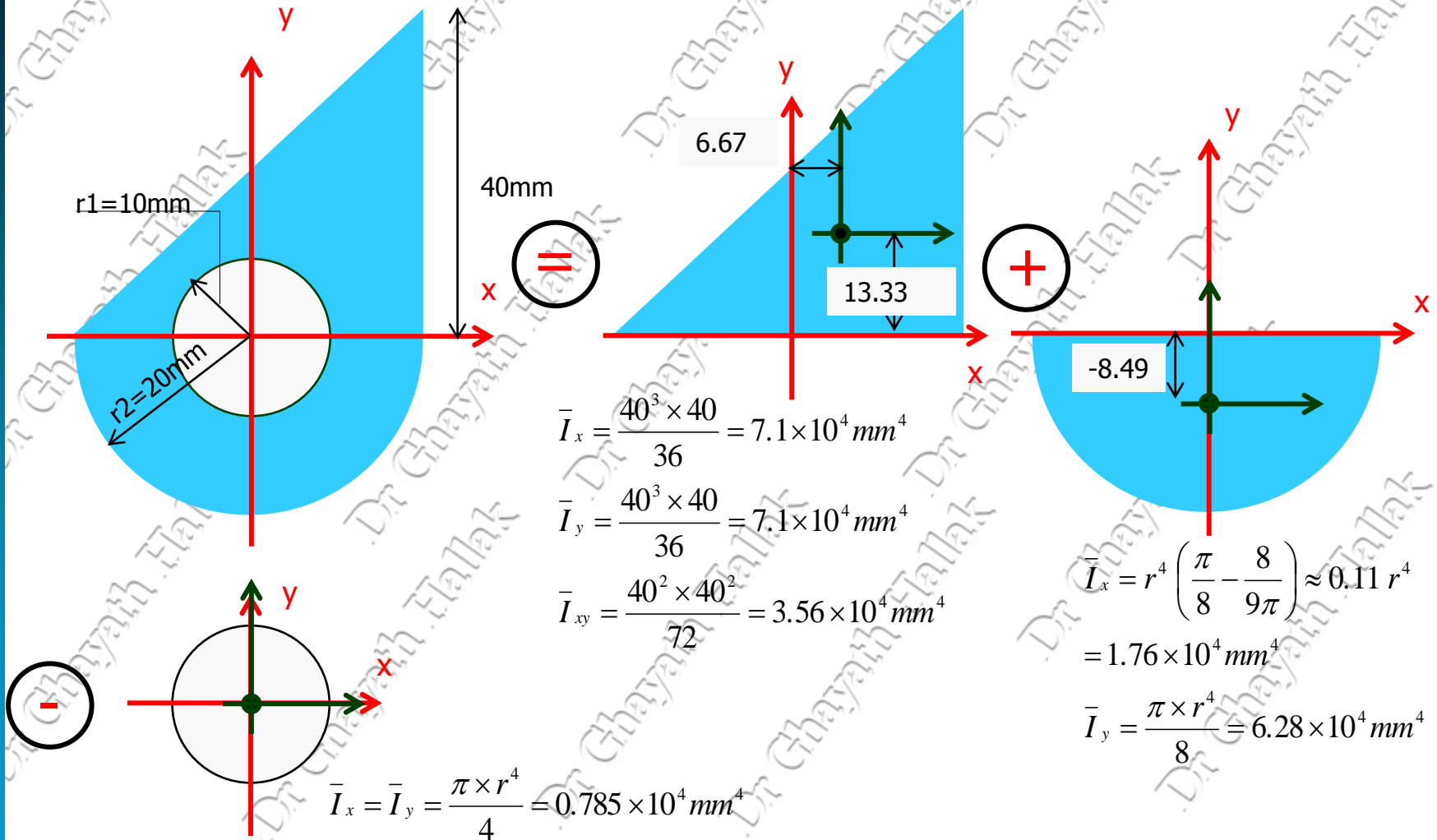
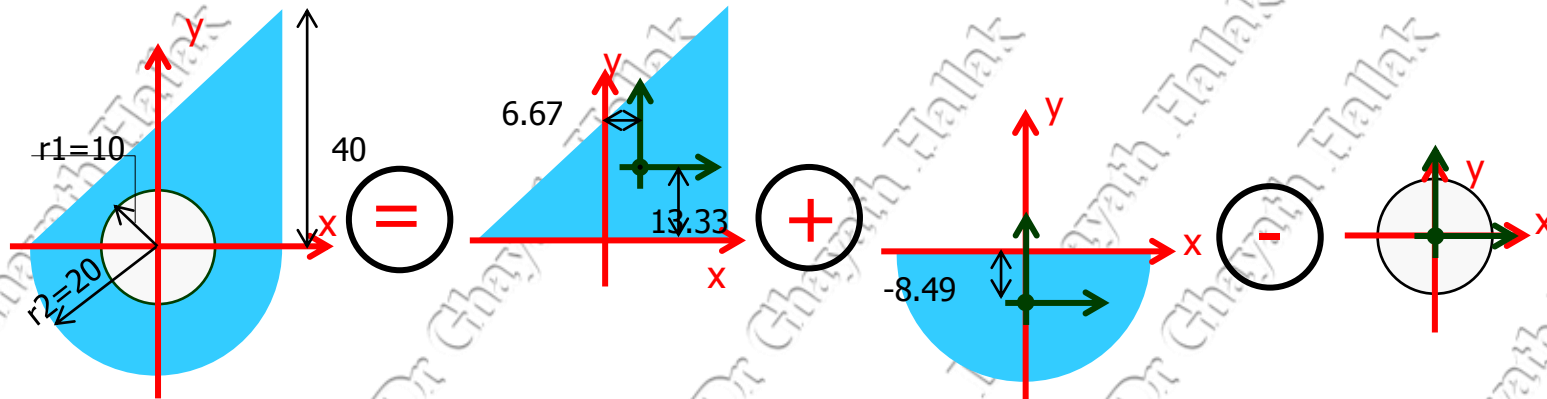


