A Novel Model for the Input Impedance of Coax-Fed Circular Microstrip Patch Antennas for CAD

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Abstract

We present a Novel model for the input impedance of a circular microstrip patch antenna. This model is well suited for computer aided design (CAD). It is based on classical methods: (a) the cavity model determining the frequency and the input resistance at resonance, (b) a new effective dielectric constant ($\varepsilon_{r,eff}$) of a circular microstrip patch antenna (to take into account the influence of the fringing field at the edges of the circular patch antenna) and (c) the resonant parallel RLC circuit with an inductive reactance. This model is valid for electrically thin and thick substrate covering the entire range of dielectric constants. The theoretical results are in good agreement with the experimental data.

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1- Introduction :

Rigorous methods developed by a number of authors[1-3] enable computation of the input impedance and the radiation pattern of circular microstrip patch antenna with good precision.

However, these methods involve complex analysis of physical phenomena, which often take considerable computation time and do not easily yield the equivalent circuit. Sometimes, less accurate results suffice, and can be obtained much faster with the help of simpler methods such as the cavity model[4-6], which adequately describe the resonant frequency, input impedance, bandwidth and radiation pattern by a simple design equation.

An analytical expression is given here for the input impedance of a circular microstrip patch antenna excited by a coaxial probe (Fig. 1a) using the cavity model and the equivalent resonant circuits. It shows explicitly the dependence of the input impedance on the characteristic parameters of a patch antenna, and is valid for electrically thin and thick substrates.

2- Analysis :

The cavity model is a bidimensional model that can be applied to patches whose geometries are specified simply by curvilinear orthogonal co-ordinate systems(circular, rectangular, etc..). It is possible to consider either the dominant mode or the complete spectrum of modes.

The circular microstrip patch antenna can be considered in the fundamental mode, modeled by a simple resonant parallel RLC circuit (Fig.1b). To take the coax-feed probe into account, it is necessary to modify the input impedance by an inductive term[7] :

$$X_L = \frac{377 \cdot f \cdot H}{c_0} Ln\left(\frac{c_0}{\pi \cdot f \cdot d_0 \sqrt{\varepsilon_r}}\right) \qquad (1)$$

Where c_o is the velocity of light in vacuum and d_o is the diameter of the probe. All expression presented hereafter are valid for the dominant mode. The input impedance is then obtained as[4] :

$$Z(f)=\frac{R(\rho)}{1+Q_T^2\left[\frac{f}{f_R}-\frac{f_R}{f}\right]^2}+J\left[X_L-\frac{R(\rho)\cdot Q_T\cdot\left[\frac{f}{f_R}-\frac{f_R}{f}\right]}{1+Q_T^2\left[\frac{f}{f_R}-\frac{f_R}{f}\right]^2}\right] \quad (2)$$

Where (Q_T) is the quality factor associated with system losses and R(ρ) is the input resistance at the resonance (the resistance R of the R – L - C circuit). Q_T includes radiation losses Q_R, dielectric losses Q_D and conductor losses Q_C[4] :

$$\frac{1}{Q_T}=\left[\frac{1}{Q_R}+\frac{1}{Q_C}+\frac{1}{Q_D}\right]^{-1} \quad (3)$$

1- Q_R has the following form from the expression by Bahl [8,eq.3-27, eq.3-44] :

$$Q_R=\frac{4\cdot a\cdot(\alpha_{11}^2-1)\cdot\varepsilon_r^{3/2}}{H\cdot\alpha_{11}^3\cdot F(\alpha_{11}/\sqrt{\varepsilon_r})} \quad (4)$$

Where $F(X)$ is given by the following expression with the help of the development of Bessel function J_0[9] :

$$F(X)=\frac{4}{X^3}\left\{2\cdot X\cdot J_0(2X)+(X^2-1)\int_0^{2X}J_0(t)dt\right\} \quad (5)$$

$$F(X)=\left[\begin{array}{l}2.666667378-1.066662519X^2+0.209534311X^4\\-0.019411347X^6+0.001044121X^8-0.000049747X^{10}\end{array}\right] \quad (6)$$

2 – The dielectric losses have been obtained by[10] :

$$Q_D=1/(\tan\delta) \quad (7)$$

Where (tanδ) is the dielectric loss tangent.

