A Theoretical Study of the Effect of Vibration on Heated Annulus Pipe[١]

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Abstract

The dynamics of annulus pipe conveying fluid is described by means of transfer matrix method. This paper provides a numerical technique for solving two dimensional incompressible equation of forced and free vibration of annulus pipe conveying fluid. The dynamics behavior for any point located a long the annular pipe which is divided into nodes and elements, are computed, taking into account the type of support and heat flux ranging from (6.2-12.5kW/m^2) for flow rate level ranging from (50-200L/hr).

A computer program written in FORTRAN 90 Languages has been developed to embrace the theoretical work. The results show that the thermal forces have predominance effect on the natural frequencies of the vibrated system as well as the effect of heat flux is greater than the fluid velocity effect on the natural frequencies of the system . Also results show that the mode shapes of vibration are greatly affected by heat flux increasing.

[١] For the paper in Arabic see pages (١٧٩-١٨٠).
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Nomenclature.

A_f Cross-sectional area of the fluid (m^2).
A_o Cross-sectional area of the outer pipe (m^2).
A_i Cross-sectional area of the inner pipe (m^2).
D Diameter of pipe(m)
E_o,E_i Modulus of elasticity for outer and inner pipe(N/m^2).
F Excitation force (N).
Fi Field matrix.
f Fraction factor.
G_o,G_i Modulus of rigidity for outer and inner pipe (N/m^2).
I_o,I_i Moment of inertia for outer and inner pipe (m^4).
L Length of the pipe (m).
Le Element length (m).
L_m Mean element length (m).
M_o,M_i Bending moment per unit length for outer and inner pipe($N.m/m$)
m_{po} Mass of outer pipe per unit length (Kg/m)
m_{pi} Mass of inner pipe per unit length (Kg/m)
m_f Mass of fluid per unit length (Kg/m)
N_o , N_i Thermal forces (N).
n Number of stations
K_o , K_i Stiffness for outer and inner pipe
K_1 , K_2 Stiffness of outer and inner support
P Fluid pressure
P_i Point matrix
V_o , V_i Shear force for outer and inner pipe (N).
t Time (s).
$\overline{U}$ Dimensionless fluid velocity
U_f Fluid velocity (m/s) .
W Compressive + Coriolis forces (N).

ω Circular frequency (rad/s)
X Longitudinal coordinate
Y_o, Y_i Deflection in Y-direction for outer and inner pipe (m)
Z_i State vector
ρ Fluid density (Kg/m^3)
τ Dimensionless time.
R Radius of pipe (m).
r Inner radius of pipe (m).
ΔT Temperature change
δt Thermal elongation (m).
αo,αi Coefficient of linear expansion for outer and inner pipe (C^o/m).
θ_o,θ_i Slope in Y-direction for outer and inner pipe (degree)
β,γ Dimensionless fluid parameter
Notation
- Dimensionless notation
a Annulus gap.
L,R Left and right .
oo Outer diameter of the outer pipe
oi Inner diameter of the outer pipe
io Outer diameter of the inner pipe
ii Inner diameter of the inner pipe
/ ∂/∂x & (. = ∂/∂τ)

1-Introduction.

Annulus pipe conveying fluids have many practical applications, such as, their use in heat exchangers, hydraulic control lines and aircraft fuel lines. In some applications, as in jet engine fuel line, these tubes are exposed to high temperatures. Normally this leads to thermal stresses which may be of a catastrophic nature when coincide with vibration.

A fluid flowing through a pipe can impose pressures on the pipe walls causing it to deflect. The deflections of a pipe produced by an accelerating fluid flow are called water hammer [1] .A steadiness of fluid flow through a pipe can also influence the deflection of the pipe .A steady high velocity flow through a thin walled pipe can either buckle the pipe or cause it to flail about. These deflections are called instabilities of fluid conveying pipes. The stability of fluid conveying pipes is of practical importance because the natural frequency of a pipe generally decreases with increasing velocity of the fluid flow [1].Initial work on the transverse vibration and the dynamics of pipe conveying fluid has received considerable attention in the transport of oil in pipes [2] during the early of 1950. Housner [3] derived the differential equation of motion of pipe line containing the flowing fluid using Hamilton's principle.Niordson [4] arrived at the same equation using" shell theory". Long [5] considered the tube as a beam and calculated the frequencies by a power series method. Amabili [6] studied and investigated the non-linear dynamics and stability of simply supported, circular cylindrical shells containing inviscid incompressible fluid flow, two different boundary conditions are applied to the fluid flow beyond the shell corresponding(i)infinite baffles (rigid extensions of the shell),and (ii)connection with a flexible wall of infinite extent in longitudinal direction.

