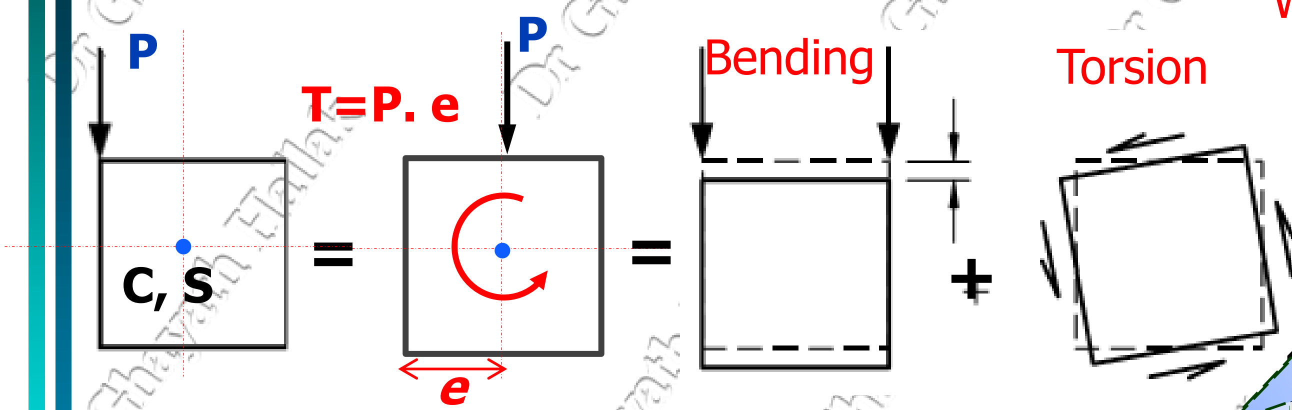
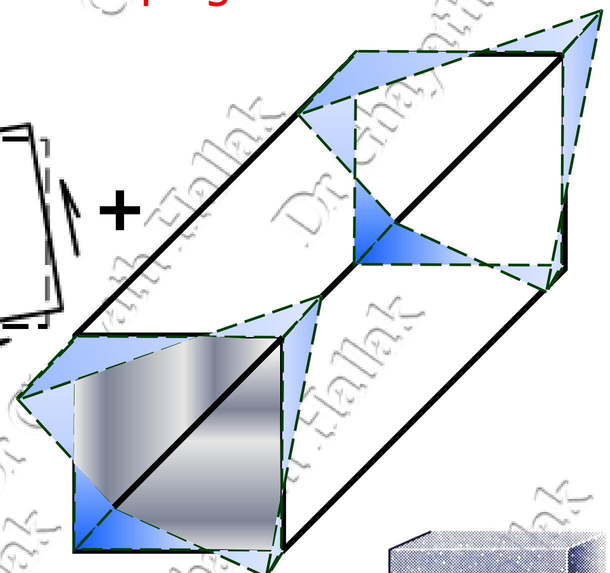
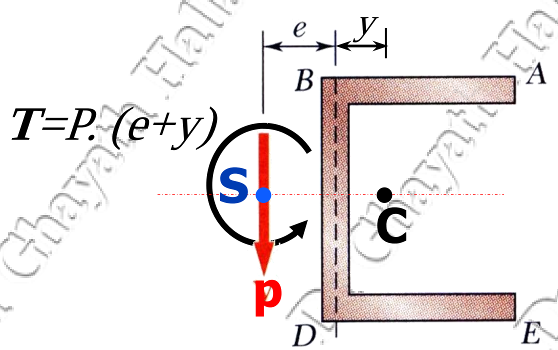
Torsion

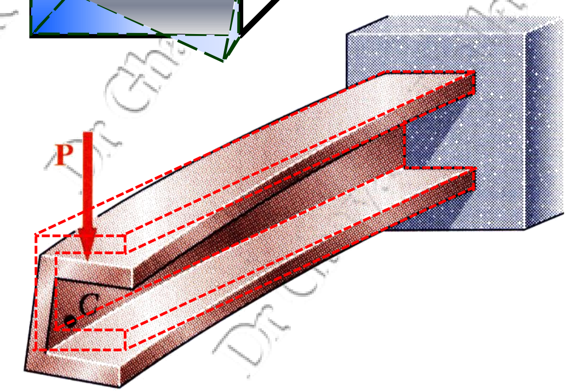
Beams subjected to loads which do not act through the point on the cross-section known as the shear centre normally suffer some twisting.



Warping due to Torsion







Torsion

Generally, when a member is subjected to a torsional moment T, the cross sections rotate around the longitudinal axis of the member (axis that is defined by the shear centre of the cross sections) and warp, that is, they undergo differential longitudinal displacements, and plane sections no longer remain plane. **If warping is free**, which happens when the supports do not prevent it and **the torsional moment is constant**, the member is said to be under **uniform torsion** or **St. Venant torsion**. Conversely, if **the torsional moment is variable or warping is restrained** at any cross section (usually at the supports), the member is under **non-uniform torsion**

Torsion

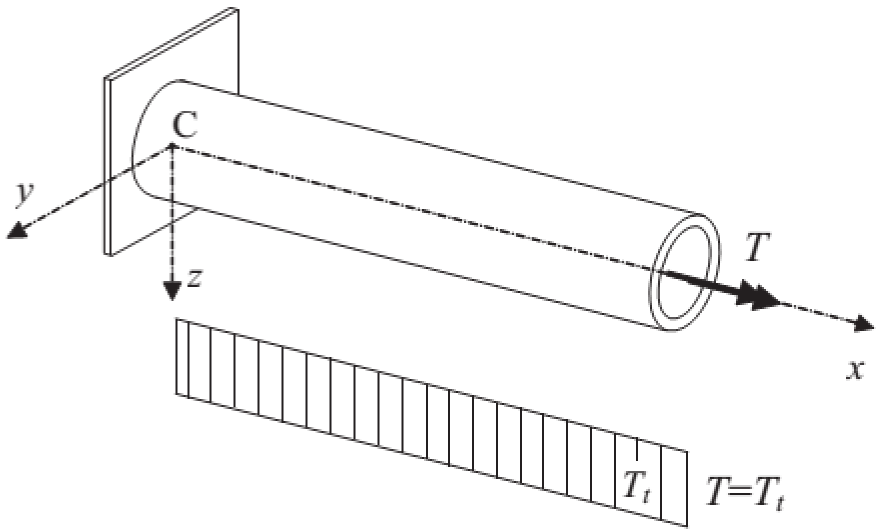
Uniform torsion induces distortion that is caused by the rotation of the cross sections around the longitudinal axis. As a consequence, shear stresses appear which balance the applied torsional moment T; under these circumstances, the resistance to the torsional moment T exclusively results from St Venant's torsion, T_t. Although longitudinal warping displacements may exist, they do not introduce stresses.

In non-uniform torsion, besides the St. Venant shear stresses, longitudinal strains also exist (because warping varies along the member). These longitudinal strains generate self-equilibrating normal stresses at the cross sectional level that, depending on the level of restriction to warping, vary along the member. The existence of varying normal stresses implies (by equilibrium in the longitudinal direction) the existence of additional shear stresses that also resist to torsional moments, leading to: $T = T_t + T_w$.

Torsion

The applied torsional moment T is thus balanced by two terms, one due to the torsional rotation of the cross section (T_t) and the other caused by the restraint to warping, designated by warping torsion (T_w).

In cross sections of circular shape, because they exhibit rotational symmetry with respect to the shear centre S (that coincides with the centroid G), only **uniform torsion** exists.

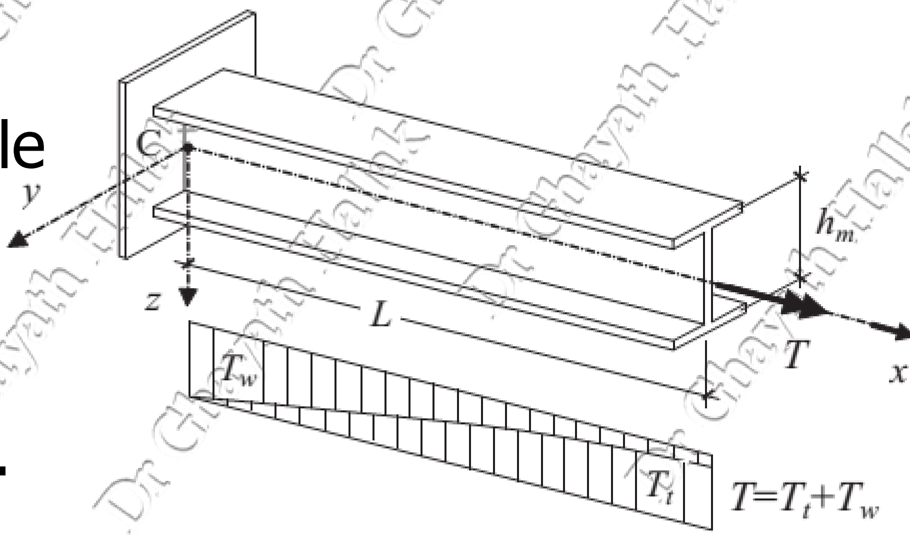


Torsion

In thin-walled closed cross sections (the most appropriate to resist torsion), *uniform torsion* is predominant. Therefore, in the analysis of thin-walled closed cross sections subjected to torsion, the warping torsion (T_w) is normally neglected.

In members with thin-walled open cross sections (such as I or H sections), so that only the uniform torsion component appears, it is necessary that the supports do not prevent warping and that the torsional moment is constant.

On the opposite, if the torsional moment is variable or warping is restrained at some cross sections (usual situation), the member is under *non-uniform torsion*.



Torsion

Shear stresses and torsion constant for typical steel cross section shapes

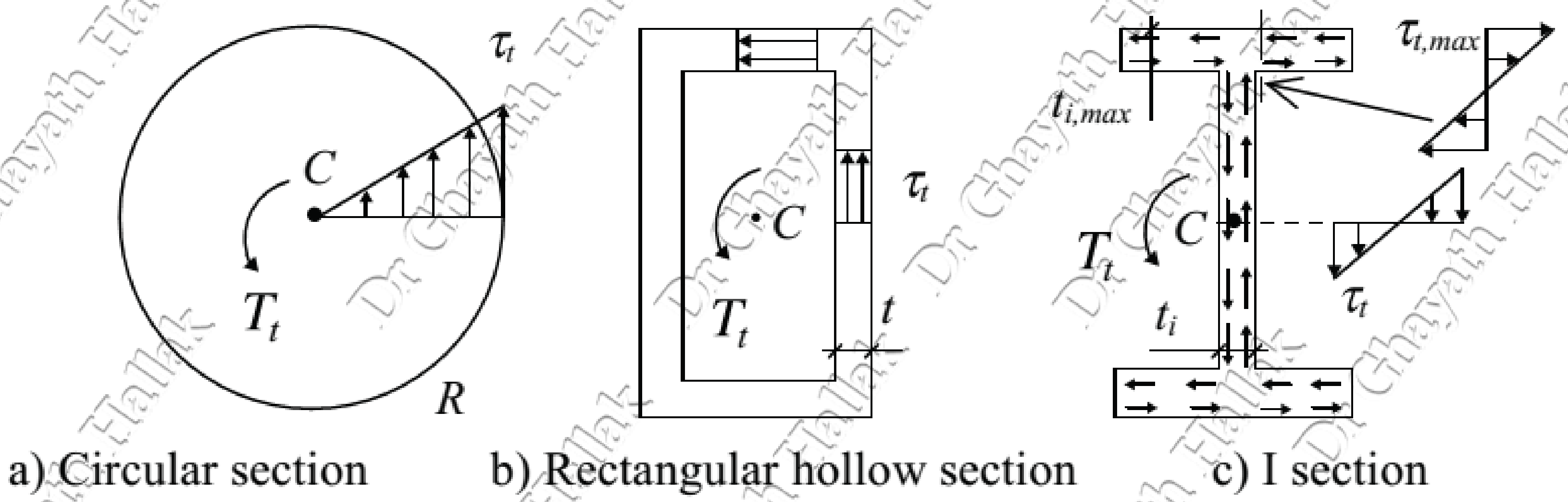
Section	Shear stress	Torsion constant
Circular (solid or hollow)	$\tau_t = \frac{T}{I_p} r$	$I_T = I_p$
Thin-walled closed	$\tau_t = \frac{T}{2A_m t}$	$I_T = \frac{4A_m^2}{\oint \frac{ds}{t}}$
Thin-walled open	$\tau_{t,\max} \approx \frac{T}{I_T} t_{i,\max}$	$I_T \approx \frac{1}{3}\sum_{i=1}^{n} h_i t_i^3$