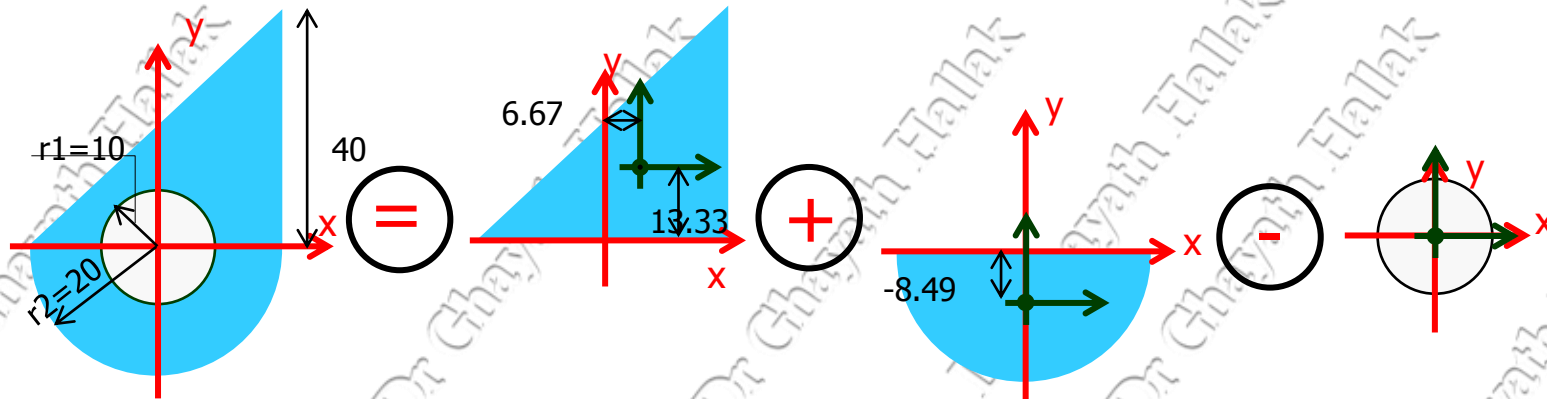
مثال : اوجد عزوم العطالة وجداء العطالة للمحاور المركزية للمحاور المفروضة . وكذلك اوجد عزوم العطالة وجداء العطالة للمحاور المركزية .





Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ x10 ³ mm ³	$A_i \cdot y_i$ X10 ³ mm ³	\bar{I}_x x10 ⁴ mm ⁴	\bar{I}_y x10 ⁴ mm ⁴	$A_i \cdot x_i^2$ x10 ⁴ mm ⁴	$A_i \cdot y_i^2$ x10 ⁴ mm ⁴	\bar{I}_{xy} mm ⁴	$A_i \cdot x_i \cdot y_i$ x10 ⁴ mm ⁴
triang	800	6.67	13.33	5.336	10.66	7.1	7.1	3.56	14.22	3.56	7.113
Semi-cir	628	0	-8.49	0	-5.331	1.76	6.28	0	4.526	0	0
circle	-314	0	0	0	0	0.785	0.785	0	0	0	0
Σ	1111			5.336	5.329	8.075	12.595	3.56	18.746	3.56	7.113

$$X_G = \frac{\sum A_i \times x_i}{\sum A_i} = \frac{5.336 \times 10^3}{1111} = 4.8 \text{ mm} \quad , \quad Y_G = \frac{\sum A_i \times y_i}{\sum A_i} = \frac{5.329 \times 10^3}{1111} = 4.8 \text{ mm}$$



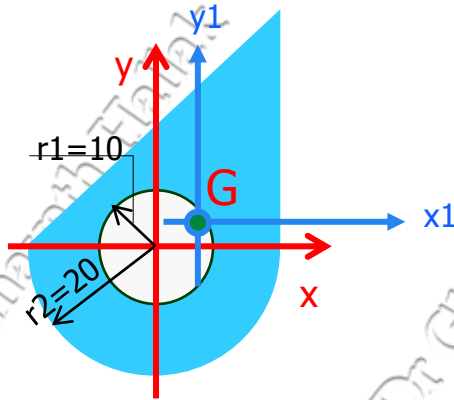
Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	\bar{I}_x $\times 10^4$ mm ⁴	\bar{I}_y $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴	\bar{I}_{xy} mm ⁴	$A_i \cdot x_i \cdot y_i$ $\times 10^4$ mm ⁴
triang	800	6.67	13.33	5.336	10.66	7.1	7.1	3.56	14.22	3.56	7.113
Semi-cir	628	0	-8.49	0	-5.331	1.76	6.28	0	4.526	0	0
circle	-314	0	0	0	0	0.785	0.785	0	0	0	0
Σ	1111			5.336	5.329	8.075	12.595	3.56	18.746	3.56	7.113

$$I_x = \sum \bar{I}_x + \sum A_i \times y_i^2 = 8.075 \times 10^4 + 18.746 \times 10^4 = 26.821 \times 10^4 \text{ mm}^4$$

$$I_y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 12.595 \times 10^4 + 3.56 \times 10^4 = 16.155 \times 10^4 \text{ mm}^4 ,$$

$$I_{xy} = \sum \bar{I}_{xy} + \sum A_i \times x_i \times y_i = 3.56 \times 10^4 + 7.113 \times 10^4 = 10.673 \times 10^4 \text{ mm}^4$$

