Improved Geoid Model for Syria Using the Local Gravimetric and GPS Measurements[1]

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Abstract

"The objective of this paper is to discuss recent research done by the author to develop a new improved geoid model for Syria and the related deflection of verticals. This research could be considered as a trial to develop more accurate local model for Syrian geoid. When global geoid model was extracted from space techniques many related local or regional parameters were neglected or at least local gravity measurements were not accurately determined. In this paper a hybrid technique is used in which combination of local gravity measurements, global model and GPS measurements are used in some regions to improve the geoidal model in Syria. Existing gravity data and GPS measurements were obtained from some oil companies used to work in Syria. Although the main objectives of the oil companies from gravity points were different from the geodetic perspectives but it could be used for geoid modeling, geoidal heights determination and deflection of vertical computations. The technique of "remove and restore» is used in the calculations. Software is developed by the author for extensive computations in addition to ArcGIS. Comparison between the final local geoid and EGM96 is proved to be in the range of few decimeter levels. As a further step and based on the resulted geoid, the deflection of verticals in Syria were developed. The resulted local model could be improved further by adding much more observation points and taking the terrain effect into consideration."

Keywords: geoidal height, deflection of verticals, remove-restore technique, geopotential model, geoidal residual, orthometric height

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[1] For the Abstract in Arabie see pages (75-76).

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1- Introduction:

Determination of a national geoid, especially in developing countries, is very problematic and more difficult task to implement. This difficulty is caused by the lack of national survey of local gravity, absence of well defined national datum in many underdeveloped countries and lack of regional and national co-operation. It is well known that most accurate determination of geoid is obtained by dense and well distributed national and regional gravity points. Conducting national survey is very expensive and time consuming. Therefore, there are two alternatives, the first one is to do nothing and the other is to use a global geoid model which does not fulfill the actual needs for accurate and reliable maps. However, the first one is a bad choice and has an adverse effect on developing of national updated and reliable maps. In most cases local geodetic networks in many countries have no consistency and reliability in different regions of the national geodetic networks. In addition to this fact, these geodetic networks are not ready to adopt new technologies including GPS or GNSS in general because of the lack of precise local geoid. The other alternative has a defect resulted from the fact that the global geoid is not accurate enough for large scale mapping. In fact many developed countries in Europe and North America have used gravimetric observations using gravimeters to establish accurate national and regional geoid. In Syria small scale gravity campaigns were executed by oil companies. Theses campaigns were not accurate enough for geodetic and topographic mapping. The normal procedure to obtain reliable geoid is to compute the gravity anomalies and the related gravity reductions, such as free-air anomaly or Bougure anomaly taking into consideration the local topography. From gravity anomalies one can use Stok's integral to obtain both geoidal heights and the deflection of verticals. It is well known that Stock's integral has to be performed (theoretically) on whole surface of the earth. This paper attempts to improve the geoid determination by using existing gravity and GPS measurements collected in Syria. The local gravity anomalies computed by General Petroleum Company were developed based on Clark 1880 ellipsoid. The author has calculated the free air gravity anomalies to be compatible with global model in addition to the use of the EGM96 global geopotential model to degree 360 in order to determine the long wavelength effect of the geoid surface.

The geometric relation between the geoid, ellipsoid and Earth surface is shown in figure (1), where the separation between orthometric height (H) and ellipsoidal height (h) is known as the geoid undulation (N) (Heiskanen and Moritz, 1967).

1. Available Data and Procedures:

The following data is used in this research:

1- Gravity Map of Syria -General Petroleum Company (GPC)

2- GPS measurements by MAM project with collaboration of the ministry of local Administrative and environment in Syria.

3- Data of gravity measurements and parameters of EGM96 Geoid model.