3 – The losses in the conductor can be obtained by[10-11] :

$$Q_c=H/\delta_s,\delta_s=(\pi\cdot f\cdot\mu_0\cdot\sigma)^{-1/2} \quad (8)$$

δ_s is the skin depth where σ is the conductivity of the conductor and μ_0 is the permeability of the dielectric. $R(\rho)$ is the resonant resistance of the resonant parallel *R-L-C* circuit[12,eq.10] which is given by :

$$R(\rho)=\frac{1}{G}\cdot\frac{J_1^2(K\cdot\rho)}{J_1^2(K\cdot a)} \quad (9)$$

Where ρ is the feed position referred to the center of the disc of radius *a*. *K* is the propagation constant. The fundamental mode corresponds to *Ka* equal to α_{11} as 1.84118 and G_T includes the conductances due to ohmic, dielectric and radiation losses :

$$G_T = G_R + G_D + G_C \tag{10}$$

1 – The conductance due to radiation losses is given by[11] :

$$G_R = \frac{2.39}{4 \cdot \mu_0 \cdot H \cdot f_R \cdot Q_R} \tag{11}$$

2 – The conductance due to dielectric losses is given by[12,eq.8] :

$$G_D = \frac{2.39 \cdot \tan\delta}{4 \cdot \mu_0 \cdot f_R \cdot H} \tag{12}$$

3 – The conductance due to ohmic losses is given by[12,eq.9] :

$$G_C = \frac{2.39 \cdot \pi \cdot (\pi \cdot f_R \cdot \mu_0)^{-3/2}}{4 \cdot H^2 \cdot \sqrt{\sigma}} \tag{13}$$

The Bessel function of order one, J_1 , is expanded in terms of polynomial[9] , for $-3 < t < 3$:

$$J_1(t) = \begin{bmatrix} t(0.5 - 0.56249985(t/3)^2 + 0.21093573(t/3)^4 - 0.03954289(t/3)^6 \\ + 0.00443319(t/3)^8 - 0.0031761(t/3)^{10}) \end{bmatrix} \tag{14}$$

An improved formulation is derived for the resonant frequency[13] of the TM modes in a circular microstrip antenna given by :

$$f_{r,nm} = \frac{\alpha_{nm} \cdot c_0}{2\pi \cdot a_{eff} \cdot \sqrt{\varepsilon_{r,eff}}} \tag{15}$$

Where α_{nm} is the mth zero of the derivative of the Bessel function of order *n* , the dominant mode is the TM_{11} (n = m =1). For this mode, $\alpha_{11} = 1.84118$. c_0 is the velocity of light in vacuum, a_{eff} is the effective radius of circular patch defined through (16), and $\varepsilon_{r,eff}$ is defined as :

$$\varepsilon_{r,eff} = \frac{4\varepsilon_r \cdot \varepsilon_{r,dyn}}{(\sqrt{\varepsilon_r} + \sqrt{\varepsilon_{r,dyn}})^2} \tag{16}$$

The term $\varepsilon_{r,eff}$ is introduced to take into account the effect of ε_r in combination with the dynamic dielectric constant $\varepsilon_{r,dyn}$ [14] to improve the model . $\varepsilon_{r,eff}$ is deduced as (16) to yield the resonant frequency as an average of the frequencies resulting from (15) by substituting ε_r and $\varepsilon_{r,dyn}$ separately in place of $\varepsilon_{r,eff}$.

