Recession Curve Modeling for Diyalah, Greater Zab and Lesser Zab Rivers[1]

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Abstract

Recession curves of yearly stream flow hydrographs are analyzed by a probabilistic approach suggested by (Akosy, H. & Bayazit M, (2001)) and development of a spatial program with advanced language programming (Visual Basic 6) compatible with Microsoft excel for this purpose. Flow of a month on a recession curve is calculated by multiplying the previous month's flow with a value of *K* smaller than one; *K*, defined as the ratio of the flows of successive months on the recession curve, was determined from observed monthly flow time series. The range of *K* is divided into three class intervals. A procedure using the concept of gradually increasing values of *K* is adopted. For this, transition probabilities and average values of *K* are determined for each class interval of the year. A recession curve can be generated, once the peak flow is known, by the probabilities approach. The procedure allows nonlinear, seasonal and stochastic effects in flow recession of a river to be considered.

Keywords: recession curve, probabilistic approach, Markov chain, transition probability, Tigris Tributaries, recession curve generation.

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Objectives

The objective of this research is to develop a computer simulation program to generate inflow recession curve for (Diyalah, Greater Zab and Lesser Zab) rivers in Iraq with probabilistic approach methodology suggested by (Akosy, H. & Bayazit M, 2001) and using visual basic 6.0 language programmer for this purpose.

Probabilistic Approach

The following probabilistic approach aims to consider the random (stochastic) structure of flow recessions (Akosy, H. & Bayazit M, 2001). During Recession of the hydrograph, flow on a Month t, Qt, can be expressed as the product of a value of K, smaller than one, and flow on the previous month, Q_{t-1}:

$$Q_t = KQ_{t-1} \qquad \text{………………..……….. 1}$$

Here, t denotes the number of months the proceeding peak of the hydrograph. In this probabilistic approach, K is considered as a random variable with values between zero and one. Zero is obtained if the stream curve is intermittent (no flow); otherwise K is always greater than zero and less than one.

Transition probabilities

Once the number of class intervals has been reduced to three, there are only three alternatives for a value of K to be taken: it is either between zero and 1 ($0 \leq K \leq 1$) . If 1, 2 and 3 denote the intervals, respectively, and then the following Markov chain (Daniel P. & Maynard T. 1996) can be constructed:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \qquad \text{……………………….. 2}$$

The matrix has been probabilities (P) of transition from one class interval to another. If nij is the number of months of observed recessions, having a value of Kj with the previous value of Ki, then the transition probability from i to j can be calculated as:

$$P_{ij} = nij / \sum_{j} nij \qquad i, j = 1 , 2 , 3 \qquad \ldots\ldots\ldots\ldots 3$$

Methodology

Akosy, H. & Bayazit M, (2001) adopted the following procedure for a recession curve analysis for rivers inflow. The recession curve of a typical hydrograph has a gradually decreasing slope, i.e. *K* gradually increases. The procedure adopted in the study follows this basic concept. In other words values of *K* to be used in the generation of the recession should be ranked from the smallest to the largest. The length of recession is determined using updating model for generating daily flows (Akosy, H. & Bayazit M, 2001) to monthly flows for the rivers in this study. For an n-month long recession curve:

$$K(t) \leq K(t+1) \qquad t = 1, .., n-1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Where t is time in months after the peak of hydrograph. In the present case, since a recession is to be generated by only three values of *K*, the inequality given in equation (4) can be rewritten as:

$$\underbrace{K_1 = \ldots = K_1}_{i} < \underbrace{K_2 = \ldots = K_2}_{l} < \underbrace{K_3 = \ldots = K_3}_{m} \qquad \ldots\ldots\ldots\ldots 5$$

Where sum of *K, l* and *m* is equal to *n*, the recession length. According to the proposed probabilistic approach (Fig. 2), a recession curve starts decaying with a value of K_1. There is no increase in the flow (if recession is still continuing) the next month of the recession is calculated by either K_1 or K_3. Once *K* if the recession curve is still continuing. Therefore, *k, l* and *m* in equation (5) satisfy:

$$K \geq 1; \quad l \geq 0; \quad m \geq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

Once limits of class interval of *K* are determined, transition probabilities are calculated by using equation (3). The recession curve cannot decay a value of K_3, after decaying with a value of K_1. Therefore P_{13} is set to zero.