Figure (2) depicts the gravimetric traverses used by GPC in which the main traverses located in central Syria. It is clear that the points are not dense enough to produce very accurate geoid. In addition, no attempt was done by the company to close these traverses as it should be. The gravity anomalies were based on Clark 1880 ellipsoid. In this paper calculation of the free air gravity anomalies and the EGM96 global geopotential model to degree 360 is used in order to determine the long wavelength effect of the geoid surface. The remove-restore technique is programmed outside Arc- GIS environment to produce the geoid undulation.

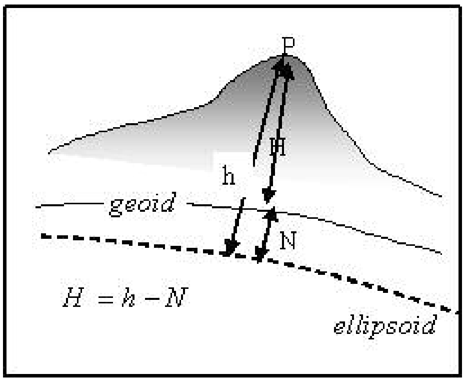
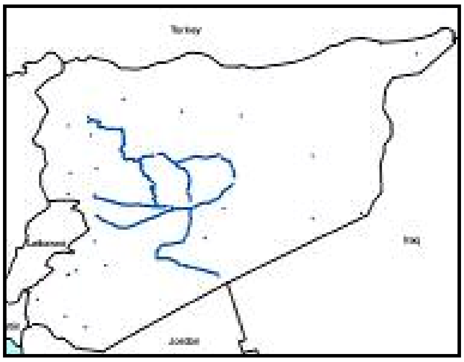


Figure (1) the geoid undulation (N)



Figure(2) gravimetric traverses in Syria used by GPC

Only 800 gravity points were used which include information on (topographic coordinates, altitude, terrestrial gravity values) for each point. The terrestrial free-air gravity anomaly.

$$\Delta g_{(FA)} = g - \gamma + 0.308 \times H + t \qquad (1)$$

Where g is the actual gravitational measurement, γ is the theoretical normal gravity value, H altitude, t is the topographic correction (neglected). GPC listed the results in its report as it is shown in table (1) and the values of the gravity anomalies is computed in mgal to be added to a constant 979 mgal. Due to the fact that original data is based on Clarke ellipsoid, a conversion to WGS-84 is performed for calculating γ_{wgs} using equation (2), from which the free-air anomalies are computed using equation (1), the results were listed in table (1).

$$\gamma = \gamma_e \frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \qquad (2)$$

g Terrestrial gravity

γ Theoretical normal gravity on ellipsoid Wgs84

ϕ Point latitude

k $k = \dfrac{b \times \gamma_p}{a \times \gamma_e} - 1$

γ_e , γ_p Theoretical gravity at the equator and poles.

e^2 square of the first ellipsoidal eccentricity

KM	$X_{(m)}$	$Y_{(m)}$	$H_{(m)}$	$d\gamma_{Clarke}$	dg	$d\gamma_{Wgs}$	$\Delta g_{(FA)-WGS}$
115	274600	294420	670	711.6	527.9	699.7	2.165
116	273620	294720	674	711.8	527.6	699.9	2.733
117	272720	294960	671	712.0	528.3	700.1	2.374
118	271760	295360	679	712.3	526.4	700.4	2.488
119	270820	295720	662	712.6	529.7	700.6	0.654
120	269880	296160	670	712.9	527.2	701.0	0.198
121	268920	296540	672	713.2	527.9	701.3	-0.820
122	267840	296960	688	713.5	521.1	701.6	-1.271
123	266800	297280	687	713.8	520.7	701.8	-2.158
124	265880	297640	732	714.0	510.5	702.1	0.735
125	264840	298000	745	714.3	508.8	702.4	2.545
126	263920	298340	745	714.6	510.2	702.6	3.583
127	262960	298680	734	714.8	513.8	702.9	3.832
128	262000	299060	732	715.1	515.2	703.2	4.315
129	260960	299360	769	715.4	507.9	703.4	7.459
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Table(1) Results of calculating free air gravity anomaly on WGS-84