I_p is the polar moment of inertia, $\pi R^4/2$ in the case of a circular solid section, R is the radius of the circular section

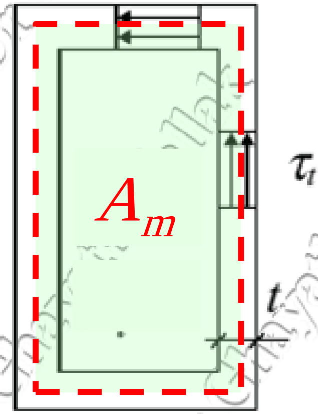
A_m is the area defined by the middle line in a thin-walled closed cross section;

s is a coordinate that is defined along the outline of a thin-walled closed section

Torsion

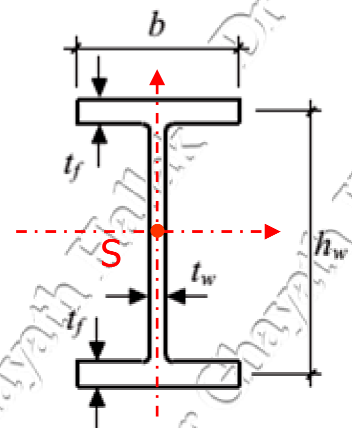
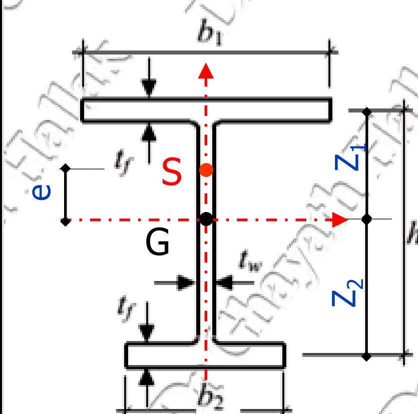


Shear stresses due to *uniform torsion* for typical steel cross section shapes

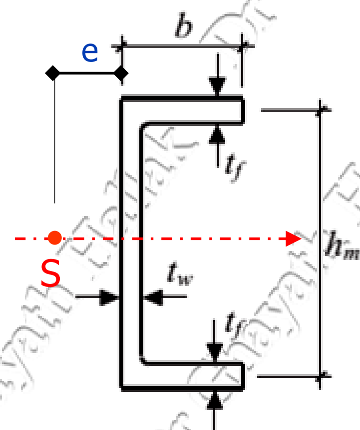
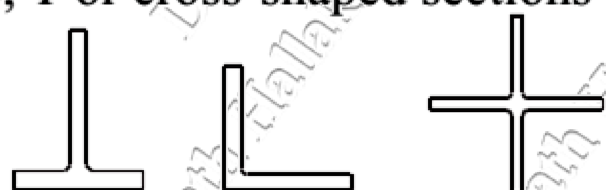


Torsion

Warping constant for typical cross sections

Section		I_W
Circular (solid or hollow)		0
Thin-walled closed		≈ 0
I or H of equal flanges		$\dfrac{t_f\, h_m^2\, b^3}{24}$
I or H of unequal flanges		$\dfrac{t_f\, h_m^2}{12}\,\dfrac{b_1^3\, b_2^3}{b_1^3+b_2^3}$ $e=\dfrac{t_f\left(z_1\, b_1^3+z_2\, b_2^3\right)}{t_w\, h_w^3+t_f\left(b_1\, z_1^2+b_2\, z_2^2\right)}$

Warping constant for typical cross sections

Section		I_W
Channel		$\frac{t_f\, b^3\, h_m^{\;2}}{12}\,\frac{3b\,t_f + 2h_m\,t_w}{6b\,t_f + h_m\,t_w}$ $e = \frac{3b^2 t_f}{t_w\, h_w + 6b\,t_f}$
L, T or cross-shaped sections 		0

Torsion

✓- Open sections normally used as beams are inherently weak in resisting torsion.

✓- In circumstances where beams are required to withstand significant torsional loading, consideration should be given to the use of a torsionally more efficient shape such as a structural hollow section.

✓- The constant of uniform torsion I_t and the warping constant I_w for standard cross sections are usually supplied by steel producers, in tables of profiles

Unrestrained Beams

Introduction

The design of a beam subject to bending and shear must be performed in two steps:

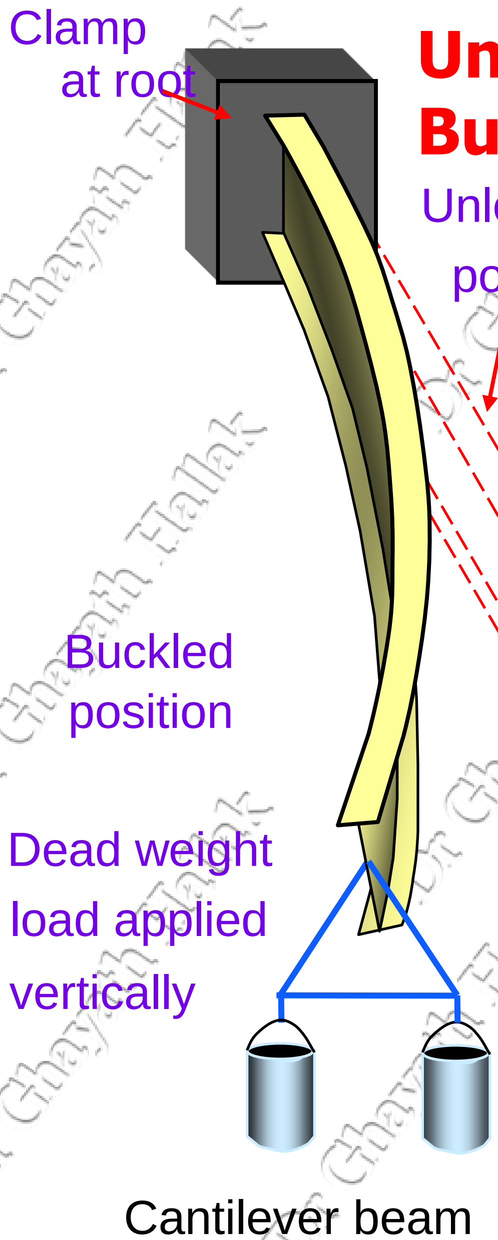
i) verification of the resistance of the cross section

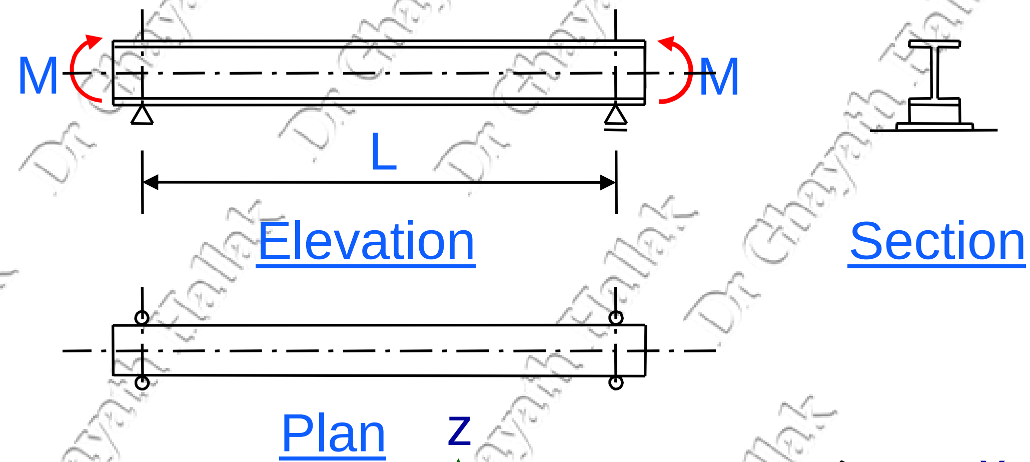
- cross sectional shape → instability caused by shear forces, shear buckling & yielding
- cross section class → local cross section instability

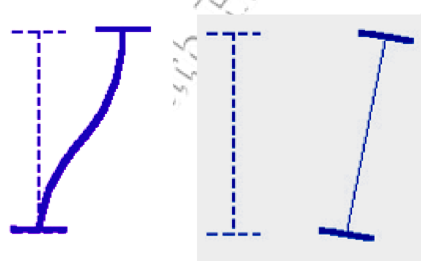
ii) Check on member stability

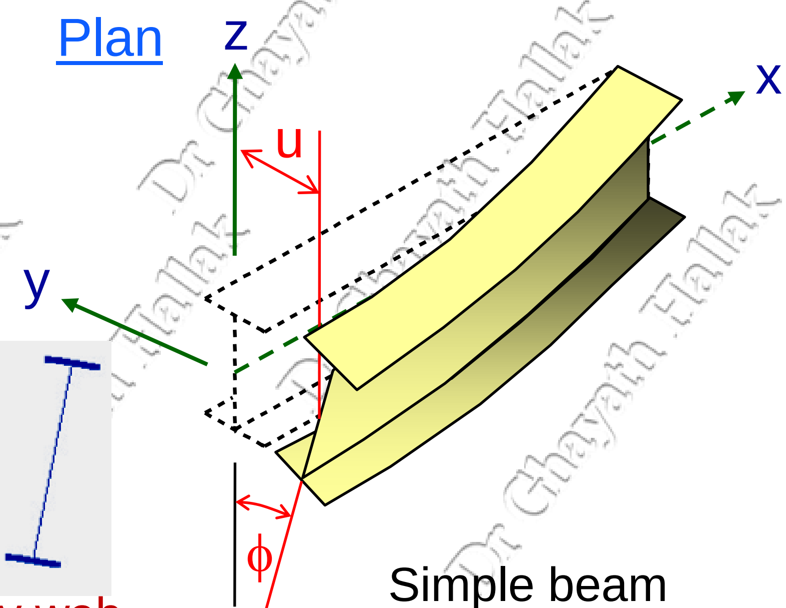
- Beams without continuous lateral restraint and subjected to bending moment are prone to buckling about their major axis, this mode of buckling is called **lateral torsional buckling (LTB).**

Unrestrained Beams- Lateral Torsional Buckling









Unrestrained Beams

❑- Consider a member subject to bending about the strong axis of the cross section (the y axis). Lateral-torsional buckling is characterised by lateral deformation of the compressed part of the cross section (the compressed flange in the case of I or H sections). This part behaves like a compressed member(column), but one continuously restrained by the part of the section in tension, which initially does not have any tendency to move laterally.

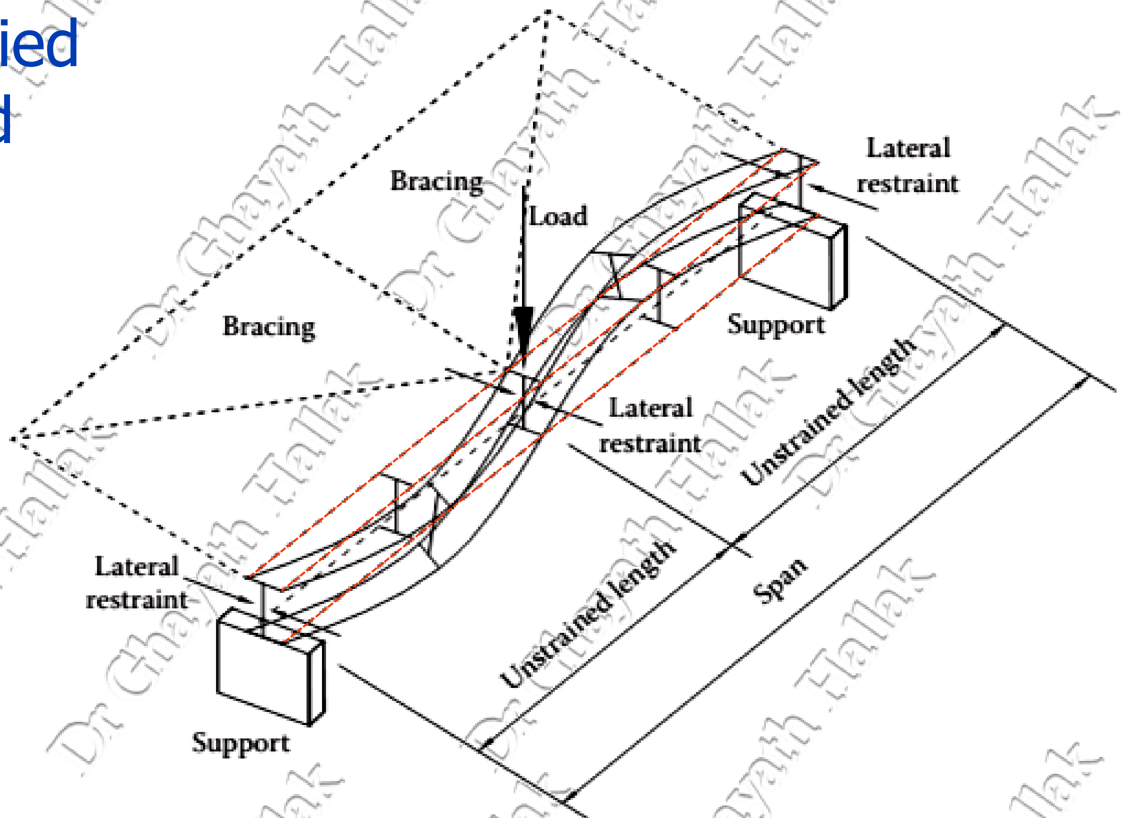
❑- As seen in the following Figure, where this phenomenon is illustrated for a cantilever beam, the resulting deformation of the cross section includes both lateral bending and torsion. This is why this phenomenon is called lateral-torsional buckling.