Additionally, once a recession curve has started to decay with K_3, it is not allowed to decay with neither K_1 nor K_2. Then $P_{21} = P_{31} = P_{32} = 0$. The probability of P_{33} is obviously equal to 1 (100%). Therefore, the transition probabilities matrix in equation (2) changes to:

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 \\ 0 & P_{22} & P_{23} \\ 0 & 0 & P_{33} \end{bmatrix} \qquad \text{.......................... 7}$$

As the sum of the probabilities equals one for each row of the matrix,

$$P = \begin{bmatrix} P_{11} & 1-P_{11} & 0 \\ 0 & P_{22} & 1-P_{22} \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{.......................... 8}$$

is obtained. It is obvious from the matrix that only P_{11} and P_{22} are needed to define the matrix.

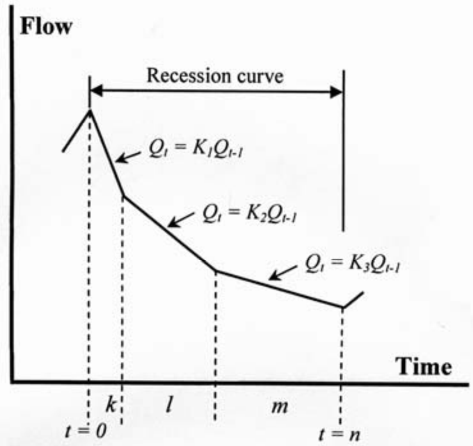


Fig.(1): Schematic representation of the probabilistic approach suggested by (Akosy, H. & Bayazit M, 2001).

In addition to the transition probabilities (P_{11}, P_{22}) three values of K (K_1, K_2, K_3) are needed for the approach. These values are computed as weighted average for each class interval and are assumed to be constant throughout the year.

Data

The probabilistic approach was applied to records of monthly river flow for three stations in the Diyalah, Greater Zab and Lesser Zab rivers respectively. These data collected from the records of the Ministry of water resources in Iraq. The records are between 67 – 75 years in length. Information and number of recession used in this study is given in table (1).

Table (1): Characteristics of data.

River name	Drainage area (km^2)	Record period	Number of recession curve
Greater Zab	26473	1925 - 1999	75
Lesser Zab	22250	1925 - 1996	71
Diyalah	31896	1924 - 1991	67

Computer Program

A recession curve of length n months starting with a peak discharge Q_o can be easily be generated by applying spatial program with advanced language programming (Visual Basic 6), once values of K and transition probabilities are determined by the following algorithm. The flowchart of the algorithm is given in Figure (2).