The X, Y the coordinates of each point are in Lambert projection (m)

$$\gamma = 979 + d\gamma \qquad (mgal)$$

$$g = 979 + dg$$

$$\gamma_{WGS} = 979 + d\gamma_{WGS}$$

The spherical harmonic expansion of the gravitational potential, geoid height or gravity anomaly could be obtained from WGS 84 EGM96 Earth Gravitational Model. At any point the geoid height $N_{(GGM)}$ and the gravity anomaly $\Delta g_{(GGM)}$ could be calculated using equations (3) & (4):

$$N_{(GGM)} = \frac{T}{\gamma} = \frac{GM}{r\gamma} \sum_{n=2}^{N} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta) \qquad (3)$$

$$\Delta g_{(GGM)} = \frac{GM}{r^2}\sum_{n=2}^{M}\left(\frac{a}{r}\right)^n (n-1)\sum_{m=0}^{n}(\delta\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\theta) \qquad (4)$$

Where θ □are the geocentric co-latitude and longitude of the point. The coefficients C nm , S nm , are the fully normalized spherical geopotential coefficients of the anomalous potential Pnm are the fully normalized associated Legendre functions and M is the maximum degree of the geo-potential model. When using a global geopotential model for regional gravimetric geoid determination, the long and medium wavelength component of the geoid must not be added to the solution twice (Rapp and Rummel, 1975; Kearsley, 1988). This can be avoided using the so called remove-restore method. The gravity anomalies implied by the geo-potential model (equations 3 &4) are subtracted or removed from the terrestrial free-air gravity anomalies ($\Delta g_{(FA)}$) to yield residual gravity anomalies:

$$\Delta g_r = \Delta g_{(FA)} - \Delta g_{(GGM)} \qquad (5)$$

These residual gravity anomalies are utilized in a Stokesian geoid computation to provide residual geoid undulations based on the same spherical harmonic expansion of the same geopotential model. The corresponding long and medium wavelength geoid component from the geopotential model (equation 3) is subsequently added or restored to these residual geoid undulations.

Due to the fact that the global geo-potential model is available, Stokes' integral can be modified to integrate gravity anomalies over small cap σ (Area of interest) which is in our case Syria and neighboring countries (Lebanon, Jordon, Palestine, Iraq and Turkey). The geoidal residual N_r could be calculated using:

$$N_r = \frac{R}{2\pi\gamma}\iint_{\sigma}(\Delta g_{(FA)} - \Delta g_{GM})S(\psi)d\sigma \qquad \textbf{(6)}$$

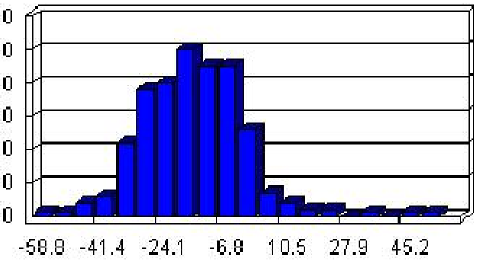
The total geoidal height N is resultant of undulation N_M obtained directly from GGM (EGM96 model), the residual undulation N_r after removal of the contribution of GGM and residual terrain effect N_t:

$$N_{final} = N_{(GGM)} + N_r + N_t \qquad \textbf{(7)}$$

Unfortunately, regional gravity measurements were not available or at least scares therefore this research is confined inside Syria with the available gravity measurements obtained from oil companies. The contribution of terrain effect is relatively small and neglected in this study.