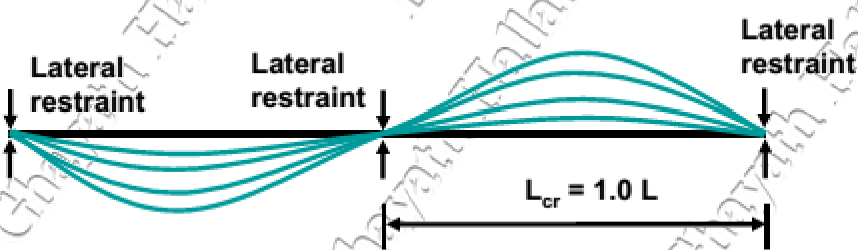
Unrestrained Beams

- The load at which the beam buckles can be **much less** than that causing the **full moment** capacity to develop.
- Beams bent about their minor principal axis will respond by deforming in that plane i.e. there is no tendency when loaded in a weaker direction to buckle by deflecting in a stiffer direction.
- If the sort of deflections illustrated in the following Figure are prevented by the form of construction e.g. by attaching the beam's top flange to a laterally very stiff concrete slab, then buckling of this type cannot occur.
- Finally, if the beam's cross-section is torsionally very stiff, as is the case for all SHS, its resistance to lateral torsional buckling for all practical arrangements will be so great that it will not influence the design.

Unrestrained Beams- factors influencing LTB

Checks should be carried out on all unrestrained segments of beams (between the points where lateral restraint exists).



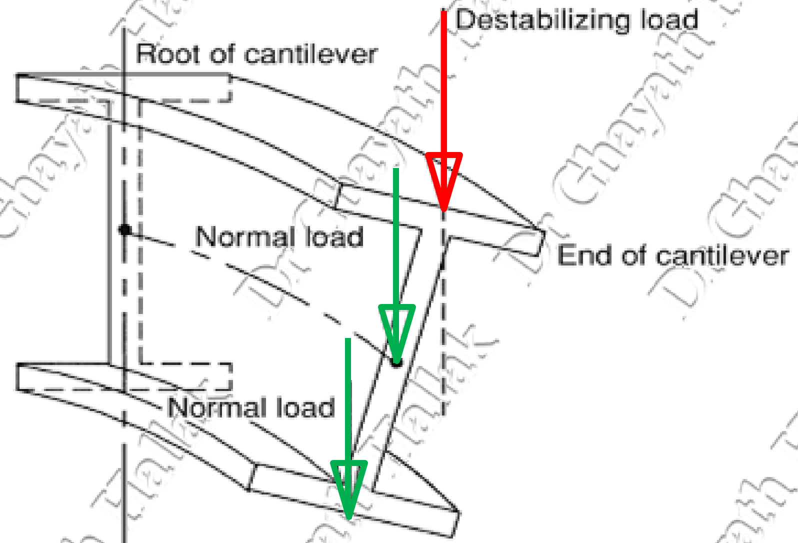


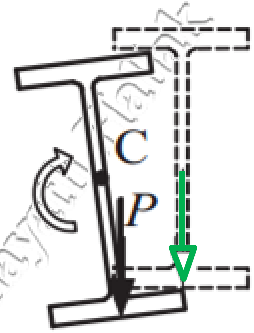
Beam on plan

Unrestrained Beams- factors influencing LTB

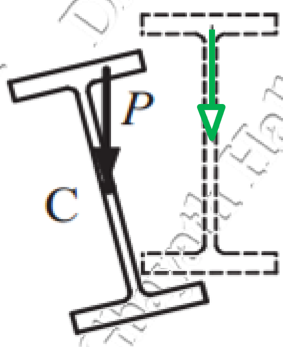
- - loading (shape of the bending moment diagram)
- - support conditions
- - length of the member between laterally braced cross sections
- - lateral bending stiffness I_z
- - torsion stiffness I_t
- - warping stiffness I_w
- - the point of application of the loading .A gravity load applied below the shear centre C (that coincides with the centroid, in case of doubly symmetric I or H sections) has a stabilizing effect, whereas the same load applied above this point has a destabilizing effect

Unrestrained Beams- factors influencing LTB

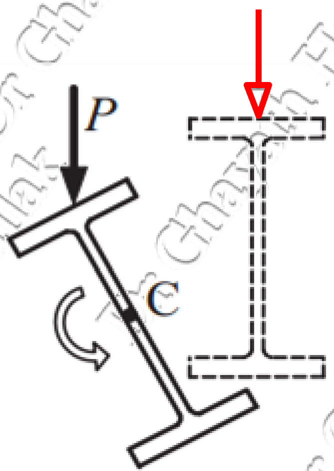




stabilizing load



stabilizing load



Destabilizing load

LATERAL-TORSIONAL BUCKLING

EN 1993-1-1 contains three methods for checking the lateral-torsional stability of a structural member:

1. The primary method adopts the lateral buckling curves given in Clause 6.3.2.2 (general case) and Clause 6.3.2.3 (just for rolled sections and equivalent welded sections).
2. The second is a simplified assessment method for beams with restraints in buildings and is set out in Clause 6.3.2.4 of EN 1993-1-1.
3. The third is a general method for lateral and LTB of structural components, such as single members with monosymmetric cross sections, built-up, non-uniform or plane frames and subframes, given in Clause 6.3.4.

LATERAL-TORSIONAL BUCKLING

The design buckling resistance $M_{b,Rd}$ of a laterally unrestrained beam (or segment of beam) should be taken as:

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} , \quad \frac{M_{Ed}}{M_{b,Rd}} \leq 1.0$$

W_y:
- $W_{pl,y}$ For class 1 & 2
- $W_{el,y}$ For class 3
- $W_{eff,y}$ For class 4

$\gamma_{M1} = 1.0$

χ_{Lt}: Reduction factor for LTB

Reduction Factor χ_{Lt}

The primary method

i) General case: (Clause 6.3.2.2 of BS EN 1993-1-1)

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$$

LATERAL-TORSIONAL BUCKLING

Reduction Factor χ_{Lt}

i) General case: (Clause 6.3.2.2 of BS EN 1993-1-1)

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{2}\right]$$

Imperfection factor

Plateau length

Table 6.4 of BS EN 1993-1-1 Buckling curves for LTB (General method)

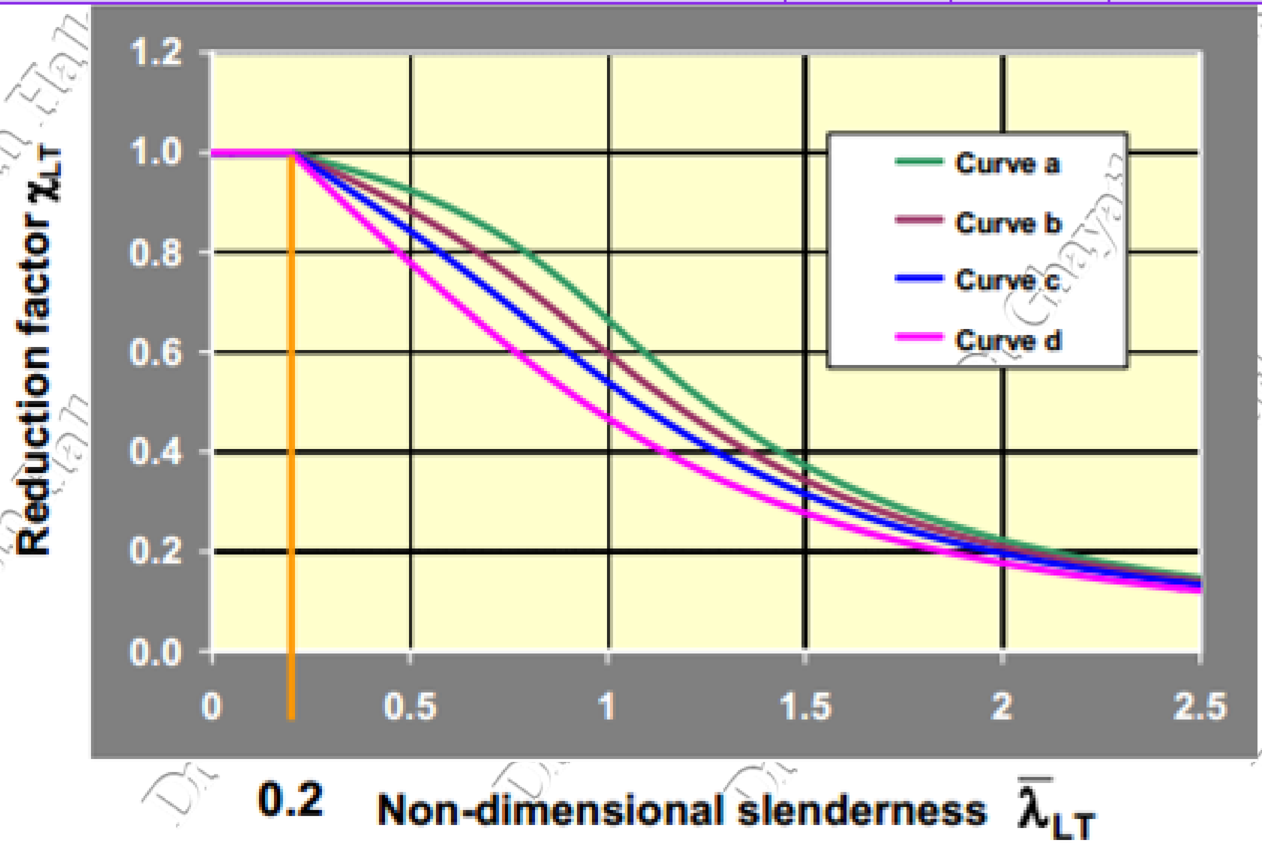
Section	Limits	Buckling curve
I or H sections rolled	h/b≤ 2	a
	h/b > 2	b
I or H sections welded	h/b ≤ 2	c
	h/b > 2	d
Other sections	---	d

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

i) General case: (Clause 6.3.2.2 of BS EN 1993-1-1)

Table 6.3 of BS EN 1993-1-1: imperfection factors for LTB curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76



LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

i) General case: (Clause 6.3.2.2 of BS EN 1993-1-1)

** - Lateral torsional buckling effects may be **ignored** and only cross sectional checks apply. IF:

$$1-\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0} \qquad where\ \bar{\lambda}_{LT,0} = \lambda_{LT,0}/\lambda_1 =$$

$$2-Or\ \frac{M_{Ed}}{M_{cr}} \leq \bar{\lambda}_{LT,0}^2 \qquad = \left(0.4\sqrt{\frac{\pi^2 E\,\gamma_{M0}}{f_y}}\right)\Bigg/ 93.9\varepsilon = 0.4$$

3- CHS, SHS and [RHS(Table13-BS5950-2000)]

Cross-Section (From NA.2.17- NA to BS EN 1993-1-1:2005)	$\bar{\lambda}_{LT,0}$
For rolled sections and hot-finished and cold-formed hollow sections:	0.4
For welded sections:	0.2

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

The primary method

ii) Rolled sections and equivalent welded sections.
Clause 6.3.2.3 of BS EN 1993-1-1

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \quad but \begin{cases} \chi_{LT} \le 1.0 \\ \chi_{LT} \le \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases}$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}^2\right]$$

Cross-Sections (From NA.2.17- NA to BS EN 1993-1-1:2005)	$\bar{\lambda}_{LT,0}$	β
For rolled sections and hot-finished and cold-formed hollow sections:	0.4	0.75
For welded sections:	0.2	1.0