$\varepsilon_{r,dyn}$ is a function of the static main and static fringing capacitances and the mode of resonance as given by[14] :

$$\varepsilon_{r,dyn} = \frac{C_{dyn}\left(\varepsilon = \varepsilon_0 \varepsilon_r\right)}{C_{dyn}\left(\varepsilon = \varepsilon_0\right)} \qquad (17)$$

$C_{dyn}(\varepsilon)$ can be written[14,eq.16] as:

$$C_{dyn}(\varepsilon) = C_{0,dyn}(\varepsilon) + C_{e,dyn}(\varepsilon) \qquad (18)$$

$C_{0,dyn}(\varepsilon)$ is the dynamic main capacitance of the dominant mode TM_{11} related to the static main capacitance $C_{0,stat}$ of the patch without considering the fringing field. It is given by[14,eq.14] :

$$C_{0,dyn}(\varepsilon) = 0.3525 \cdot C_{0,stat}(\varepsilon) \qquad (19)$$

$C_{e,dyn}(\varepsilon)$ represents the dynamic fringing capacitance of the dominant mode given by[14,eq.15] :

$$C_{e,dyn}(\varepsilon) = \frac{1}{2} C_{e,stat}(\varepsilon) \qquad (20)$$

A comparatively recent formulation for the static capacitance of a circular microstrip disc obtained by Wheeler[15] is applied to calculate $C_{0,stat}$ and $C_{e,stat}$ since the result in [15] is much improved over the earlier ones [14], [16], [17], [18] and is widely applicable to the entire range of dielectric constants and to all H/a values of the antenna. The expression of the capacitance given by Wheeler[10,eq.15] can be more explicitly written as:

$$C = \frac{\varepsilon_0 \varepsilon \, \pi \quad a^2}{H_T}(1+q) \qquad (21)$$

Where:

$$q = u + v + uv \quad (22)$$

$$u = \frac{1+\varepsilon_r}{\varepsilon_r} \cdot \frac{4}{\pi \, a/H} \quad (23)$$

$$v = \frac{2}{3t} \cdot \frac{\ln(p)}{8 + \pi \, a/H} + \frac{1/t - 1}{g} \quad (24)$$

$$t = 0.37 + 0.63\varepsilon_r \quad (25)$$

$$P = \frac{1 + 0.8(a/H)^2 + (0.31a/H)^4}{1 + 0.9\,a/H} \quad (26)$$

$$g = 4 + 2.6\,a/H + 2.9\,H/a \quad (27)$$

In (21),the first term is equal to the static main capacitance $C_{0,stat}$ and the term q arises due to the fringing field at the edge of the disc capacitor. The fringing capacitance $C_{e,stat}$ thus is defined as:

$$C_{e,stat} = C_{0,stat}(\varepsilon) \cdot q \quad (28)$$

Where:

$$C_{0,stat}(\varepsilon) = \frac{\varepsilon_0 \varepsilon_r \pi \cdot a^2}{H} \quad (29)$$

Equation(7) also defines the effective radius of the microstrip disc

$$a_{eff} = a \cdot \sqrt{(1+q)} \quad (30)$$

Results:

In this section computations concerning the fundamental mode (m = 1,n = 1) are presented and compared with measurements and previous computations. table I detail the experimental results for resonance frequency. In table II We present computed values of the resonant frequency for thin and thick substrates. We observe that our results fall among those calculated by other methods, and are closer to measured values in most cases.

The agreement between our theoretical and experimental values of the resonant frequency is typically within 0.5 % for values of electrical thickness ($H/\lambda \approx 0.1$).

Fig.2 shows the input impedance for a thinner patch substrate operating at about 1.90GHz. The results computed by Babu[24] and the measurements of Kumprasert[25] are reported.

Fig.3 presents the input impedance for a thicker substrate operating about 3.60GHz. Our computations are compared with the computed and measurements of Lo[26] . The parameter $\varepsilon_{r,eff}$ introduced in the present theory thus becomes significant for all large and small values of a/H .