Transfer matrix method is a suitable technique to compute the dynamic behavior, the natural frequencies of the vibrating system and system mode shapes.

Hence, this paper presents a numerical technique for solving two-dimensional incompressible equation of forced and free vibration of pipe conveying fluid by adopting the cited transfer matrix method. A heat flux on the outer pipe was accounted for.

2-Equation of motion:

The equation of motion for forced vibration of an annulus pipe conveying fluid is given by [7],

$$(E_o I_o + E_i I_i)\overset{iv}{Y} + (\overline{N_o} + \overline{N_i})Y'' + (P A_p + m_f . U_f^2)Y'' + 2.m_f .U.Y' +$$
$$(m_f + m_{po} + m_{pi})Y = F(X,t) \qquad (1)$$

Where: $F(X,t)$, is the external force applied normal to the pipe axis in the y-direction

and (I) is the moment of inertia pipe .

This equation is different from the usual beam equation by additional three terms. The first and last terms of the left hand side of the equation are the usual stiffness and mass terms which would be presented regardless of flow. The second term represented the thermal forces, third term represented the centrifugal force required to change the direction of fluid to conform to the curvature of the pipe material normally this force is considered equivalent to compressive force. The fourth term from the left represents the Coriolis force which is a result of the rotation of the fluid element due to the system lateral motion as each point in the span rotates with angular velocity. Equation (1) can be written in dimensionless form by introducing the following quantities [1, 8].

$$\overline{X}=\frac{X}{L},\quad \overline{Y}=\frac{Y}{L},\quad \overline{U}=\left[\frac{m_f}{EI}\right]^{1/2}U_f.L_m\quad ,B=\left[\frac{m_f}{(m_f+m_p)}\right]^{1/2}\quad \gamma=\left[\frac{L_m^2}{EI}\right].P.A_p$$

$$\tau=\left[\frac{EI}{m_t}\right]^{1/2}.\left[\frac{t}{L_m^2}\right]\qquad ,\ and\ \ let\ \ \eta=\left(\frac{E_iI_i}{E_o\,I_o}\right)$$

After substitution Eq (1) becomes,

$$(1+\eta)\overline{Y}^{iv}+(N_i+N_0)\overline{Y}^{//}+(\overline{U}^2+\gamma)\overline{Y}^{//}+2\frac{\overline{U}(B)^{1/2}}{\tau}.\overset{./}{Y}+\frac{\overset{./}{Y}}{\tau^2}=\overline{F}(\overline{x},\tau)\qquad (2)$$

Where, $\overline{F}(\overline{x},\tau)$ is the non-dimensional external force applied normal to the pipe axis in the y-direction .

The pressure drop due to flow in annular pipe of any uniform cross-section is given by [9].

$$\Delta P_a=\frac{f_a.L}{D_h}.\frac{\rho_a.U_a^2}{2}\qquad (3)$$

Where, (f) is the friction factor for laminar flow in annular pipe which is given by;

$$\frac{1}{f}=\frac{R_{ea}}{64}\left(\frac{1+k^2}{1-k}+\frac{1+k}{\ln k}\right)$$

Where: $k=\frac{D_{io}}{D_{oi}}$, $\quad \mathrm{Re}_a=\frac{U_a.D_{ea.}}{\nu_a}$, $\quad U=\frac{\overset{o}{m}_a}{\rho_a.A_a}$ $,D_{ea}=D_{oi}+D_{io}$

3 –Solutions.