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

The primary method

ii) Rolled sections and equivalent welded sections.
Clause 6.3.2.3 of BS EN 1993-1-1

LTB curve for rolled and equivalent welded cases (From NA.2.17- NA to BS EN 1993-1-1:2005)

Section	Limits	Buckling curve
rolled doubly symmetric I and H sections and hot-finished hollow sections	h/b≤ 2	b
	2<h/b ≤ 3.1	c
	h/b > 3.1	d
Welded doubly symmetric sections and cold-formed hollow sections	h/b ≤ 2	c
	2<h/b ≤ 3.1	d
Angles (for moments in the major principal plane)	---	d
All other hot-rolled sections	---	d

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

The primary method

ii) Rolled sections and equivalent welded sections. Clause 6.3.2.3 of BS EN 1993-1-1

According to this method, the shape of the bending moment diagram, between braced sections, can be taken into account by considering a modified reduction factor $\chi_{LT, mod}$:

$$\chi_{LT,\mathrm{mod}} = \frac{\chi_{LT}}{f} \; but \; \chi_{LT,\mathrm{mod}} \leq 1.0$$

$$f = 1 - 0.5(1 - K_c)\left[1 - 2.0\left(\bar{\lambda}_{LT} - 0.8\right)^2\right] but \; f \leq 1.0$$

K_c is a correction factor- Table 6.6 BS EN 1993-1-1

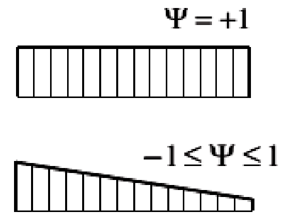
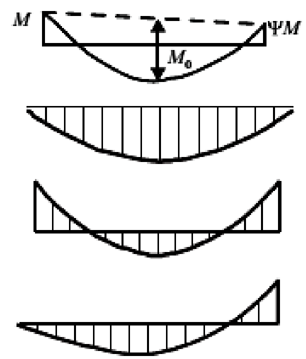
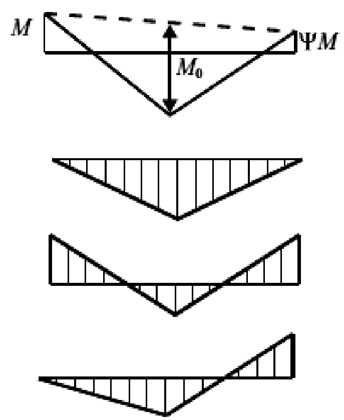
$$M_{b,Rd} = \chi_{Lt,\mathrm{mod}} \frac{W_y f_y}{\gamma_{M1}}$$

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

Table 6.6 – k_c correction factors

$k_c = 1/\sqrt{C_1} \Rightarrow$

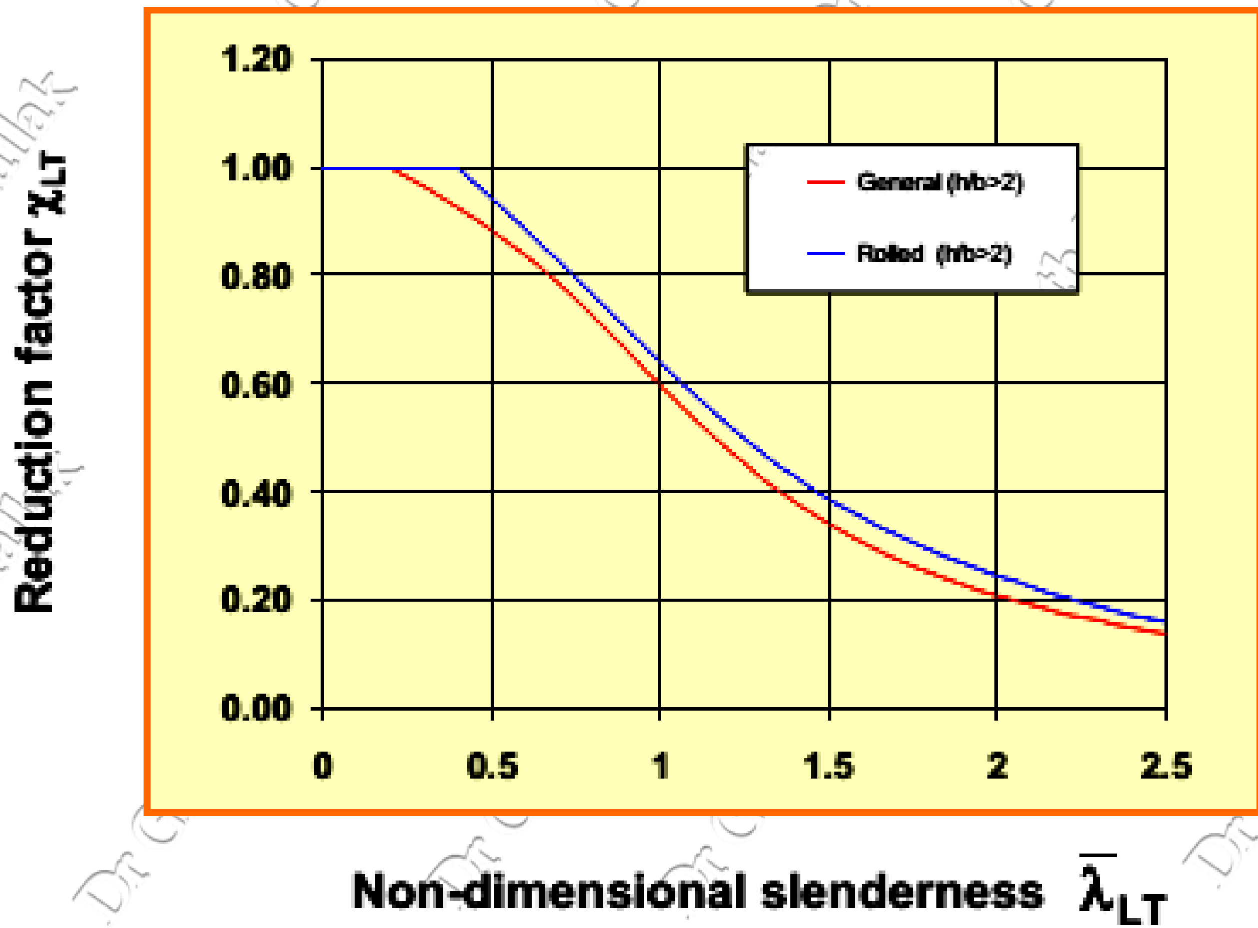
$C_1 = 1/k_c^2$

Diagram of bending moments	k_c
	1.0 $\frac{1}{1.33 - 0.33\Psi}$
	0.94 0.90 0.91
	0.86 0.77 0.82
Ψ - ratio between end moments, with $-1 \le \Psi \le 1$.	

LATERAL-TORSIONAL BUCKLING- Reduction Factor χ_{Lt}

The primary method

Comparison between general curves and curves for rolled and equivalent welded sections (I-sections – h/b>2)



Rolled method more Economy

LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Method 1 -Method for doubly symmetric sections

Access Steel Document SN003.This method only applies to:

- ❑- uniform straight members
- ❑- the cross-section is symmetric about the bending plane.
- ❑- The conditions of restraint at each end are at least : ☐

  *- restrained against lateral movement (lateral restraints are defined as arrangements that only prevent lateral deflection of the compression flange i.e. Lateral deflection of the tension flange and twisting are still possible.)

  **- restrained against rotation about the longitudinal axis (torsional restraints are defined as arrangements that prevent both lateral deflection and twisting e.g. restraint to both the tension and compression flanges

LATERAL-TORSIONAL BUCKLING

Calculation of M_{cr} Elastic critical moment

Method 1 -Method for doubly symmetric sections

Access Steel Document SN003

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(KL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_T}{\pi^2 E I_z} + (C_2 Z_g)^2} - C_2 Z_g \right\}$$

E is the Young modulus (E = 210000 N/mm2)

G is the shear modulus (G = 80770 N/mm2)

I_z is the second moment of area about the weak axis

I_T is the torsion constant

I_w is the warping constant

L is the beam length between points which have lateral restraint

k and kw are effective length factors

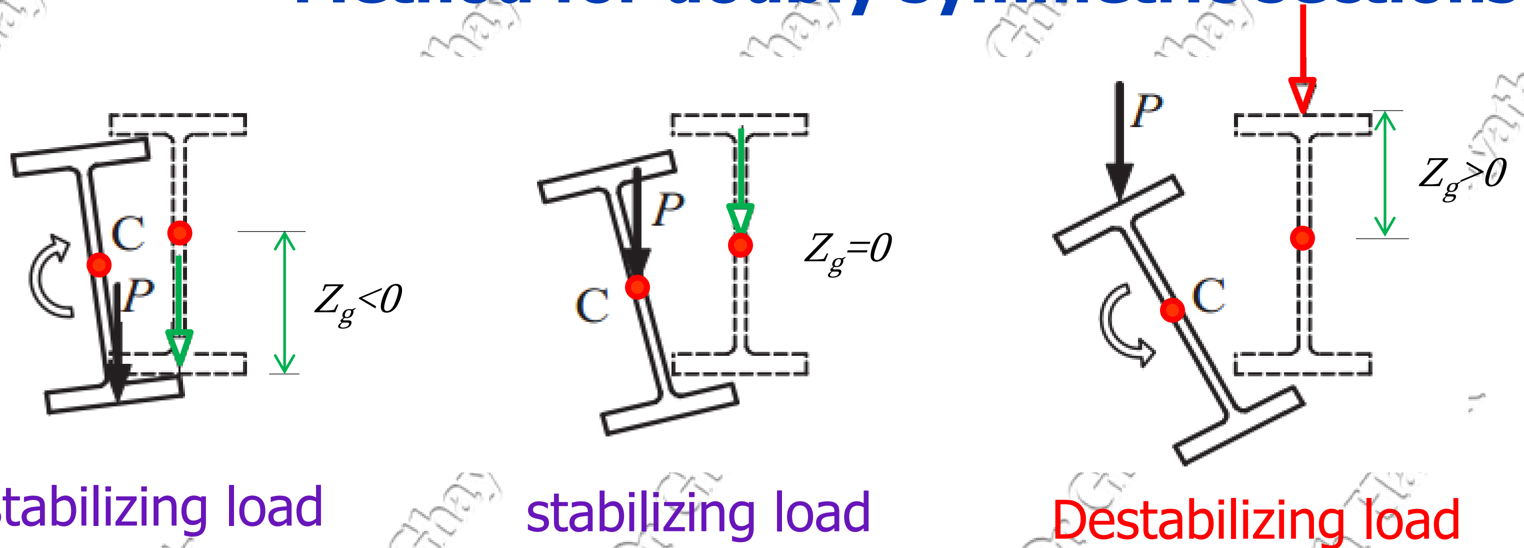
Zg is the distance between the point of load application and the shear centre.

C_1 and C_2 are coefficients depending on the loading and end restraint conditions

LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Method 1 -Method for doubly symmetric sections



In the common case of normal support conditions at the ends (fork supports), $k = k_w = 1.0$.

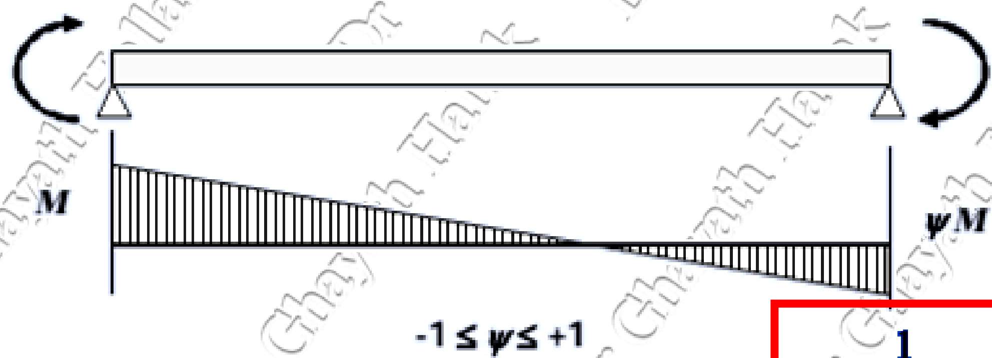
When the bending moment diagram is linear along a segment of a member delimited by lateral restraints, or when the transverse load is applied in the shear centre, $C_2 z_g = 0$.