2.Results of Analysis:

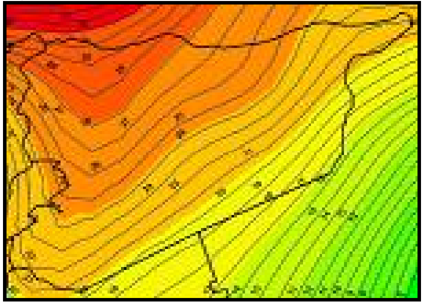
Statistical parameters for the gravity residual Δg_r (mgal) were computed using EGM96, by applying equation (5)



Minimum = -58.767
Maximum = 56.188
Mean = -14.019
Standard deviation = 14.424

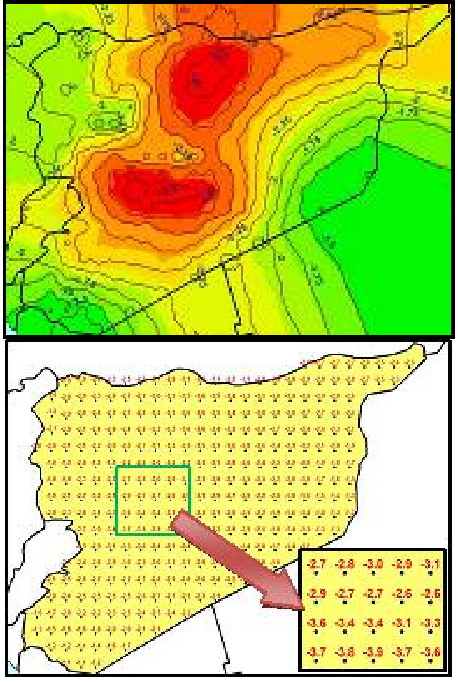
Fig (3) statistical parameters for the residual gravity anomalies (mgal)

The EGM96 Geoid model as it is shown in figure (4) is used as reference to this work. Many research concluded that EGM96 behaves better than OSU91 Geoid model and it is more accurate than the OSU91A due to the lack of gravity data of the Middle East area within the OSU91A model and the long wavelength error propagation in the GGM (Al-Bayari and Al-Zoubi, 2007).

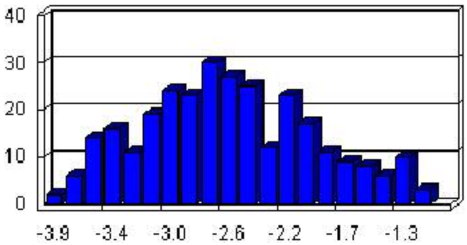


Figure(4) EGM96 Geoid Model in Syria

Figure (5) depicts the variations of Gravimetric Syrian-Geoid-Model (Nr) calculated by applying Stokes' integral equation (6) where the coordinates in UTM system and residual geoid undulation calculated in meter. Figure (6) presents the statistical parameters of these values in meter.



Fig(5) the residual Syrian-Geoid-Model undulation calculated in Syria Nr



Minimum = -3.8

Maximum = -1.1

Mean = -2.5

Standard deviation = 0.62

Fig(6) statistical parameters for the residual Syrian geoid undulation Nr(meter)

The computation of Syrian-Geoid-Model was done in accordance with software developed in Arc-GIS environment using GM96 Geoid model equation (7), accordingly the results are shown in fig.(7).

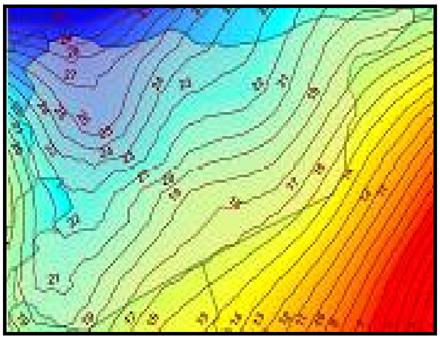


Figure (7) The final Syrian geoidal heights calculated Model N_{final}

In addition to the gravimetric measurements GPS was used in number of points (few points in different places in Syria) where the orthometric heights are known and the values of geoidal heights are computed from:

$$N_{GPS} = h_{GPS} - H_{orthometric} \qquad (8)$$

These values were compared with computed N_{final} from the algorithm developed by the author, and with the values of height in EGM96 model and the results were presented in fields ($N_{final} - N_{GPS}$) and ($N_{EGM96} - N_{GPS}$) in table (2).