المحاور المركزية



$$I_X = \bar{I}_{x1} + (\sum A_i) \times Y_G^2 \Rightarrow \bar{I}_{x1} = I_X - (\sum A_i) \times Y_G^2 = 26.821 \times 10^4 - (1111) \times (-4.8)^2 = 24.26 \times 10^4 \text{ mm}^4$$

$$I_Y = \bar{I}_{y1} + (\sum A_i) \times X_G^2 \Rightarrow \bar{I}_{y1} = I_Y - (\sum A_i) \times X_G^2 = 16.155 \times 10^4 - (1111) \times (-4.8)^2 = 13.595 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \bar{I}_{x1y1} + (\sum A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x1y1} = I_{XY} - (\sum A_i) \times X_G \times Y_G = 10.673 \times 10^4 - (1111) \times (-4.8) \times (-4.8)$$

$$\bar{I}_{x1y1} = 8.11 \times 10^4 \text{ mm}^4$$

المحاور الرئيسية

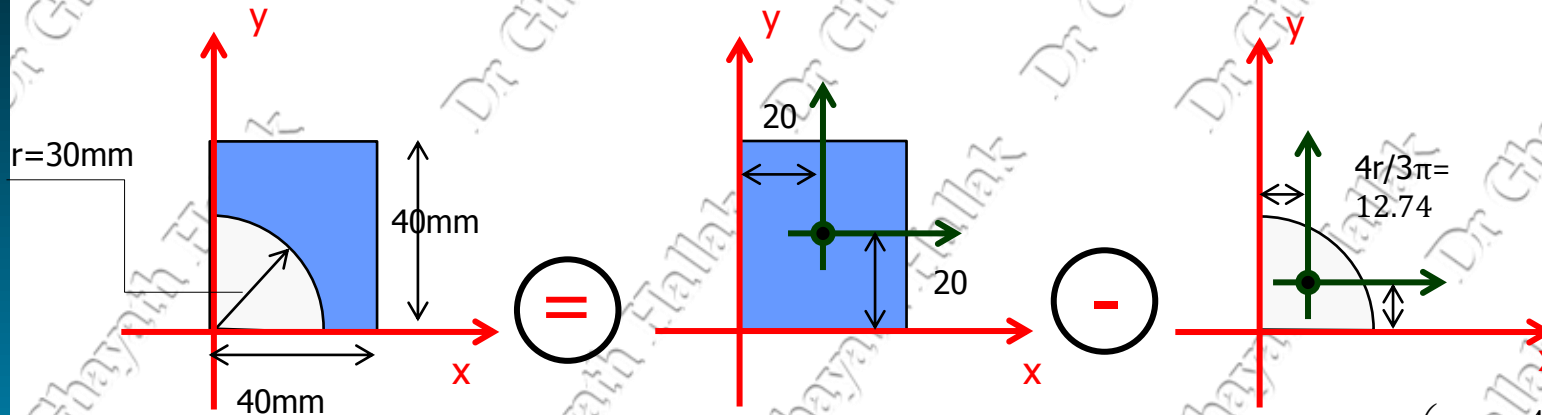
$$\tan 2\phi_p = \frac{2\bar{I}_{x1y1}}{\bar{I}_{y1} - \bar{I}_{x1}} = \frac{2(8.11 \times 10^4)}{(13.595 - 24.26) \times 10^4} = -1.52 \Rightarrow \phi_p = -28.34 \text{ degree}$$

$$I_{xp} = \bar{I}_{x1} \cos^2 \phi_p + \bar{I}_{y1} \sin^2 \phi_p - \bar{I}_{x1y1} \sin 2\phi_p = 28.634 \times 10^4 \text{ mm}^4$$

$$I_{yp} = \bar{I}_{y1} \cos^2 \phi_p + \bar{I}_{x1} \sin^2 \phi_p + \bar{I}_{x1y1} \sin 2\phi_p = 9.221 \times 10^4 \text{ mm}^4$$

$$I_{max,min} = \frac{\bar{I}_{x1} + \bar{I}_{y1}}{2} \pm \frac{1}{2} \sqrt{(\bar{I}_{x1} - \bar{I}_{y1})^2 + 4\bar{I}_{x1y1}^2} = \{28.634 \times 10^4, 9.221 \times 10^4\} \text{ mm}^4$$