Calculation of M_{cr} Elastic critical moment

Method 1 -Method for doubly symmetric sections

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

Values of $1/\sqrt{C_1}$ and C_1 for various moment conditions (load is not destabilising)

End Moment Loading	ψ	$\frac{1}{\sqrt{C_1}}$	C_1
	+1.00	1.00	1.00
	+0.75	0.92	1.17
	+0.50	0.86	1.36
	+0.25	0.80	1.56
	0.00	0.75	1.77
	-0.25	0.71	2.00
	-0.50	0.67	2.24
	-0.75	0.63	2.49
	-1.00	0.60	2.76

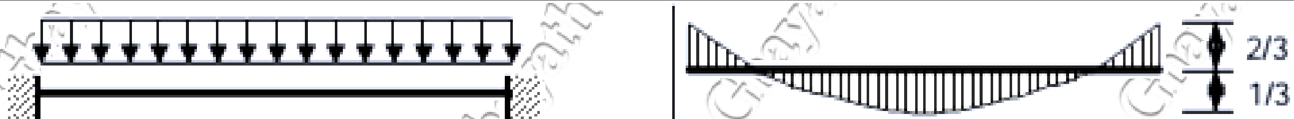
$$\frac{1}{1.33 - 0.33\Psi}$$

Access Steel Document SN003

LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Values of $1/\sqrt{C_1}$ and C_1 for various moment conditions (load is not destabilising)

Intermediate Transverse Loading	$1/\sqrt{C_1}$	C_1
	0.94	1.13
	0.62	2.60
	0.86	1.35
	0.77	1.69

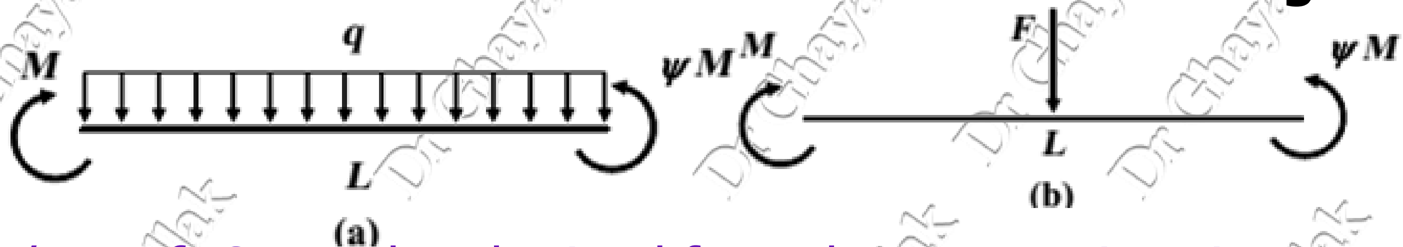
$$k_c = 1/\sqrt{C_1} \Rightarrow C_1 = 1/k_c^2$$

Access Steel Document SN003

LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Member with end moments and transverse loading



Values of C_1 may be obtained from the curves given in Access Steel Document SN003. The moment distribution may be defined using two parameters :

ψ is the ratio of end moments. By definition, M is the maximum end moment, and so : $-1 \le \psi \le 1$ ($\psi = 1$ for a uniform moment)

μ is the ratio of the moment due to transverse load to the maximum end moment M

Case a) (end moments with a uniformly distributed load) $\mu = qL^2/8M$

Case b) (end moments with a concentrated load at mid-span) $\mu = FL/4M$

$\mu > 0$ if M and (q or F), bend the beam in the same direction. As shown above

Access Steel Document SN003

LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Method 2

The value of M_{cr} may be determined using the software *'LTBeam'* available from www.cticm.com

Method 3

As an alternative to calculating M_{cr} and hence $\bar{\lambda}_{LT}$, the value of $\bar{\lambda}_{LT}$ may be calculated directly from the expression:

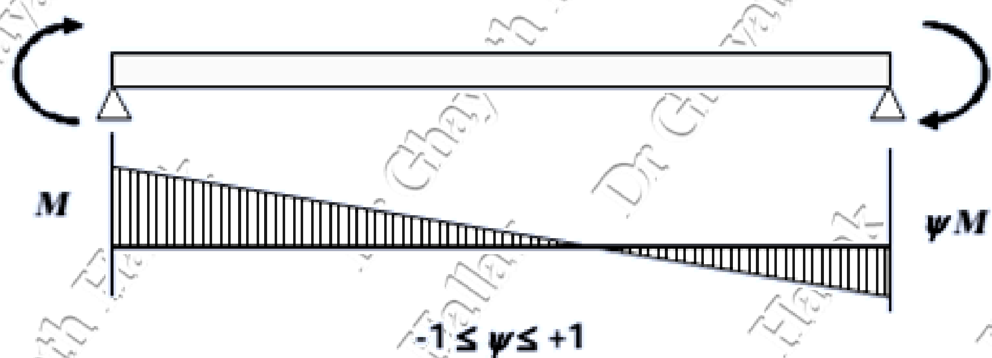
$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD\bar{\lambda}_z \sqrt{\beta_w}$$

C_1 is a factor that allows for the shape of the bending moment diagram. It may be conservatively taken as equal to 1.0. For cantilevers C_1 should be taken as 1.0 The factors in the following Table assume that the load is not destabilising. Where the load is destabilising *C*1 should be taken as 1.0.

LATERAL-TORSIONAL BUCKLING-
Calculation of M_{cr} Elastic critical moment
Method 3

Values of $1/\sqrt{C_1}$ and C_1 for various moment conditions

End Moment Loading



ψ	$\frac{1}{\sqrt{C_1}}$	C_1
+1.00	1.00	1.00
+0.75	0.92	1.17
+0.50	0.86	1.36
+0.25	0.80	1.56
0.00	0.75	1.77
-0.25	0.71	2.00
-0.50	0.67	2.24
-0.75	0.63	2.49
-1.00	0.60	2.76

$$\frac{1}{1.33-0.33\Psi}$$

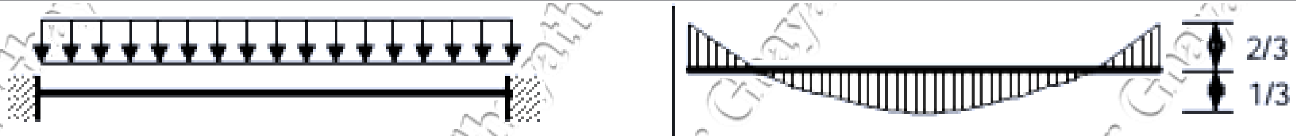
LATERAL-TORSIONAL BUCKLING-

Calculation of M_{cr} Elastic critical moment

Method 3

Values of $1/\sqrt{C_1}$ and C_1 for various moment conditions

Intermediate Transverse Loading

Loading and moment diagram	$1/\sqrt{C_1}$	C_1
	0.94	1.13
	0.62	2.60
	0.86	1.35
	0.77	1.69

LATERAL-TORSIONAL BUCKLING

Calculation of M_{cr} Elastic critical moment

Method 3

U is a parameter that depends on section geometry

$$U = \sqrt{\frac{W_{pl,y}\, g}{A}\sqrt{\frac{I_z}{I_w}}}$$

$$\text{Where} \quad g = \sqrt{\left(1 - \frac{I_z}{I_y}\right)}, \text{or may conservatively} = 1.0.$$

U = 0.9 conservative upper bound for UKB and UKC sections

V is a parameter related to slenderness:

$$V = \frac{1}{\sqrt[4]{\left(\frac{k}{k_w}\right)^2 + \dfrac{\lambda_z^2}{\dfrac{\pi^2 E}{G}\dfrac{A}{I_T}\dfrac{I_w}{I_z}} + \left(C_2 Z_g\right)^2 \dfrac{I_z}{I_w}}}$$

LATERAL-TORSIONAL BUCKLING

Calculation of M_{cr} Elastic critical moment

Method 3

For doubly-symmetric hot-rolled UKB and UKC sections, and for cases where the loading is not destabilizing:

$$V = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{\lambda_z}{h/t_f}\right)^2}}$$

For all sections symmetric about the major axis and not subjected to destabilizing loading, $V = 1.0$ conservatively

LATERAL-TORSIONAL BUCKLING

Calculation of M_{cr} Elastic critical moment

Method 3

Values of slenderness parameter V

λ_z	h/t_f									
	5	10	15	20	25	30	35	40	45	50
30	0.77	0.91	0.96	0.97	0.98	0.99	0.99	0.99	0.99	1.00
50	0.64	0.82	0.90	0.93	0.96	0.97	0.98	0.98	0.99	0.99
75	0.53	0.72	0.82	0.88	0.91	0.93	0.95	0.96	0.97	0.97
100	0.47	0.64	0.75	0.82	0.86	0.90	0.92	0.93	0.95	0.96
125	0.42	0.58	0.69	0.76	0.82	0.86	0.88	0.91	0.92	0.93
150	0.38	0.53	0.64	0.72	0.77	0.82	0.85	0.88	0.90	0.91
175	0.36	0.50	0.60	0.67	0.73	0.78	0.82	0.85	0.87	0.89
200	0.33	0.47	0.56	0.64	0.70	0.75	0.79	0.82	0.84	0.86
225	0.31	0.44	0.53	0.61	0.67	0.72	0.76	0.79	0.82	0.84
250	0.30	0.42	0.51	0.58	0.64	0.69	0.73	0.76	0.79	0.82
275	0.28	0.40	0.49	0.56	0.61	0.66	0.70	0.74	0.77	0.79
300	0.27	0.38	0.47	0.53	0.59	0.64	0.68	0.72	0.75	0.77

β_w is a parameter that allows for the classification of the cross-section; for Class 1 and 2 sections, $\beta_w = 1$ while for Class 3 sections $\beta_w = W_{el,y} / W_{pl,y}$.

LATERAL-TORSIONAL BUCKLING

Method 3

Calculation of M_{cr} Elastic critical moment

$\bar{\lambda}z$ is the minor axis non-dimensional slenderness of the member, given by $\bar{\lambda}z = \lambda_z / \lambda_1$, in which $\lambda_1 = \pi\sqrt{E/f_y} = 93.9\varepsilon$, $\lambda_z = kL/i_z$, where *k* is an effective length parameter

K & D parameters for simply supported beam

Conditions of restraint at supports		Parameters	
		k	*D*
Compression flange laterally restrained: Nominal torsional restraint against rotation about longitudinal axis	Both flanges fully restrained against rotation on plan	0.70	1.2
	Compression flange fully restrained against rotation on plan	0.75	1.2
	Both flanges partially restrained against rotation on plan	0.80	1.2
	Compression flange partially restrained against rotation on plan	0.85	1.2
	Both flanges free to rotate on plan	1.00	1.2

LATERAL-TORSIONAL BUCKLING

Method 3

Calculation of M_{cr} Elastic critical moment

Effective length kL for Normal and destabilising Loading for simply supported beams.

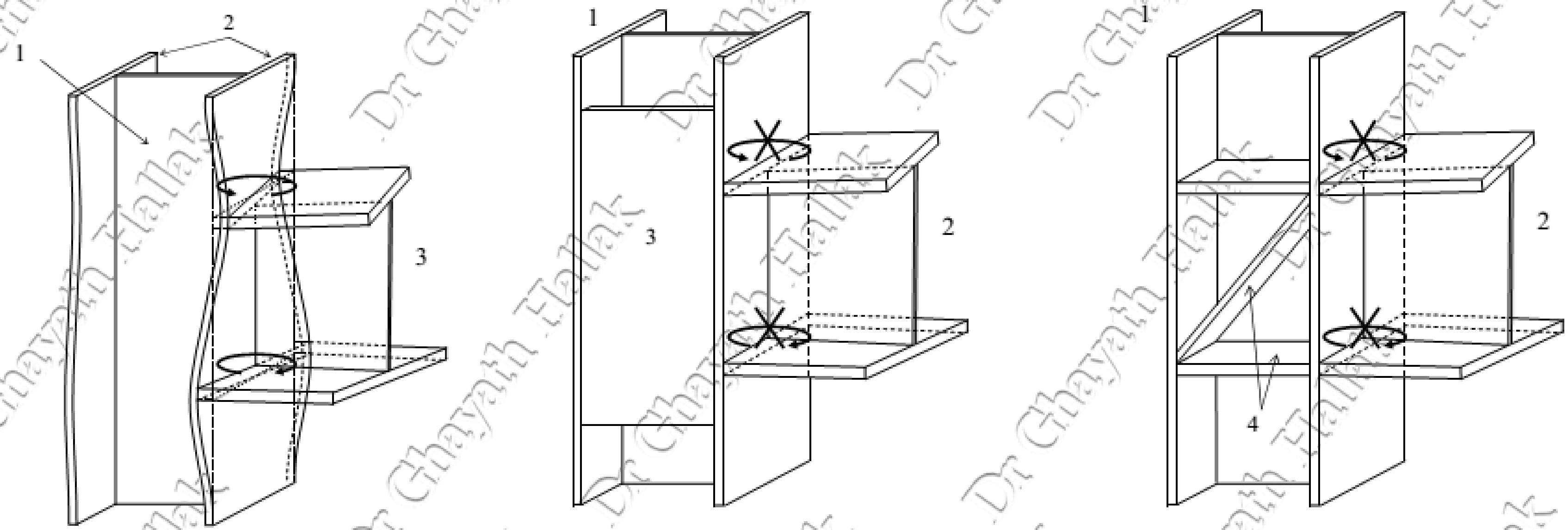
Conditions of restraint at supports		kL Loading condition: Normal	kL Loading condition: Destabilizing
Compression flange laterally unrestrained. Both flanges free to rotate on plan.	Partial torsional restraint against rotation about longitudinal axis provided by connection of bottom flange to supports.	$1.0L + 2h$	$1.2L + 2h$
	Partial torsional restraint against rotation about longitudinal axis provided only by pressure of bottom flange onto supports.	$1.2L + 2h$	$1.4L + 2h$

h = depth of beam.