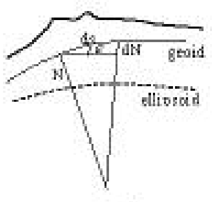
Table.(2) Results of the difference between geoidal height GPS and geoidal height calculated

λ	ϕ	$h_{(GPS)}$	H_{ortho}	$N_{(GPS)}$	N_{final}	$N_{(EGM96)}$	$N_{final} - N_{GPS}$	$N_{EGM96} - N_{GPS}$
36.71355	34.69339	567.79	543.32	24.47	23.39	25.86	-1.08	1.39
36.76342	34.73567	561.65	537.29	24.36	23.47	26.15	-0.88	1.79
35.75499	35.55509	34.84	12.34	22.50	22.62	24.84	0.12	2.34
38.77439	34.08147	717.83	699.91	17.92	18.21	21.37	0.29	3.45
38.69338	33.99871	766.78	748.93	17.84	18.24	21.33	0.40	3.49
35.90678	34.87167	93.54	72.40	21.14	21.57	22.88	0.42	1.74

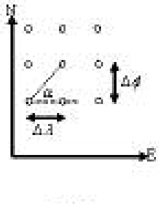
4. Deflection of Verticals Map:

Based on the result of the Syrian geoid, the deflection of verticals could be computed based on the following:

The Syrian local geoid values is converted to a grid with 20x20 km resolution (ϕ,λ) at every point the geoidal height is read and the difference between two adjacent points in both directions is obtained. Figure (8) and (9) show the methodology of developing equations (13) and (14).



Fig(8



Fig(9)

where: η is the deflection of vertical component in East-West direction ξ is the deflection of vertical component in North-South direction α is the azimuth between two adjacent points

ε is the resultant deflection of vertical angle

$\Delta\phi$ increment in latitude

$\Delta\lambda$ increment in longitude

The general approach of the deflection of verticals computation is to perform the integration all over the earth surface noting that:

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \qquad (9)$$

In a north-south direction the azimuth angle $\alpha = 0$ and equation (9) becomes:

$$\varepsilon = \xi \qquad \& \qquad ds = \mathrm{R} \times \mathrm{d}\phi$$

Similarly in east-west direction $\alpha = \pi / 2$ and equation (9) becomes:

$$\varepsilon = \eta \qquad \& \qquad ds = R \times \cos\phi \times d\lambda$$

From Figure (10) and after substituting in (9), the general equations are:

$$\xi = -\frac{1}{R}\frac{\partial N}{\partial \phi} = \frac{1}{4\pi G}\iint_{\sigma} \Delta g \frac{ds}{d\psi} \cos\alpha . d\sigma \qquad (10)$$

$$\eta = -\frac{1}{R\cos\phi}\frac{\partial N}{\partial \lambda} = \frac{1}{4\pi G}\iint_{\sigma} \Delta g \frac{ds}{d\psi} \sin\alpha . d\sigma \qquad (11)$$

where:

$$\frac{ds}{d\psi} = -\frac{\cos(\psi/2)}{2\sin^2(\psi/2)} + 8\sin(\psi) - 6\cos(\psi/2) - 3\frac{1-\sin(\psi/2)}{\sin(\psi)} + 3\sin(\psi).\ln[\sin(\psi/2) + \sin^2(\psi/2)] \qquad (12)$$

$$d\sigma = d\psi \times \sin\psi \times d\alpha$$

In this work another approach is proposed which benefits from the results obtained from the Syrian geoidal height model. A program was devised to perform the numerical calculations outside ArcGIS environment using :