The selected solution is written as follows [1].

$$Y(x,t)=a_1.\Psi(x).\sin\omega t+a_2.\psi(x).\cos\omega t\qquad (4)$$

Where a_1 and a_2 are interdependent. The pinned-pinned boundary conditions of the pipe span shown may be written as:

$$Y(0,t)=Y(L,t)=0\qquad (5)$$

$$(\partial^2 Y / \partial X^2)(0,t) = (\partial^2 Y / \partial X^2)(L,t) = 0 \qquad (6)$$

These boundary conditions can be satisfied by the set of sinusoidal mode shapes.

$$\psi_n(x) = \sin(n\pi\, x / l) \qquad (7)$$

Let the third and fourth terms from the left of equation of motion (1) which represent the compressive force plus the coriolis force equal to [W(x,t)].

$$W\,(x,t) = \left(\rho A_f .U^2 + PA_p\right).(\frac{\partial^2 Y}{\partial x^2}) + \left(2\rho A_f .U\right)(\frac{\partial^2 Y}{\partial x \partial t}) \qquad (8)$$

So, the expression obtained from the solution for these specified two fluid forces terms is given by [7].

$$W = \Bigg(\Bigg[\Bigg[\left\{a_1\left(PA_{p0} + \rho A_f U^2\right)\left(\frac{n\pi}{L}\right)^2 .\sin\left(\frac{n\pi x}{L}\right)\right\}^2 -$$
$$\left\{2a_1\left(PA_{p0} + \rho A_f U^2\right)\left(\frac{n\pi}{L}\right)^2 .\sin\left(\frac{n\pi x}{L}\right).2.\rho.A_f.U.a_2.w.\left(\frac{n\pi}{L}\right).\cos\left(\frac{n\pi x}{L}\right)\right\} +$$
$$\left\{a_2.2.\rho.A_f.U.w.\left(\frac{n\pi}{L}\right).\cos\left(\frac{n\pi x}{L}\right)\right\}^2\Bigg] + \Bigg[\left\{a_1.2.\rho.A_f.U.\left(\frac{n\pi}{L}\right).w.\cos\left(\frac{n\pi x}{L}\right)\right\}^2$$
$$-\left\{2a_1.2.\rho.A_f.U.\left(\frac{n\pi}{L}\right).w.\cos\left(\frac{n\pi x}{L}\right).a_2.\left(PA_{p0} + \rho.A_f.U^2\right)\left(\frac{n\pi}{L}\right)^2 .\sin\left(\frac{n\pi x}{L}\right)\right\}$$
$$+\left\{a_2.\left(PA_{p0} + \rho.A_f.U^2\right)\left(\frac{n\pi}{L}\right)^2 .\sin\left(\frac{n\pi x}{L}\right)\right\}^2\Bigg]\Bigg)^{\frac{1}{2}} \qquad (9)$$

4- Theory of State Vector and Transfer Matrix Method:

At any station on the annular pipe there are two state vectors, one to the right, and the other to the left of the station. $\overline{(Z)}_n^R$ Is the state vector to the right of station (n), similarly, $\overline{(Z)}_n^L$ is the state vector to the left of the station (n). The state vector $\overline{(Z_n)}$ for outer and inner pipe of the annular is defined by.

$$\overline{(Z_n)} = \left[Y_o, \theta_o, M_o, V_o, Y_i, \theta_i, M_i, V_i, U_f, P_f : 1 \right] \qquad (10)$$

The flexural properties of the segment are described by the field transfer matrix of the span, and the internal effect of the segment is described by the point transfer matrix of the mass. Hence, the transfer matrix of segment consists of two parts:

1. The field transfer matrix of the elastic member, due to (K_i) and (K_o), and the effect of fluid physical properties (w_e).
2. The point transfer matrix is due to the inertia of the internal and external masses (m_i) and (m_o) of the annular pipe and the flexibility of the supports.

The steady–state condition caused by a harmonic excitation is more readily solved with the aid of a particular integral of the nonhomogenous differential equation. Using this approach if the forcing term has a circular frequency (ω), the system will vibrate in its steady state with the same circular frequency, but with an amplitude and phase dependent on the value of (ω). This fact enables us to extend the application of the transfer matrix method to steady-state forced vibration.