LATERAL-TORSIONAL BUCKLING

Calculation of M_{cr} Elastic critical moment

Method 3



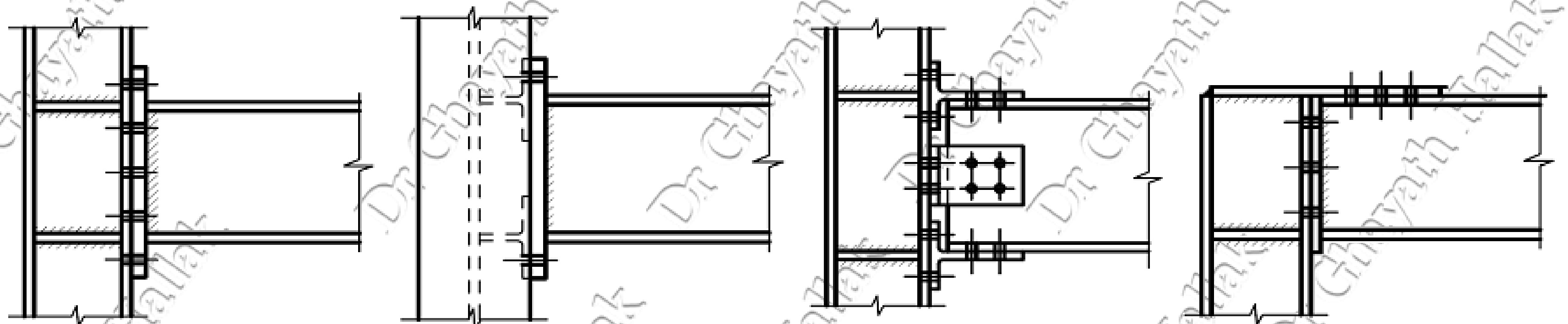
1 Column
2 Thin flanges
3 Cantilever beam (buckling)
Case of "free" warping conditions at support

1 Column
2 Cantilever beam (buckling)
3 Stiffening plate (on both sides)
4 Stiffeners (on both sides)
Case of "restrained" warping conditions at support

LATERAL-TORSIONAL BUCKLING

Method 3

Calculation of M_{cr} Elastic critical moment



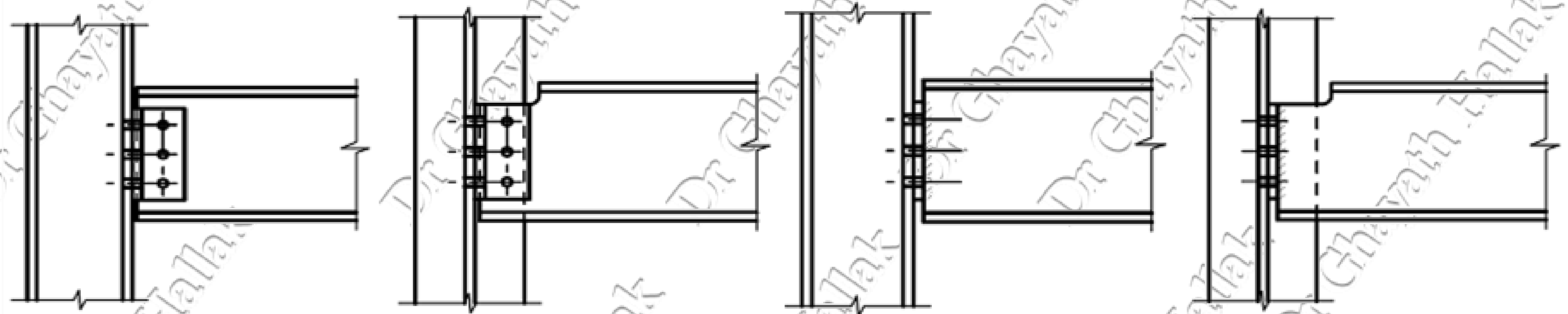
Both Flanges fully restrained against rotation on plan

Both Flanges partially restrained against rotation on plan

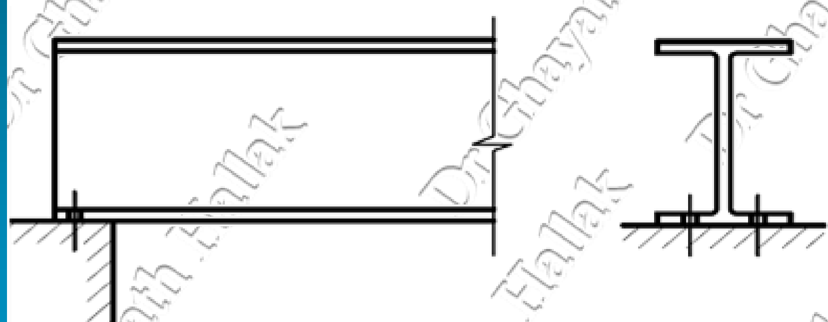
LATERAL-TORSIONAL BUCKLING

Method 3

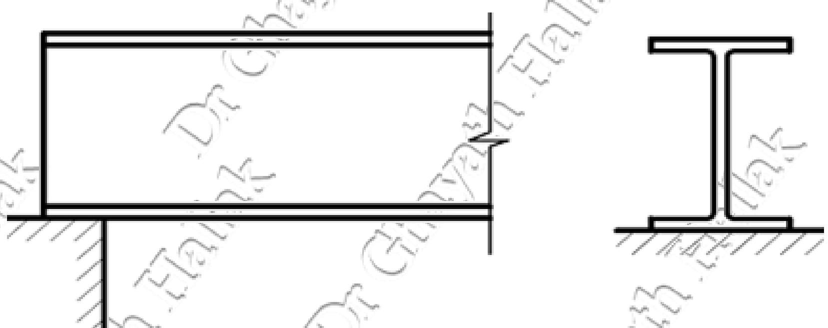
Calculation of M_{cr} Elastic critical moment



Both Flanges free to rotate on plan



Partial torsional restraint against rotation about longitudinal axis provided by connection of bottom flange to supports

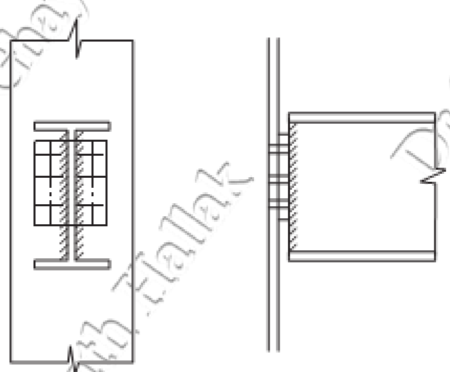


Partial torsional restraint against rotation about longitudinal axis provided by pressure of bottom flange onto supports

LATERAL-TORSIONAL BUCKLING

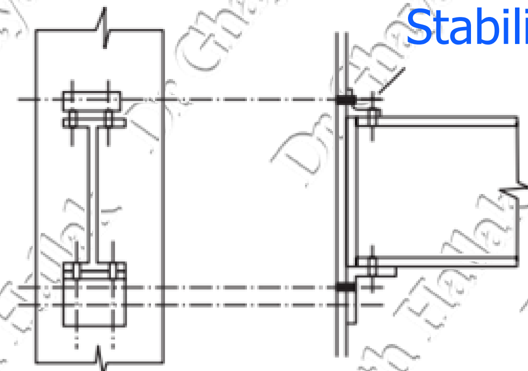
Calculation of M_{cr} Elastic critical moment

Method 3



Beam compression flange free to rotate on plan

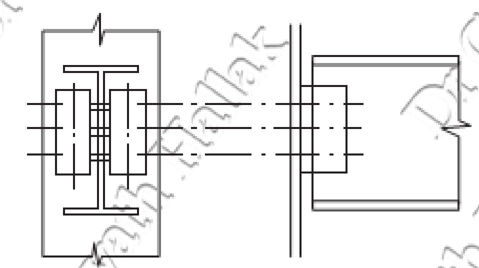
(a) Thin end plate to web only



Stability cleat

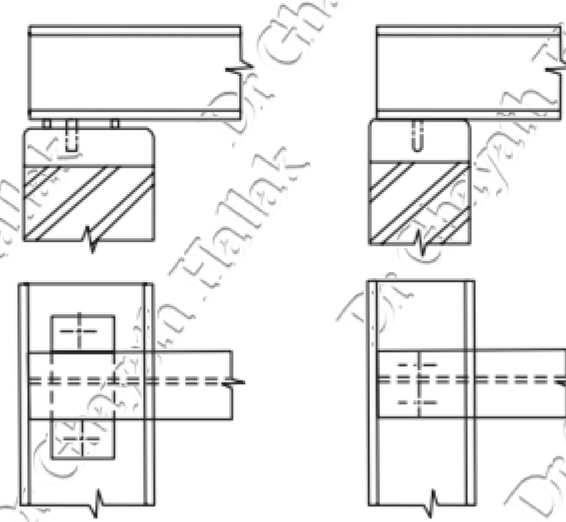
Full torsional end restraint to beam

(b) Bottom flange cleat



Beam compression flange free to rotate on plan

(c) Web cleats bolted to beam



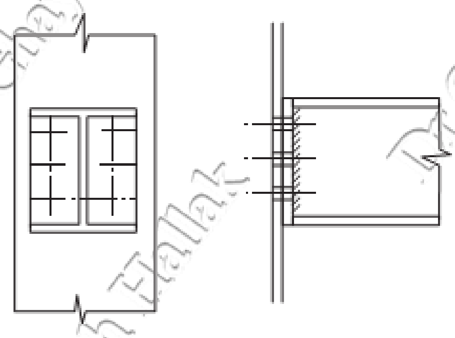
Compression flange free to rotate. No torsional restraint

(d) Flange bolted to padstone

LATERAL-TORSIONAL BUCKLING

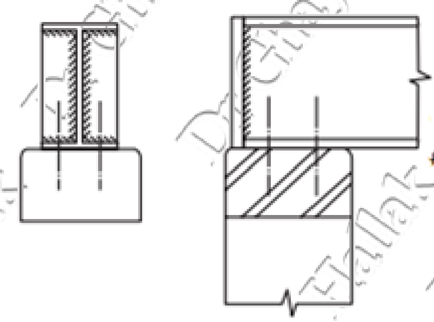
Calculation of M_{cr} Elastic critical moment

Method 3



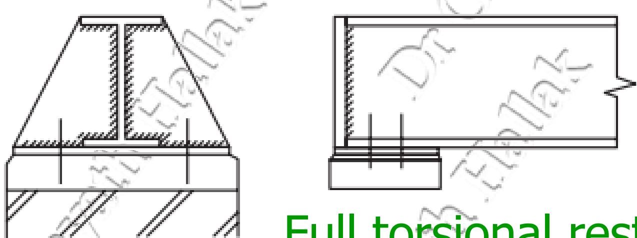
Full torsional end restraint to beam

(e) Full depth end plate welded to web and flange



Compression flange rotation reduced. Torsional end restraint

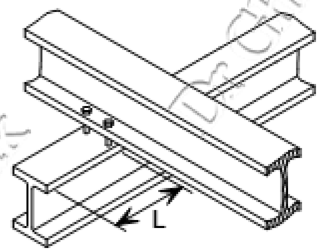
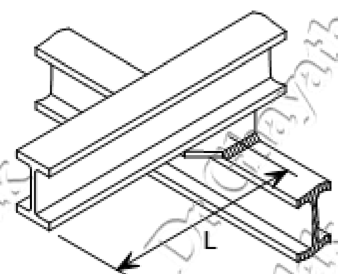
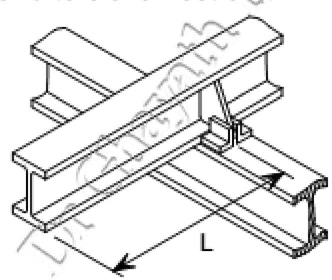
(f) Flange bolted to padstone, web stiffener