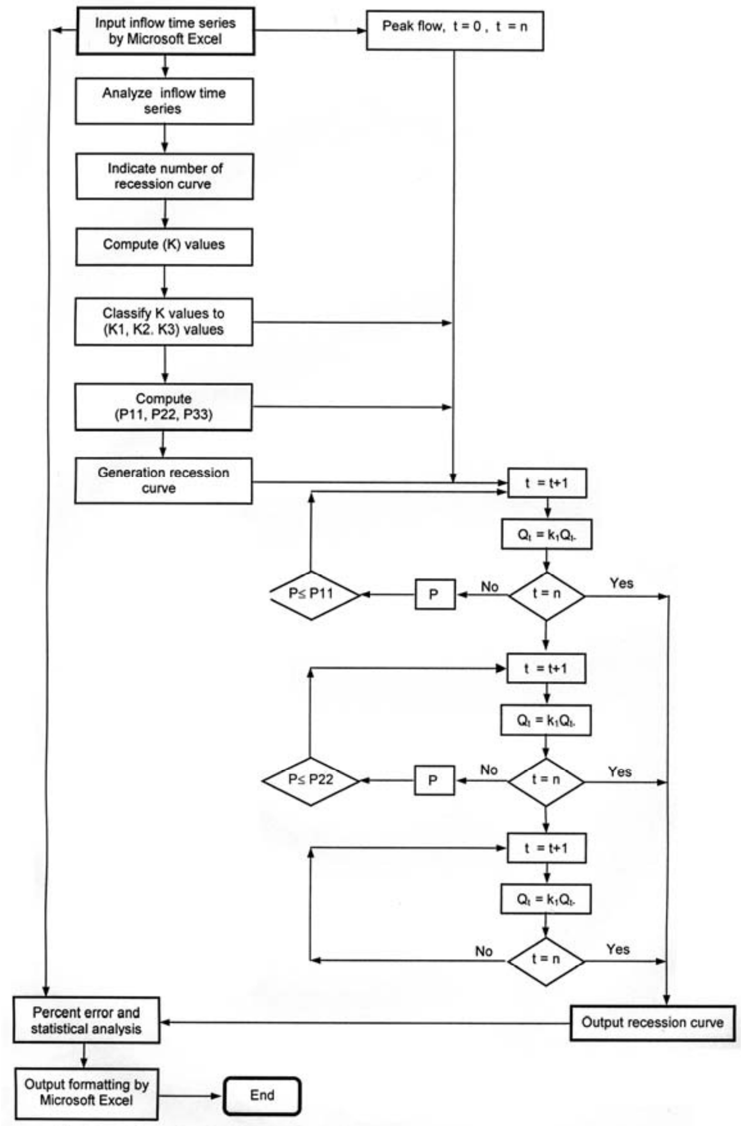


Fig.(2): Flowchart of probabilistic recession curve generation algorithm.

By using the flowchart of the algorithm the researcher development of user interface computer program in (Visual Basic 6), this program running compatible with Microsoft Excel and used Microsoft Excel to format input and output data. Data used in this study is the time series of the monthly inflow for the important Iraqi tributaries for Tigris river (Diyalah, Greater Zab and Lesser Zab) rivers. The philosophy of the mechanism of the computer program is first analyze the inflow time series by using probabilistic approach which be defined in the previous paragraph. The output of this analysis is the number of recession curve, peak flow, and K values, at this stage the program classify the K values to three class intervals by computing the average values of K in each class intervals due to distribution of K values a long the recession curve and due to K magnitude in each point in the recession curve. After this stage the program compute the values of transition probability (P11, P22, P33) for distribution for (K1, K2, K3) along the recession curve (Fig. (2)).
The third stage of program is to generate the recession curve by probabilistic approach, and using the peak flow values of inflow hydrograph as start point to generate the recession curve. The figure (3) below describe the program interface.

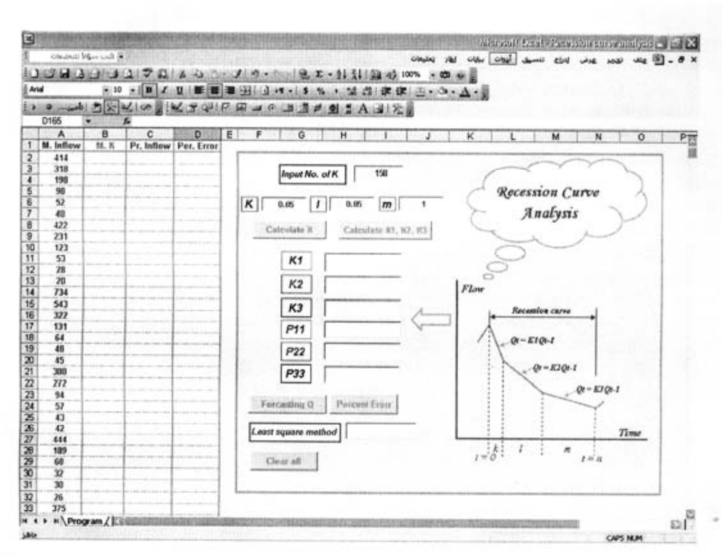


Figure (3): Computer program interface.

Study Area

The probabilistic approach was applied to records of monthly river flow for Diyalah, Greater and Lesser Zab Rivers in the North-East of Iraq. The region is of agricultural important for the country and hence irrigation water demand increases in the summer when most of the rivers in the region have low discharge. The drainage area of Diyalah, Greater Zab and Lesser Zab is (31896, 26473, and 22250) Km^2 respectively, and Figure (4) Shows the Iraq rivers map.