The simply supported pipe conveying fluid is subdivided into a number of elements and stations as shown in fig(1) which enable transfer-matrix to relate any adjacent state vector. The fluid flow forces (Compressive and Coriolies) will be concentrated at each field element, The state vectors for each field element and point station consist of the following state variables for outer and inner pipe; deflection (Y_i) and (Y_o), slope (θi) and (θ_o), moment (M_i) and (M_o), shear force (V_i) and (V_o), the fluid velocity (U), and the fluid pressure (P). The harmonic force is assumed to be imposed at mid length of the annulus pipe. .

5- Field Matrix (Element Matrix).

In order to determine the transfer matrix of any beam elements arbitrary oriented in space, a portion of pipe must be considered in (x) and (y) plane. Consider now the outer and inner pipe portion only between the points (n) and (n-1) in the (x-y) plane.

The forces and deflections are illustrated as shown in fig (2). The equilibrium of the massless pipe element length (L_e) requires that the sum of the vertical forces be zero also the sum of the moments about point (n-1) be zero, as shown in appendix (1).

6- Point Matrix:
In order to determine the transfer matrix of any beam elements arbitrary oriented in space, we need three types for point matrix
1-The point matrix for a particular node with concentrated mass for the outer and the inner pipe may be written as shown in appendix (2).
2-The point matrix for the supported node with concentrated mass shown in fig (4) may be written as shown in appendix (3).
3-Field and Point Matrix with thermal forces:
The thermal forces effect on the element and mass is shown in fig (5) &fig (6), the displacement and thermal force in the x-direction may be written as shown in appendix (4).
Using the point and field matrices due to the thermal effects for all the nodes and eliminating the intermediate nodes , then applying the boundary conditions (X_o) & (X_i) equal to zero at the supports , thermal forces at all nodes may be evaluated.
The common boundary conditions for the annular pipe problem with flexible support are:

$$\{Y_o, \theta_o, M_o, V_o, Y_i, \theta_i, M_i, V_i, U, P\} = \{Y_o, \theta_o, 0, 0, Y_i, \theta_i, 0, 0, U, P\}$$

Since the shear force and moment at boundaries of a flexible support must be zero while the slope and the deflection are unknown and nonzero, the fluid velocity and pressure are assumed to be known and nonzero.
Then applying the boundary conditions to the adjacent state vectors of the last obtained equations yields the results to find the state variables at all points in the system domain.
7-Results and discussion
A suitable FORTRAN language program has been developed to embrace the theoretical work. The pipe span was discrtized into ten elements and eleven point stations and the forced vibration at different excitation frequencies is imposed at station (6) which represented the mid length of the annular pipe. Fig (7) and Fig (8) show that the fluid force generally increases as the excitation frequency increases for different values of heat flux and fluid velocity, due to the local dynamic effect of the fluctuated force of the fluid which interacts with the Coriolis and compressive force

for the zone of natural frequencies. It can be seen from fig (9) that in general the excitation force increases as the frequency increases, this behavior is reflected directly on the behavior of the Coriolis force. Fig (10) and fig (11) show the variation of bending moment for simply supported annular pipe conveying fluid with various excitation frequencies at mid length with various heat flux and fluid velocity. This figure shows the bending moment values of the outer pipe are higher than that of the inner pipe at the same range of excitation frequencies. This is due to the higher values of the thermal forces in the outer pipe in comparison with the inner pipe. Also, it may be observed from the figure, the positions of the natural frequencies at the peak values as well as it is obvious that the fluctuated values of the bending moment increase as the flow velocities decrease due to the increasing values of the thermal forces. It can be noticed from fig (12) the following:

* The deflection in general increases as the fluid velocity increases for the same heat and range of excitation angular frequencies. This may be due to the increasing values of the Coriolis forces.
* As the heat flux increases (6.2-12.5 kW/m^2) the response increases for the same values of fluid velocities and excitation frequencies. This may be due to the increasing values of the thermal forces.