Full torsional restraint. No rotation of compression flange

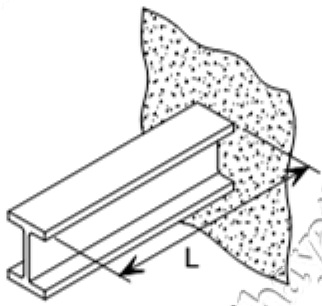
(g) Full end stiffener

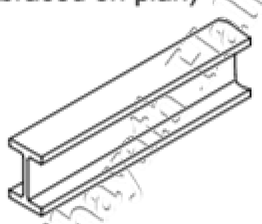
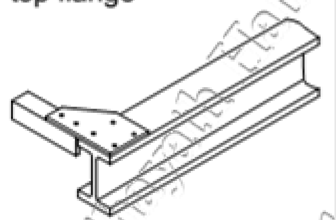
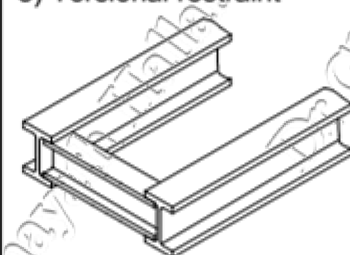
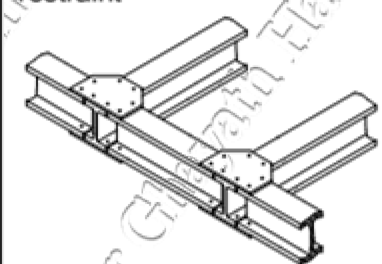
Effective length parameter k and destabilising D for cantilevers without intermediate restraint.

Restraint conditions		k	D
At support	At tip		
a) Continuous, with lateral restraint to top flange 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	3,0 2,7 2,4 2,1	2,5 2,8 1,9 1,7
b) Continuous, with partial torsional restraint 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	2,0 1,8 1,6 1,4	2,5 2,8 1,9 1,7
c) Continuous, with lateral and torsional restraint 	1) Free 2) Lateral restraint to top flange 3) Torsional restraint 4) Lateral and torsional restraint	1,0 0,9 0,8 0,7	2,5 2,8 1,9 1,7

LATERAL-TORSIONAL BUCKLING

Effective length parameter k and destabilising D for cantilevers without intermediate restraint.

Restraint conditions		k	D
At support	At tip		
d) Restrained laterally, torsionally and against rotation on plan 	1) Free	0,8	1,75
	2) Lateral restraint to top flange	0,7	2,0
	3) Torsional restraint	0,6	1,0
	4) Lateral and torsional restraint	0,5	1,0

Tip restraint conditions			
1) Free (not braced on plan) 	2) Lateral restraint to top flange  (braced on plan in at least one bay)	3) Torsional restraint  (not braced on plan)	4) Lateral and torsional restraint  (braced on plan in at least one bay)

LATERAL-TORSIONAL BUCKLING

Method 3

Calculation of M_{cr} Elastic critical moment

D is a destabilizing parameter to allow for destabilizing loads (i.e. Loads applied above the shear centre of the beam, where the load can move with the beam as it buckles), given by:

$$D = \frac{1}{\sqrt{1 - V^2 C_2 Z_g \sqrt{\frac{I_z}{I_w}}}}$$

For non-destabilizing loads, $D = 1.0$.
For destabilizing loads, $D = 1.2$ for simply supported beams. As shown in the previous Table.

In practice, destabilizing loads are only considered in cases for which the applied loading offers no resistance to lateral movement, e.g. a free standing brick wall on a beam. Normal loads from floors do not constitute a destabilizing load.

Design procedure for LTB

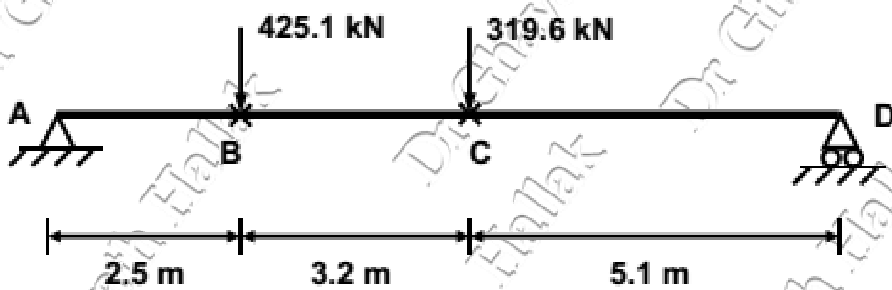
1. Determine BMD and SFD from design loads
2. Select section and determine geometry
3. Classify cross-section (Class 1, 2, 3 or 4)
4. Determine effective (buckling) length L_{cr} – depends on boundary conditions and load level
5. Calculate M_{cr} and $W_y f_y$
6. Non-dimensional slenderness $$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$
7. Determine imperfection factor α_{LT}
8. Calculate buckling reduction factor χ_{LT}
9. Design buckling resistance $$M_{b,Rd} = \chi_{LT} \frac{W_y f_y}{\gamma_{M1}}$$
10. Check for each unrestrained portion

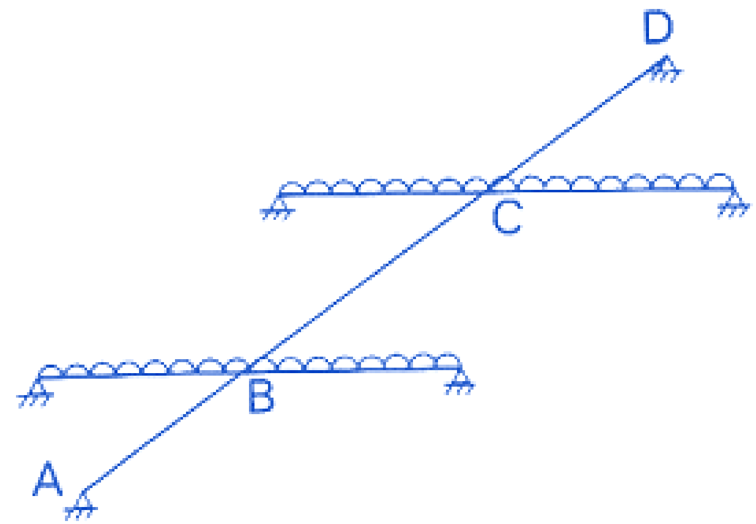
$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0$$

LTB Example

A simply-supported primary beam is required to span 10.8m and to support two secondary beams as shown below. The secondary beams are connected through fin plates to the web of the primary beam, and full lateral restraint may be assumed at these points. Select a suitable member for the primary beam assuming grade S 275 steel.

Design loading is as follows:





LTB Example

Lateral torsional buckling checks to be carried out on segments BC and CD. By inspection, segment AB is not critical.

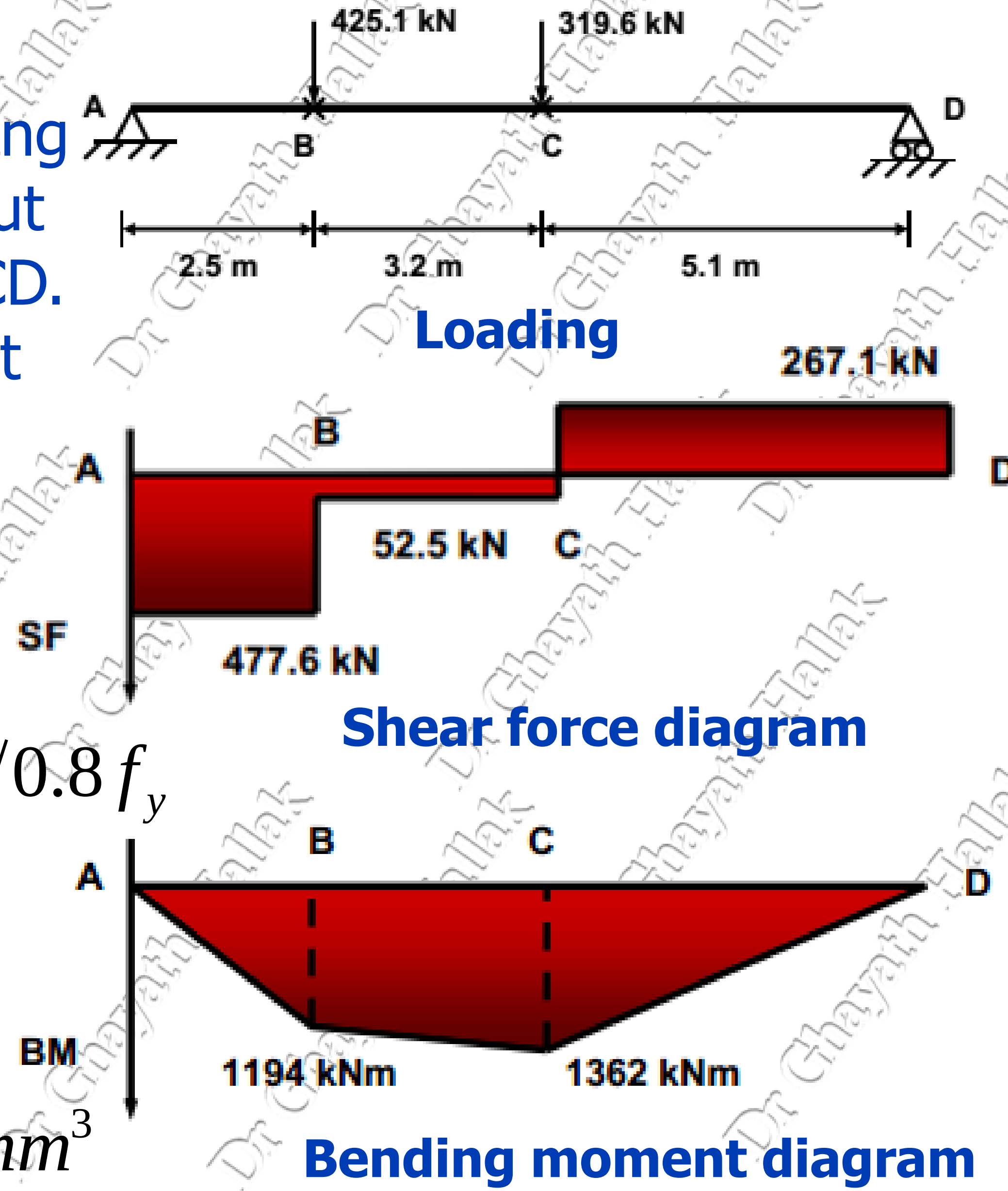
$$M_{c,Rd} = W_{y,pl} f_y / \gamma_{M0}$$

$$W_{y,pl} = M_{c,Rd} \gamma_{M0} / f_y$$

$$W_{y,pl,trial} = M_{c,Rd} \gamma_{M0} / 0.8 f_y$$

$$W_{y,pl,trial} = \frac{1362 \times 10^3}{(0.8 \times 265)}$$

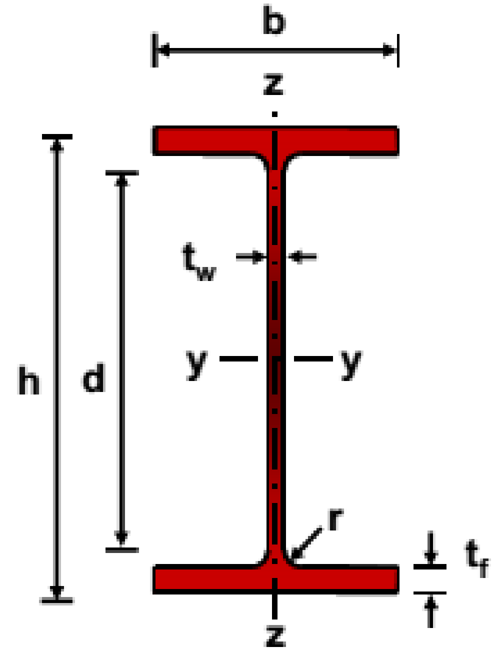
$$W_{y,pl,trial} = 6424.5E^3 mm^3$$



LTB Example

Try 762×267×173 UB in grade S 275 steel.

h = 762.2 mm , U=0.865
b = 266.7 mm , i_z=55.8mm
t_w = 14.3 mm
t_f = 21.6 mm
r = 16.5 mm
A = 22000 mm^2
$W_{y,pl}$ = 6200×10^3 mm^3
I_z = 68.50×10^6 mm^4
I_T = 2670×10^3 mm^4
I_w = 9390×10^9 mm^6



Steel Properties

40mm>t_f = 21.6 mm>16mm → For S275 (to EN 10025-2)
f_y = 265N/mm^2

LTB Example

Steel Properties

From clause 3.2.6: $E = 210000 N/mm^2$ and $G \approx 81000 N/mm^2$

Cross-section classification (clause 5.5.2):

$$\varepsilon = \sqrt{235/f_y} = \sqrt{235/265} = 0.94$$

Outstand flanges (Table 5.2, sheet 2)

$c_f = (b - t_w - 2r) / 2 = 109.7$ mm

$c_f / t_f = 109.7 / 21.6 = 5.08$

Limit for Class 1 flange = $9\varepsilon = 8.48 > 5.08$

∴ Flange is Class 1

Web – internal part in bending (Table 5.2, sheet 1)

$c_w = h - 2t_f - 2r = 686.0$ mm

$c_w / t_w = 686.0 / 14.3 = 48.0$

Limit for Class 1 web = $72\varepsilon = 67.8 > 48.0$

LTB Example

Cross-section classification (clause 5.5.2):

∴ Web is Class 1

Overall cross-section classification is therefore Class 1.