Conclusion:

We have developed a simple model yielding the input impedance of a probe-fed circular microstrip patch antenna, which gives results in accordance with experiment. This model can be used successfully in computer An improved analytical formulation based on a resonator model is presented aided design (CAD) of circular microstrip patch antenna arrays.

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Table 1 : Experimental resonance frequency of the circular patches

Ref.	Patch number	$H_{/mm}$	ε_r	Patch radius $r_{0\ /mm}$	H/λ_g	Measured resonance frequency $f_{R/GHz}$
[1]	1	0.4900	2.43	1.9698	0.07	25.600
	2	0.4900	2.43	3.9592	0.03	13.100
	3	0.4900	2.43	5.8898	0.02	8.960
	4	0.4900	2.43	8.0017	0.02	6.810
	5	0.4900	2.43	9.9617	0.01	5.470
[19]	6	1.5875	2.65	7.4000	0.06	6.634
	7	1.5875	2.65	8.2000	0.05	6.074
	8	1.5875	2.65	9.6000	0.05	5.224
	9	1.5875	2.65	10.7000	0.04	4.723
	10	1.5875	2.65	11.5000	0.01	4.425
[1]	11	1.1938	10.00	4.7752	0.07	5.455
	12	1.1938	10.00	7.1628	0.05	3.650
[4]	13	2.3500	4.55	7.7000	0.08	4.945
	14	2.3500	4.55	10.4000	0.06	3.750
	15	2.3500	4.55	20.0000	0.03	2.003
	16	2.3500	4.55	29.9000	0.02	1.360
	17	2.3500	4.55	39.7500	0.02	1.030
	18	2.3500	4.55	49.5000	0.01	0.825

Table 2 : Comparison of experimental resonance frequency with circular patch models

Patch number	Ensemble [21]	Pozar [22]	Benella [23]	Verma [20]	Our model
1	24.900	25.736	25.865	25.900	25.700
2	13.400	13.331	13.347	13.250	13.150
3	9.170	9.111	9.120	9.032	9.010
4	6.830	6.775	6.780	6.831	6.800
5	5.530	5.475	5.485	5.515	5.475
6	6.735	6.674	6.685	6.702	6.660
7	6.132	6.058	6.065	6.092	6.080
8	5.286	5.219	5.265	5.254	5.230
9	4.757	4.708	4.715	4.747	4.720
10	4.449	4.396	4.400	4.434	4.430
11	5.572	5.657	5.660	5.475	5.445
12	3.776	3.798	3.800	3.701	3.675
13	4.967	4.997	5.000	4.960	4.955
14	3.788	3.746	3.780	3.756	3.753
15	2.000	1.989	1.990	2.012	2.000
16	1.358	1.343	1.345	1.356	1.362
17	1.025	1.015	1.015	1.025	1.028
18	0.825	0.818	0.835	0.825	0.826

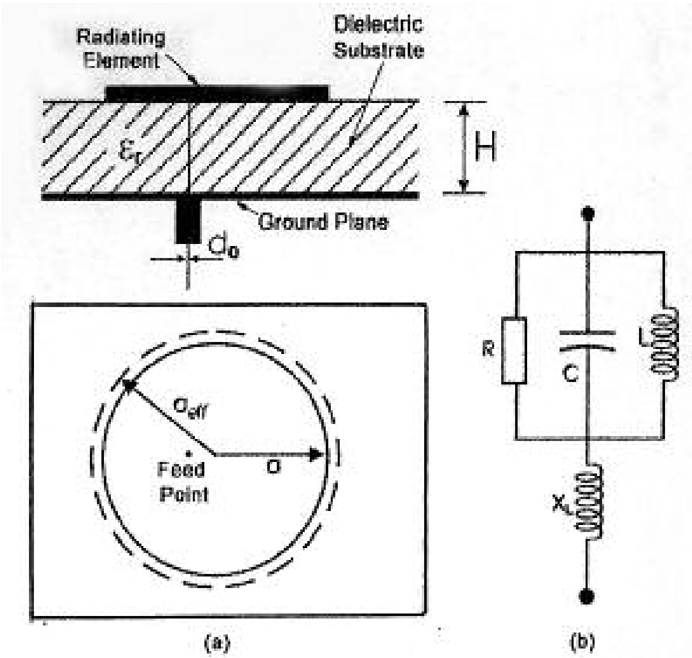

Fig. 1 (a) Geometry of a circular microstrip patch antenna.
(b) Equivalent resonant parallel R L C circuit.

