	1.00
	+0.75	0.92	1.17
	+0.50	0.86	1.36
	+0.25	0.80	1.56
	0.00	0.75	1.77
	-0.25	0.71	2.00
	-0.50	0.67	2.24
	-0.75	0.63	2.49
$\boxed{\frac{1}{1.33-0.33\Psi}}$	-1.00	0.60	2.76

$1/\sqrt{C_1}$ is a factor that accounts for the shape of the bending moment diagram

$$\psi = \frac{M_{C,Ed}}{M_{B,Ed}} = \frac{313}{382} = 0.819$$

$$\frac{1}{\sqrt{C_1}} = \frac{1}{1.33 - 0.33\,\psi} = \frac{1}{1.33 - 0.33\,\psi} = 0.94$$

$$g = \sqrt{\left(1 - \frac{I_z}{I_y}\right)} = \sqrt{\left(1 - \frac{1870}{37100}\right)} = 0.97$$

$$U=\sqrt{\frac{W_{pl,y}\,g}{A}\sqrt{\frac{I_z}{I_w}}}=\sqrt{\frac{1830\times10^3\times0.97}{104\times10^2}\sqrt{\frac{1870\times10^4}{0.922\times10^{12}}}}=0.877$$

β_w =1 class 1 & 2

$$\lambda_z=\frac{kL}{i_Z}=\frac{1\times3000}{42.3}=70.92$$

$$\bar{\lambda}_z=\frac{\lambda_Z}{\lambda_1}=\frac{70.92}{93.9\times0.92}=0.821$$

$$V=\frac{1}{\sqrt[4]{1+\frac{1}{20}\left(\frac{\lambda_Z}{h/t_f}\right)^2}}=\frac{1}{\sqrt[4]{1+\frac{1}{20}\left(\frac{70.92}{460/16}\right)^2}}=0.936$$

D=1 load applied at the bottom flange

$$\bar{\lambda}_{LT}=0.94\times0.877\times0.936\times1\times0.821\times\sqrt{1}=0.633$$

0.633 > 0.4 ($\bar{\lambda}_{LT,0}$)

Therefore the resistance to lateral-torsional buckling must be verified — 6.3.2.2(4)

$$\frac{M_{Ed}}{M_{b,Rd}}\le1.0$$

6.3.2.1(1) Eq (6.54)

The design buckling resistance moment ($M_{b,Rd}$) of a laterally unrestrained beam is determined from: — 6.3.2.1(3) Eq (6.55)

$$M_{b,Rd}=\chi_{Lt}\frac{W_y f_y}{\gamma_{M1}}$$

W_y = $W_{pl,y}$ for Class 1 and 2 cross-sections

For UKB sections, the method given in 6.3.2.3 for determining χ_{LT} for rolled sections may be used. Therefore

$$\chi_{LT}=\frac{1}{\phi_{LT}+\sqrt{\phi_{LT}^2-\beta\bar{\lambda}_{LT}^2}}\;but\begin{cases}\chi_{LT}\le1.0\\ \chi_{LT}\le\frac{1}{\bar{\lambda}_{LT}^2}\end{cases}$$

6.3.2.3(1) Eq (6.57)

$$\phi_{LT}=0.5\left[1+\alpha_{LT}\left(\bar{\lambda}_{LT}-\bar{\lambda}_{LT,0}\right)+\beta\bar{\lambda}_{LT}^2\right]$$

From the UK National Annex, $\bar{\lambda}_{LT,0}$ = 0.4 and β = 0.75 — NA.2.17

$\frac{h}{b} = \frac{460.0}{191.3} = 2.4$

2< 2.40 < 3.1, therefore use buckling curve 'c' NA.2.17

For buckling curve 'c', α_{LT} = 0.49 NA.2.16 & Table 6.5

$$\phi_{LT} = 0.5\left[1+0.49(.633-0.4)+0.75\times0.633^2\right] = .707$$

$$\chi_{LT} = \frac{1}{0.707+\sqrt{0.707^2-0.75\times0.633^2}} = 0.867$$

$$\frac{1}{\bar{\lambda}^2_{LT}} = \frac{1}{0.633^2} = 2.5$$

0.867< 1.0 < 2.5

Therefore,

χ_{LT} =0.867

To account for the shape of the bending moment distribution, χ_{LT} may be modified by the use of a factor 'f'.

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \; but \; \chi_{LT,mod} \le 1.0$$ 6.3.2.3(2)

$$f = 1-0.5(1-K_c)\left[1-2.0(\bar{\lambda}_{LT}-0.8)^2\right] but \; f \le 1.0$$

$$f = 1-0.5(1-0.94)\left[1-2.0(0.633-0.8)^2\right] = 0.97$$

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} = \frac{0.867}{0.97} = 0.89$$

$$M_{b,Rd} = \chi_{Lt,mod}\frac{W_y f_y}{\gamma_{M1}} = 0.89\times1830\times10^3\times275\times10^{-6}/1.0 = 448 kN.m$$

$$\frac{M_{A,Ed}}{M_{b,Rd}} = \frac{382}{448} = 0.85 \le 1.0$$

Therefore the design buckling resistance moment of the member is adequate.

1.10 Vertical deflection at serviceability limit state

The vertical deflections should be verified.

1.11 Resistance of the web to transverse forces

There is no need to verify the resistance of the web to transverse forces in this example, because the secondary beams are connected into the webs of the primary beams and flexible end plates are used to connect the beams to the columns.