مثال : اوجد عزوم العطالة وجداء العطالة للمحاور المركزية للمحاور المفروضة . وكذلك اوجد عزوم العطالة وجداء العطالة للمحاور المركزية .



$$\bar{I}_x = \frac{40^3 \times 40}{12} = 21.33 \times 10^4 \text{ mm}^4$$

$$\bar{I}_y = \frac{40^3 \times 40}{12} = 21.33 \times 10^4 \text{ mm}^4$$

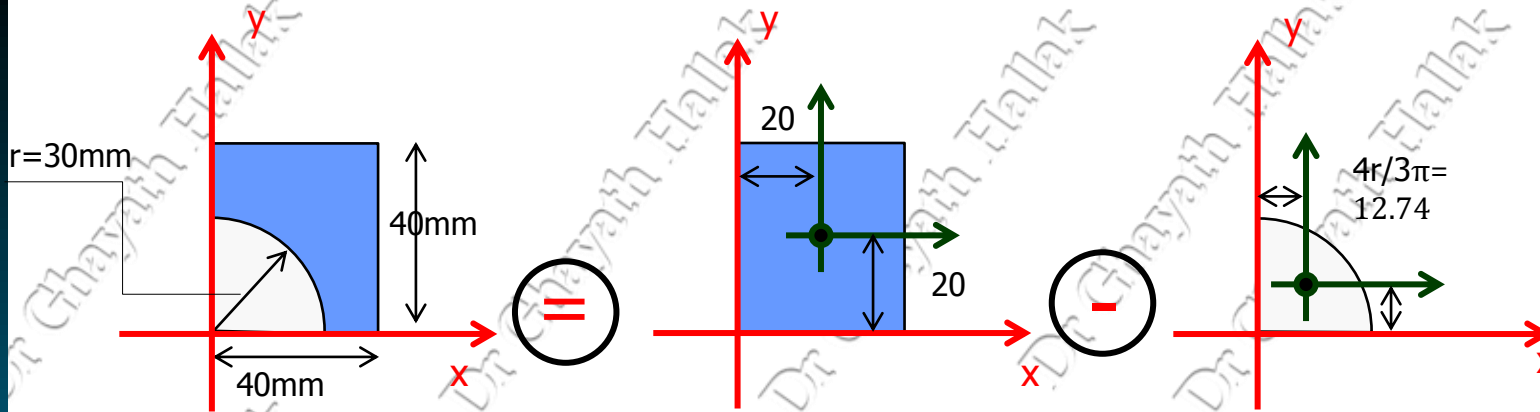
$$\bar{I}_{xy} = 0 \text{ mm}^4$$

$$\bar{I}_x = \bar{I}_y = r^4 \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) \approx 0.055 r^4$$

$$= 4.455 \times 10^4 \text{ mm}^4$$

$$\bar{I}_{xy} = r^4 \left(\frac{1}{8} - \frac{4}{9\pi} \right) = -0.0165 r^4$$

$$\bar{I}_{xy} = -1.337 \times 10^4 \text{ mm}^4$$



Seg	A_i mm ²	X_i mm	Y_i mm	$A_i \cdot x_i$ $\times 10^3$ mm ³	$A_i \cdot y_i$ $\times 10^3$ mm ³	\bar{I}_x $\times 10^4$ mm ⁴	\bar{I}_y $\times 10^4$ mm ⁴	$A_i \cdot x_i^2$ $\times 10^4$ mm ⁴	$A_i \cdot y_i^2$ $\times 10^4$ mm ⁴	\bar{I}_{xy} mm ⁴	$A_i \cdot x_i \cdot y_i$ $\times 10^4$ mm ⁴
Squar	1600	20	20	32	32	21.33	21.33	64	64	0	64
Quart er-cir	706.5	12.74	12.74	9	9	4.455	4.455	11.47	11.47	-1.34	11.47
Σ	893.5			23	23	16.875	16.875	52.53	52.53	+1.34	52.53