Fig (13) and fig (14) show the effect of heat flux on the response of the outer and inner pipe for flexible and rigid support in annular pipe conveying fluid. It is obvious from these figures the following:-

*The values of the natural frequencies in the rigid support are higher than that of flexible support since the overall stiffness of the system with flexible support will be decreased for the same value of heat flux.
* As the heat flux increases, the flexibility of the pipe and the support increases, (overall stiffness increasing), causing the value of the natural frequencies to decrease for the same support and fluid velocity, however this decreasing differs in the order of the natural frequency. It is also clear from table (1) to table (3) the first three natural frequencies calculated by transfer matrix method for a simply supported annular pipe conveying fluid with the variation of heat flux, fluid velocity and type of support.

Table (1) three lowest natural frequencies of the system with different support configurations and different fluid velocity

Heat flux (kW/m^2)	Flexible support (rad/s)			Rigid support (rad/s)		
q = 12.5	ω_{n1}	ω_{n2}	ω_{n3}	ω_{n1}	ω_{n2}	ω_{n3}
U= 0.0158 (m/s)	119.3	163.0	364.4	257.6	314	383.2
U= 0.0635 (m/s)	125.6	169.6	364.4	263.9	364.4	-

Table (2) three lowest natural frequencies of the system with different support configurations and different fluid velocity.

Heat flux (kW/m^2)	Flexible support (rad/s)			Rigid support (rad/s)		
q = 6.2	ω_{n1}	ω_{n2}	ω_{n3}	ω_{n1}	ω_{n2}	ω_{n3}
U= 0.0158 (m/s)	125.6	175.9	351.9	263.9	307.8	----
U= 0.0635 (m/s)	128.2	175.9	351.9	270	326.7	364.4

Table (3) three lowest natural frequencies of the system with different support configurations and different fluid velocity.

Heat flux (kW/m^2)	Flexible support (rad/s)			Rigid support (rad/s)		
q = zero	ω_{n1}	ω_{n2}	ω_{n3}	ω_{n1}	ω_{n2}	ω_{n3}
U= 0.0158 (m/s)	129.8	209.2	330.7	153.9	293.4	345.2
U= 0.0635 (m/s)	131.9	221.2	400.7	266.2	400.7	-----

Fig (15) to fig (17) show first, second and third bending mode shape for fluid velocity of range (0.0158 – 0.0635 m/s) with various heat fluxes. In general it can be observed from these figures as heat flux increases the annular pipe vibrates in natural frequencies less than its values without heat for high fluid velocity (0.0635 m/s), while at low fluid velocity of (0.0158 m/s) at the third mode shape the increasing effect of thermal forces is clearly observed such that the system vibrates by a higher next natural frequency and hence by its associated mode shape. i.e. the outer and inner

pipe vibrates in the next mode . This may be due to the effect of the thermal forces which decreases the natural frequency. Fig(18) and fig(19), show the first and the second bending mode of outer and inner annulus pipe conveying fluid with various heat flux and fluid velocity (U = 0.0635 m/s) and the first three lower values of natural frequencies under no heat condition. It can be seen from these figures that the effect of heat flux increasing on bending moment is very small on the second and third natural frequencies, but this effect is highly increased at the first mode shape.

Fig (20) and fig (21) show the slope of outer and inner annulus pipe conveying fluid with various heat flux and fluid velocity range (0.0158-0.0635 m/s), it can be seen that the slope increases as the heat flux and fluid velocity increase and the natural frequencies decrease due to temperature rises, however these figures show a linear vibration system since the highest value of the slope doesn't exceed 5 degree [12].

8-Conclusions:

The following can be concluded from the present work:

1- The fluid forces (Coriolis&Compressive) greatly affect the response of the undamped annular pipe under vibration.

2- The outer and the inner pipes of the annular may vibrate individually in different mode shapes.

3- The value of the fundamental natural frequency for flexible support is less than that obtained for rigid support with a maximum difference of (50%) for low frequency and (4%) for a high frequency for the adopted stiffness values.

4- The effect of heat flux of the system is greater than it's the fluid velocity effect on the natural frequency.

5- In general, the values of the bending moments in outer pipe are higher than those for the inner pipe at the same excitation frequency.

6- Increasing the heat flux may result in increasing the thermal forces which may lead the system to vibrate at a higher natural frequency (i.e. if it is vibrating in the third mode it may vibrate in the fourth mode under the same frequency).