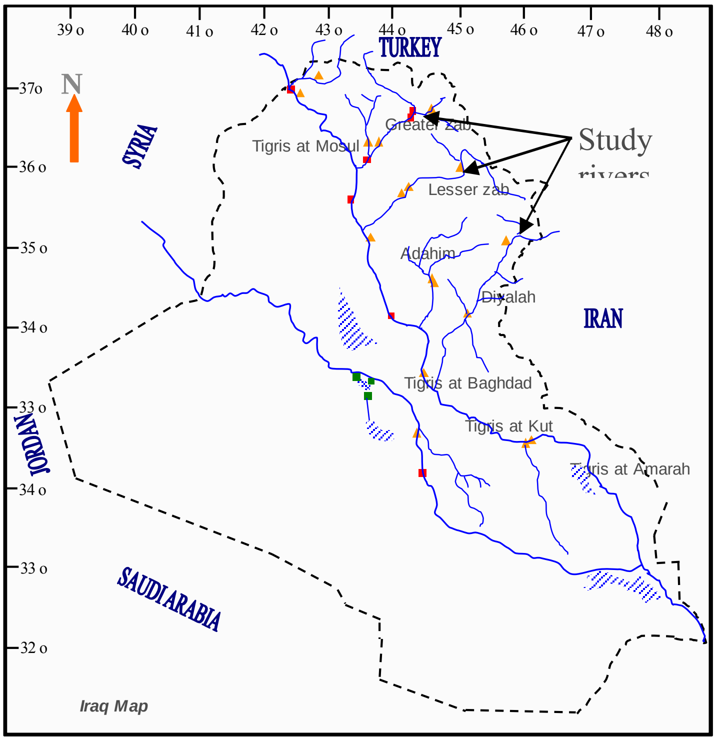


Figure (4): Location of study area.

Discussion

The best advantage of a probabilistic approach to predicated the inflow recession curve after peak flow. Flow on a recession curve of the hydrograph is founded by multiplying the previous mean monthly flow with parameter *K*, which is considered as random variable between zero and one. The range of *K* is divided into three part intervals in this study.

The applicability of the proposed approach was tested by simulation of observed recession curves of data series for Diyalah, Greater Zab and Lesser Zab rivers by applying an adopted spatial recession curve modeling program with advanced language programming (Visual Basic 6). The output of the recession curve program is the magnitude of (*K1, K2, K3*) and the (*P11, P22, P33*) for Greater Zab, Lesser Zab, Diyalah rivers. All observed recessions were recalculated by the probabilistic generation algorithm. Peaks of the recessions were taken from the records and the program simulates the recession curve of the inflow for each river. Table (2) shows the result of the simulation.

Table (2): Result of the simulation.

Rivers	K1	P11	K2	P22	K3	P33
Diyalah	0.485	53.08%	0.794	26.15%	0.918	20%
Greater Zab	0.512	48.32%	0.705	27.45%	0.927	24.23%
Lesser Zab	0.523	44.03%	0.674	35.07%	0.87	20.15%

Table (3) shows the distribution of *K* for recession curve during the year for all rivers in this study, this distribution of K determined with the computer program by analyzing the recession curves series for three rivers in this study.

Table (3): Distribution of *K*.

April	K1
May	K1
June	K1
July	K1
August	K2
September	K2
October	K3
November	K3

Figure (5) showed the comparison between the observed and generated recession curves for Diyalah, Greater Zab and Lesser Zab rivers, although overall comparison gives a much better agreement. Recession curves were mostly underestimated for high flows and overestimated for low flows, and Figure (6) showed the comparison between observed and predicated inflow for some recession curves used in this study. When overall characteristics of recession were considered, relative errors of underestimation at the high flow portion of recession curve were found smaller than those of overestimation at the lower portion. Table (4) shows various statistical analysis for the results, which included (average of relative percentage error, max relative percentage error, min relative percentage error, liner Correlation coefficient and Standard Error).
From table (4) we can be noticed that average of relative percentage error is ranged between (8 % - 9%) for three rivers and maximum relative percentage error between observed and generated recession curves is greater for Lesser Zab river with compression with the Diyalah and Greater Zab rivers. The minimum relative percentage error between observed and generated recession curves is greater for Greater Zab river with compression with the Diyalah and Lesser Zab rivers. The relative percentage error can be calculate by using the following equation (8).

$$ER\% = \frac{Q_{observed} - Q_{Generated}}{Q_{observed}} * 100 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 9$$