Bending resistance of cross-section (clause 6.2.5):

$M_{c.y,Rd}=M_{pl,\ Rd}=W_{pl.y}\ f_y/\gamma_{M0}$ for Class 1 and 2 sections

$M_{c.y,Rd}=6200\text{x}10^3\text{x}265/1.0=1643\text{x}10^6$ N.mm

$=1643$ kN.m >1362 kN.m

∴ Cross-section resistance in bending is OK.

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC:

$$M_{Ed} = 1362 kN.m$$

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} \quad ,W_y = W_{pl,y} \; For \; class \; 1\&2$$

Determine M_{cr} for segment BC (L_{cr} = 3200 mm)

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

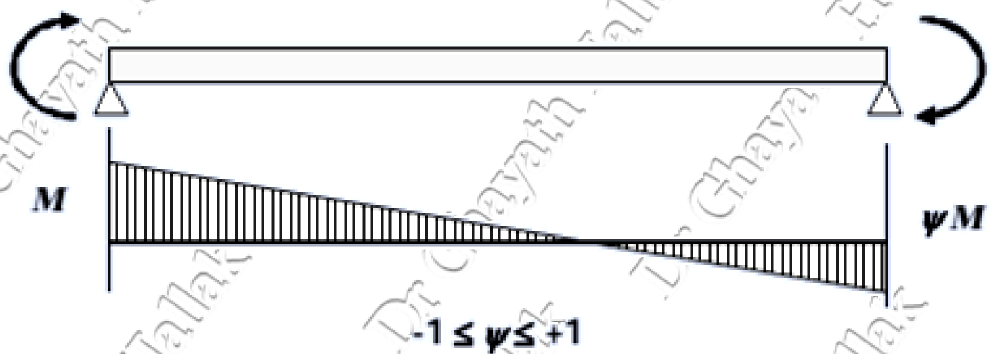
For end moment loading C1 may be approximated from:

C1 = 1.88 – 1.40ψ + 0.52ψ^2 but C1 ≤ 2.70

ψ is the ratio of the end moments =1194/1362=0.88⇒ C1= 1.05

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC:

End Moment Loading	ψ	$\frac{1}{\sqrt{C_1}}$	C_1
	+1.00	1.00	1.00
	+0.75	0.92	1.17
	+0.50	0.86	1.36
	+0.25	0.80	1.56
	0.00	0.75	1.77
	-0.25	0.71	2.00
	-0.50	0.67	2.24
	-0.75	0.63	2.49
	-1.00	0.60	2.76

According to the above table (by interpolation for ψ=0.88) C1= 1.08

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC:

According to the above table (by interpolation for ψ=0.88)

C_1= 1.08

$$M_{cr} = 1.05 \times \frac{\pi^2 \times 210000 \times 68.5 \times 10^6}{3200^2} \sqrt{\frac{9390 \times 10^9}{68.5 \times 10^6} + \frac{3200^2 \times 81000 \times 2670 \times 10^3}{\pi^2 \times 210000 \times 68.5 \times 10^6}}$$

$$M_{cr} = 5699 \times 10^6 \, N.mm = 5699 kN.m$$

Non-dimensional lateral torsional slenderness for segment BC:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{6200 \times 10^3 \times 265}{5699 \times 10^6}} = 0.54$$

Select buckling curve and imperfection factor α_{LT}:

From Table 6.4: h/b = 762.2/266.7 = 2.85

For a rolled I-section with h/b > 2, use buckling curve b

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC:

From Table 6.3 of EN 1993-1-1: For buckling curve b, $\alpha_{LT}=0.34$

Calculate reduction factor for lateral torsional buckling, χ_{LT} – Segment BC:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5 \times \left[1 + 0.34 \times \left(0.54 - 0.2\right) + 0.54^2\right]$$

$$\phi_{LT} = 0.70$$

$$\therefore \chi_{LT} = \frac{1}{0.7 + \sqrt{0.7^2 - 0.54^2}} = 0.87 \langle 1.0$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC:

Lateral torsional buckling resistance $M_{b,Rd}$ – Segment BC:

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} = 0.87 \times 6200 \times 10^3 \times \frac{265}{1.0} = 1429 \times 10^6 \, N.mm$$

$$M_{b,Rd} = 1429 kN.m$$

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{1362}{1429} = 0.95 \leq 1.0 \therefore Segment\ BC\ is\ OK.$$

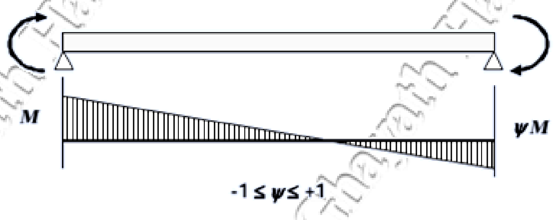
LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

Lateral torsional buckling resistance $M_{b,Rd}$ – Segment BC:

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD\bar{\lambda}_z \sqrt{\beta_w}$$

ψ is the ratio of the end moments =1194/1362=0.88



End Moment Loading	ψ	$\frac{1}{\sqrt{C_1}}$	C_1
	+1.00	1.00	1.00
	+0.75	0.92	1.17
	+0.50	0.86	1.36
	+0.25	0.80	1.56
	0.00	0.75	1.77
	-0.25	0.71	2.00
	-0.50	0.67	2.24
	-0.75	0.63	2.49
	-1.00	0.60	2.76

$\Rightarrow \frac{1}{\sqrt{C_1}} = 0.93$, U=0.865, i_z=55.8mm , D=1.0 (Normal), β_w =1

$$\lambda_z = kL/i_z = 1 \times 3200/55.8 = 57.35$$

$$\bar{\lambda}_z = \lambda_z / \lambda_1 = 57.35/(93.9 \times 0.92) = 0.66$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

$$V = \frac{1}{\sqrt[4]{1+\frac{1}{20}\left(\frac{\lambda_z}{h/t_f}\right)^2}} = \frac{1}{\sqrt[4]{1+\frac{1}{20}\left(\frac{57.35}{762.2/21.6}\right)^2}} = 0.97$$

$$\bar{\lambda}_{LT} = 0.93 \times 0.865 \times 0.97 \times 1.0 \times 0.66 \times 1.0 = 0.52$$

Select buckling curve and imperfection factor α_{LT}:

From Table 6.4: h/b = 762.2/266.7 = 2.85

For a rolled I-section with h/b > 2, use buckling curve b

From Table 6.3 of EN 1993-1-1: For buckling curve b, α_{LT}=0.34

Calculate reduction factor for lateral torsional buckling, χ_{LT} – Segment BC:

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5 \times \left[1 + 0.34 \times (0.52 - 0.2) + 0.52^2\right]$$

$$\phi_{LT} = 0.69$$

$$\therefore \chi_{LT} = \frac{1}{0.69 + \sqrt{0.69^2 - 0.52^2}} = 0.87 \langle 1.0$$

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} = 0.87 \times 6200 \times 10^3 \times \frac{265}{1.0} = 1429 \times 10^6\, N.mm$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{1362}{1429} = 0.95 \le 1.0 \therefore Segment\ BC\ is\ OK.$$

Using the second method: Rolled sections and equivalent welded sections. Clause 6.3.2.3 of BS EN 1993-1-1

From NA.2.17- NA to BS EN 1993-1-1:2005:
3.1>h/b = 762.2/266.7 = 2.85>2, use buckling curve C

From Table 6.3 of EN 1993-1-1: For buckling curve C, α_{LT}=0.49

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}^{2}\right]$$

$$\phi_{LT} = 0.5 \times \left[1 + 0.49(0.52 - 0.4) + 0.75 \times 0.52^2\right] = 0.63$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} = \frac{1}{0.63 + \sqrt{0.63^2 - 0.75 \times 0.52^2}}$$

$$\chi_{LT} = 0.934 \, but \begin{cases} \chi_{LT} \le 1.0 \; OK \\ \chi_{LT} \le \dfrac{1}{\bar{\lambda}_{LT}^2} = 3.7 \quad OK \end{cases}$$

$$f = 1 - 0.5\left(1 - K_c\right)\left[1 - 2.0\left(\bar{\lambda}_{LT} - 0.8\right)^2\right]$$

$$f = 1 - 0.5 \times (1 - 0.93)[1 - 2(0.52 - 0.8)] = 0.95 \le 1.0$$

$$\chi_{LT,\mathrm{mod}} = \frac{\chi_{LT}}{f} = \frac{0.934}{0.95} = 0.983 \le 1.0$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment BC: using Method 3

$$M_{b,Rd} = \chi_{Lt,mod} \frac{W_y f_y}{\gamma_{M1}} = 0.983 \times 6200 \times 10^3 \times \frac{265}{1.0} = 1615\, kN.m$$

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{1362}{1615} = 0.84 \leq 1.0 \therefore Segment\ BC\ is\ OK.$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment CD:

$$M_{Ed} = 1362 kN.m$$

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} \quad , W_y = W_{pl,y} \; For \; class \; 1\&2$$

Determine M_{cr} for segment CD (L_{cr} = 5100 mm)

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

For end moment loading C1 may be approximated from:

C1 = 1.88 – 1.40ψ + 0.52ψ^2 but C1 ≤ 2.70

ψ is the ratio of the end moments =1194/1362=0.88⇒ C1= 1.05

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment CD:

Determine ψ from Table:

ψ is the ratio of the end moments =0/1362=0 ⇒

C1= 1.88

$$M_{cr} = 1.88 \times \frac{\pi^2 \times 210000 \times 68.5 \times 10^6}{5100^2} \sqrt{\frac{9390 \times 10^9}{68.5 \times 10^6} + \frac{5100^2 \times 81000 \times 2670 \times 10^3}{\pi^2 \times 210000 \times 68.5 \times 10^6}}$$

$$M_{cr} = 4311 \times 10^6 \, N.mm = 4311 kN.m$$

Non-dimensional lateral torsional slenderness for segment CD:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{6200 \times 10^3 \times 265}{4311 \times 10^6}} = 0.62$$

The buckling curve and imperfection factor α_{LT} are as for segment BC.

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment CD:

Calculate reduction factor for lateral torsional buckling, χ_{LT} – Segment CD:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right] = 0.5 \times \left[1 + 0.62 \times \left(0.62 - 0.2\right) + 0.62^2\right]$$

$$\phi_{LT} = 0.76$$

$$\therefore \chi_{LT} = \frac{1}{0.76 + \sqrt{0.76^2 - 0.62^2}} = 0.83 \langle 1.0$$

LTB Example

Lateral torsional buckling check (clause 6.3.2.2) – Segment CD:

Lateral torsional buckling resistance $M_{b,Rd}$ – Segment CD:

$$M_{b,Rd} = \chi_{Lt} \frac{W_y f_y}{\gamma_{M1}} = 0.83 \times 6200 \times 10^3 \times \frac{265}{1.0} = 1363.7 \times 10^6 \, N.mm$$

$$M_{b,Rd} = 1363.7 kN.m$$

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{1362}{1363.7} = 1.0$$

Segment CD is critical and marginally fails LTB check.