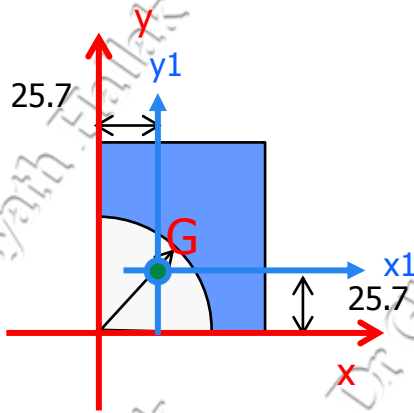
$$X_G = \frac{\sum A_i \times x_i}{\sum A_i} = \frac{23 \times 10^3}{893.5} = 25.7 \text{ mm} \quad , \quad Y_G = \frac{\sum A_i \times y_i}{\sum A_i} = \frac{23 \times 10^3}{893.5} = 25.7 \text{ mm}$$

$$I_X = \sum \bar{I}_x + \sum A_i \times y_i^2 = 16.875 \times 10^4 + 52.53 \times 10^4 = 69.405 \times 10^4 \text{ mm}^4$$

$$I_Y = \sum \bar{I}_y + \sum A_i \times x_i^2 = 16.875 \times 10^4 + 52.53 \times 10^4 = 69.405 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \sum \bar{I}_{xy} + \sum A_i \times x_i \times y_i = 1.34 \times 10^4 + 52.53 \times 10^4 = 53.87 \times 10^4 \text{ mm}^4$$

المحاور المركزية



$$I_X = \bar{I}_{x1} + (\sum A_i) \times Y_G^2 \Rightarrow \bar{I}_{x1} = I_X - (\sum A_i) \times Y_G^2 = 69.405 \times 10^4 - (893.5) \times (-25.7)^2 = 10.39 \times 10^4 \text{ mm}^4$$

$$I_Y = \bar{I}_{y1} + (\sum A_i) \times X_G^2 \Rightarrow \bar{I}_{y1} = I_Y - (\sum A_i) \times X_G^2 = 69.405 \times 10^4 - (893.5) \times (-25.7)^2 = 10.39 \times 10^4 \text{ mm}^4$$

$$I_{XY} = \bar{I}_{x1y1} + (\sum A_i) \times X_G \times Y_G \Rightarrow \bar{I}_{x1y1} = I_{XY} - (\sum A_i) \times X_G \times Y_G = 53.87 \times 10^4 - (893.5) \times (-25.7) \times (-25.7)$$

$$\bar{I}_{x1y1} = -5.145 \times 10^4 \text{ mm}^4$$

المحاور الرئيسية

$$\tan 2\phi_p = \frac{2\bar{I}_{x1y1}}{\bar{I}_{y1} - \bar{I}_{x1}} = \frac{2(-5.145 \times 10^4)}{(10.39 - 10.39) \times 10^4} = -\infty \Rightarrow \phi_p = -45 \text{ degree}$$

$$I_{xp} = \bar{I}_{x1} \cos^2 \phi_p + \bar{I}_{y1} \sin^2 \phi_p - \bar{I}_{x1y1} \sin 2\phi_p = 5.245 \times 10^4 \text{ mm}^4$$

$$I_{yp} = \bar{I}_{y1} \cos^2 \phi_p + \bar{I}_{x1} \sin^2 \phi_p + \bar{I}_{x1y1} \sin 2\phi_p = 15.535 \times 10^4 \text{ mm}^4$$

$$I_{max,min} = \frac{\bar{I}_{x1} + \bar{I}_{y1}}{2} \pm \frac{1}{2} \sqrt{(\bar{I}_{x1} - \bar{I}_{y1})^2 + 4\bar{I}_{x1y1}^2} = \{15.535 \times 10^4, 5.245 \times 10^4\} \text{ mm}^4$$