Where, *ER%* = relative percentage error, $Q_{observed}$ = Observed inflow and $Q_{Generated}$ = Generated inflow.
The linear correlation coefficient between observed and generation recession curves which can be calculated for the three rivers in this study, correlation coefficient used to determine the relationship between two sets of data such as two sets of recession curves (observed and generation). The equation for the liner correlation coefficient is explained in the following equation.

$$Correl(x, y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}} \qquad \ldots\ldots\ldots \; 10$$

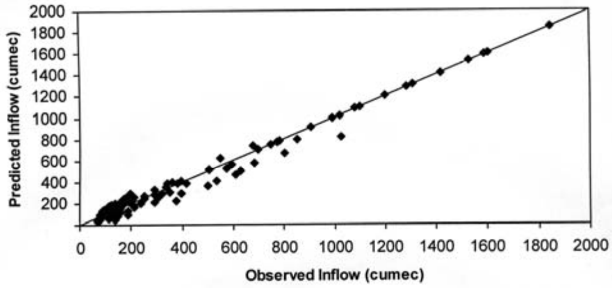
Where, *Correl (x,y)* = Liner correlation coefficient, x and y represent the observed and generation recession curves, $\bar{x}$ and $\bar{y}$ represent the arithmetic mean for observed and generation recession curves respectively.
From table (4) we can be noticed that liner correlation coefficient between observed and generated recession curves is ranged between (0.98 – 0.99) for three rivers in this study , this mean very good correlation coefficient.
The important statistical parameter in this type of researches is standard error to measure the amount of error in the prediction of inflow in recession curve for individual observed inflow in observed recession curve. The following equation can be used to calculate the standard error.

$$STE. = \sqrt{\frac{1}{(n-2)}\left[\sum(y-\bar{y})^2 - \frac{\left[\sum(y-\bar{y})(y-\bar{y})\right]^2}{\sum(x-\bar{x})^2}\right]} \quad \ldots\ldots\ldots 11$$

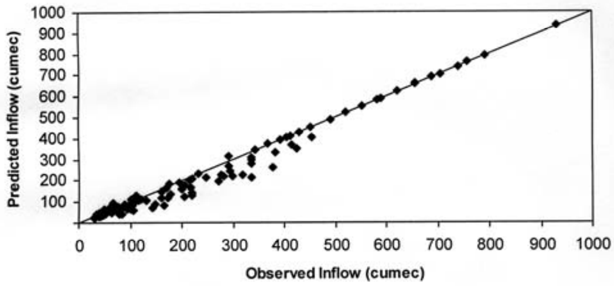
Where, STE. = standard error,
The value of standard error for Diyalah river is (48.31) which can be considered the greatest wsh in comparison to the Greater Zab and Lesser Zab rivers which have standard error values (27.05 and 35.94) respectively.

Table (4): Statistical analysis for the results.

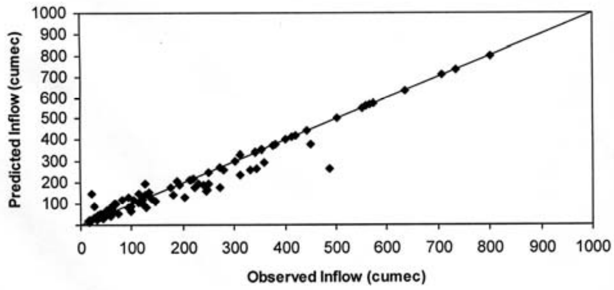
Statistical parameters	Diyalah River	Greater Zab River	Lesser Zab River
Average of relative percentage error	9%	8%	8%
Maximum relative percentage error	32%	39%	-45%
Minimum relative percentage error	-17%	-37%	21%
Liner Correlation coefficient	0.99	0.99	0.98
Standard Error	48.31	27.05	35.94