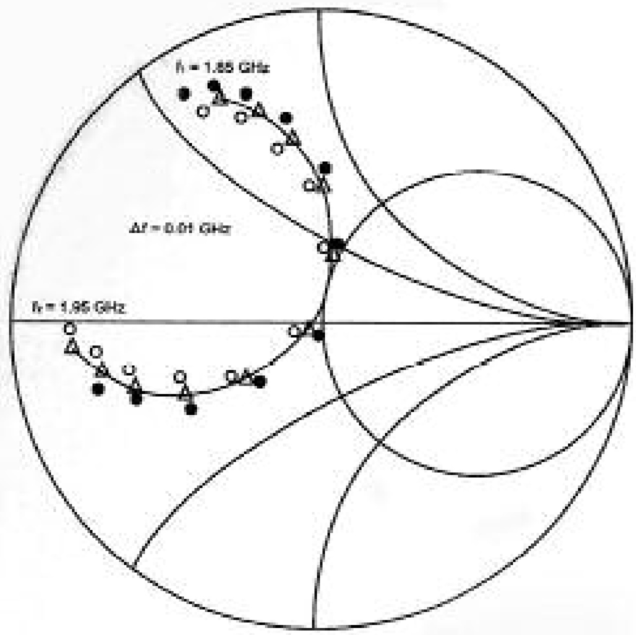

Fig. 2 : Input impedance of coax-fed microstrip patch antenna.
$\varepsilon_r = 2.2$; $tg\delta = 0.001$; $a = 0.31cm$; $H = 0.159cm$;
$d_0 = 0.125\ cm$; $Z_0 = 50\ \Omega$; $\rho = 0.55\ cm$;
mode (m =1, n = 1)
● : calculated [24] Δ : measured[25] ○ : our model

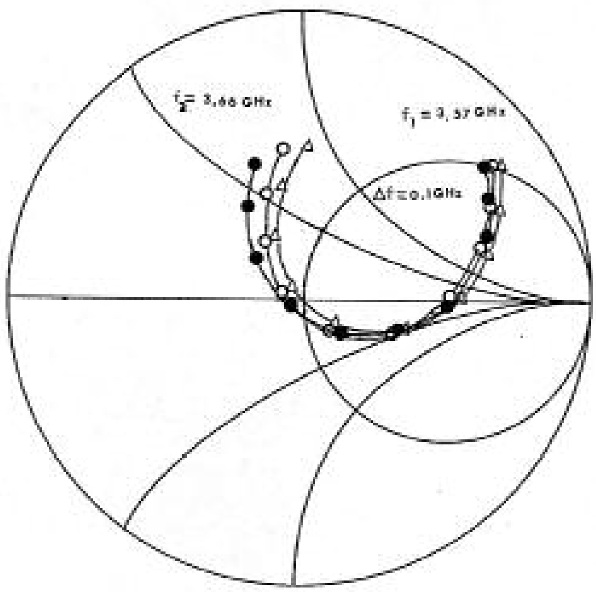

Fig. 3 : Input impedance of coax-fed microstrip patch antenna.
$\varepsilon_r = 2.62$; $tg\delta = 0.001$; $a = 1.3cm$; $H = 0.47cm$;
$d_0 = 0.125\ cm$; $Z_0 = 50\ \Omega$; $\rho = 0.66\ cm$;
mode (m =1, n = 1)
● : calculated [26] Δ : measured[26] ○ : our model

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