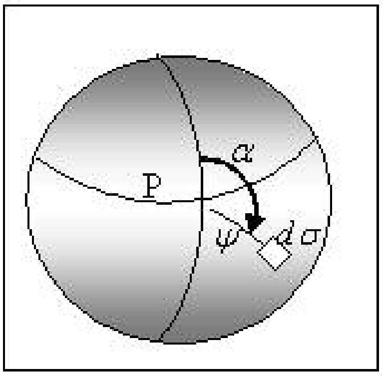


Figure (10)

$$\xi = -\frac{1}{R}.\frac{\Delta N}{\Delta \phi} \qquad (13)$$

$$\eta = -\frac{1}{R\cos\phi}\frac{\Delta N}{\Delta \lambda} \qquad (14)$$

Where ΔN is the difference in geoidal undulation in north-south or east-west cells and could be read from the Syrian geoidal model file. The computations is performed in ϕ direction (East-West) and λ direction (North-South) to produce the deflection of vertical components the computation of η, ξ .

In figures (11) and (12) a grid with spacing $\Delta\phi = \Delta\lambda \approx 20\ km$ and R=6371 Km are used for depicting the results of computing of both components. The result of the computations are shown in Figure (11) and figure(12).

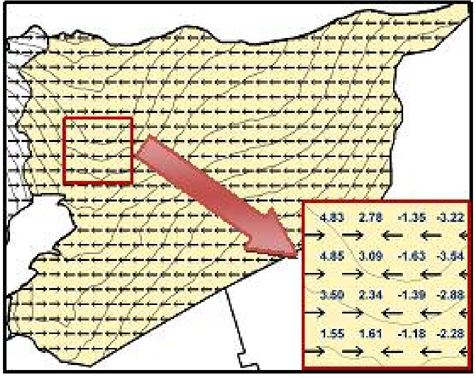


Fig (11) ξ is the deflection of vertical component in North-South direction(arc-sec)

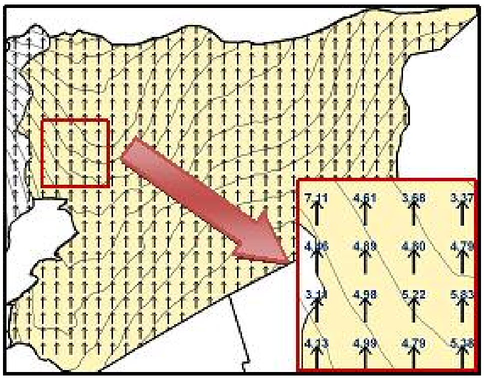


Fig (12) η is the deflection of vertical component in East-West direction(arc-sec)

From figure (11) one can conclude that the values of the deflection of vertical are small in general (-8.9 to 11.8 arc-second) and change direction in east - west of Syria. In figure (12) all deflection of vertical components are in north direction and the absolute values are relatively small (0.04 to 11.3 arc-second).

5. Discussion and Conclusion:

Few remarks could be discussed if the final goal is a very accurate geoid for Syria:

1- The number of measured points available in order to extract more accurate local geoid has to be increased in the future and the existing data from oil companies could be part of it.

2-For the existing data more geodetic reductions are needed to improve the accuracy of the measurements especially closing the gravity traverses. This task was not taking care of by GPC.

3-It might be more efficient if a hybrid technique is used which combines conventional gravimetric measurements with astro-geodetic methods.

4-The global geodetic reference is used in this research, however there is an urgent need to transform this global datum into the Syrian local datum. This is accomplished by a national gravity campaign to extract the local geoidal undulation (heights). This could not be implemented without a national surveying body that is capable to do this task. Otherwise Syria has to be divided into patches of areas and for each small one extensive GPS measurements have to be used to perform the required transformation. The resulted parameters are different for each area. Here again, this approach is not acceptable for an accurate regional geoidal height determination.

5- This research could be considered as a first step to improve the local geoid for Syria and further steps have to be followed.

6-Acknowledgment:

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