a- Greater Zab River

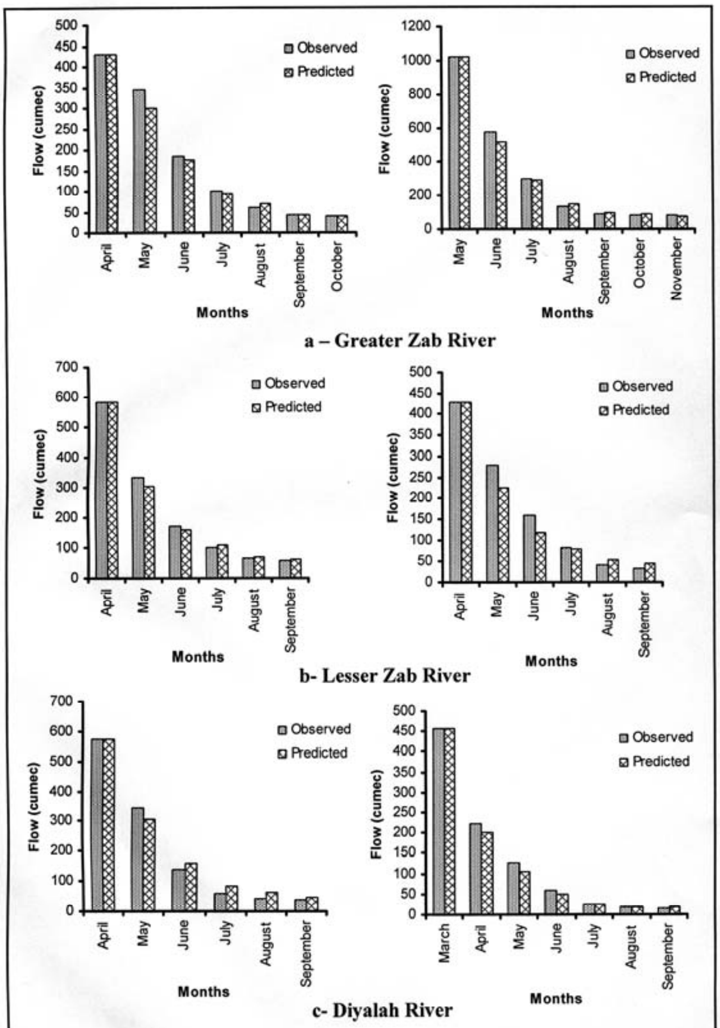


b- Lesser Zab River



c- Diyalah River

Fig(5): Comparison between observed and predicated inflow for all recession curves in this study.



Fig(6): Comparison between observed and predicated inflow for some recession curves in this study.

Conclusion

The recession curve predicating and forecasting the mean monthly inflow after peak is very important in water resources studying and operating of dams' reservoirs and irrigation projects. In this research a probabilistic approach is suggested for the non- increasing portion (recession curve) of a monthly stream flow hydrograph for Diyalah, Greater and Lesser Zab Rivers in the North-East of Iraq.

Flow on a recession curve of the hydrograph is found by multiplying the previous month flow with a parameter K, which is considered as a random variable between zero and one. The range of K is divided into three class intervals. The weighted average of K for each interval is assumed to be constant through the year but transition probabilities are changed due to seasonal effects of the hydrograph cycle. Therefore, three value of K together with three probabilities (one for each class intervals) make out the parameter set of the probabilities approach. The approach is found be useful for the generation of recession curve as shown by the results of application on monthly stream flow series.

The generation of recession curves are very difficult, therefore, the development a computer program interface by using (visual basic 6) language programmer compatible with Microsoft Excel for analyze and generation recession curves series in this research is very important to predicated the recession curve in the future for management the water resources in Tigris river in Iraq.

The good results of the statistical analysis between observed and generation recession curves series in this research indicated the good confidence to use a computer program which be suggested in this research to generate and predicated the inflow recession curve for other Iraqi rivers.

References

1. Akosy, H. & Bayazit M. (2001) Probabilistic approach to modeling of recession curves. IAHS Publ. no. 2. Vol. 46. PP. 269-285.
2. Daniel P. & Maynard T. (1996) Finite Mathematics. McGraw. Hill Book Company Inc., New York, N.Y. Fourth Edition. PP. 345-406.
3. Evora, N. D. & Rousselle, J. (2000) Hybrid stochastic model for daily flows simulation in semiarid climates. J. Hydrol. Engine. ASCE 5(1), 33-42.
4. Kelman, J. (1980) A stochastic model for daily stream flow. J. Hydrol. 47, 235-249.
5. Kavvas, M. L. & Delleur, J. W. (1984) A statistical analysis of the daily stream flow hydrograph . J. Hydrol. 71, 253-275.
6. Sargent, D. M. (1979) A simplified model for the generation of daily stream flows. J. Hydrol. 24(4), 509-527.

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