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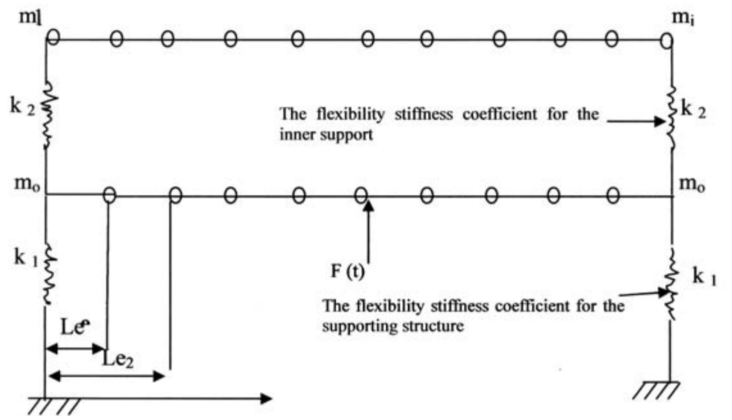


Fig (1) annular pipe with lumped elements and masses

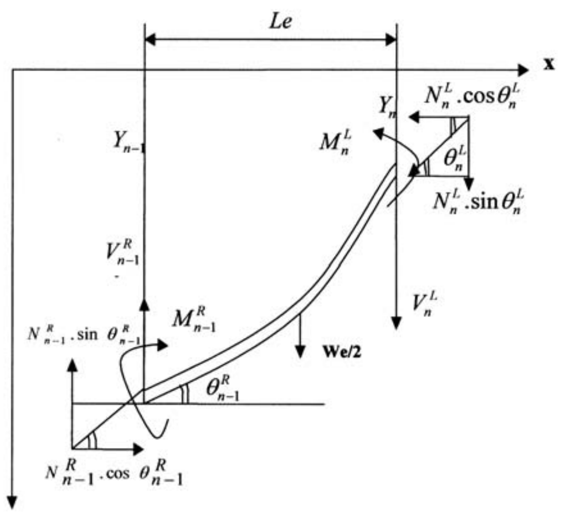


Fig (2) End forces, moments, thermal forces and deflections for massless outer and inner beam (free body sketch of span)

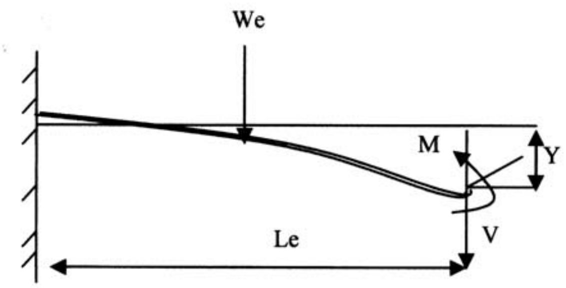


Fig (3) cantilever subjected to end moment and shear

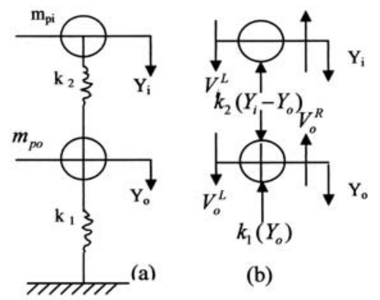


Fig (4) a: Flexible support; b: free-body diagram for the flexible support.

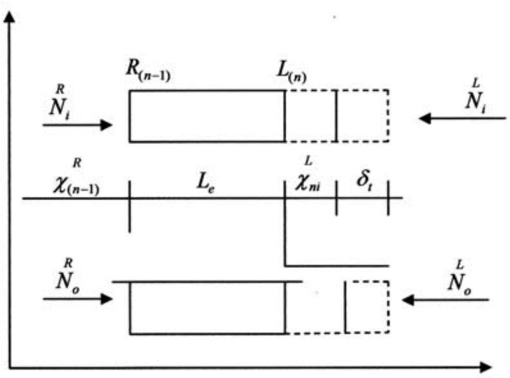


Fig (5) pipe element under thermal forces

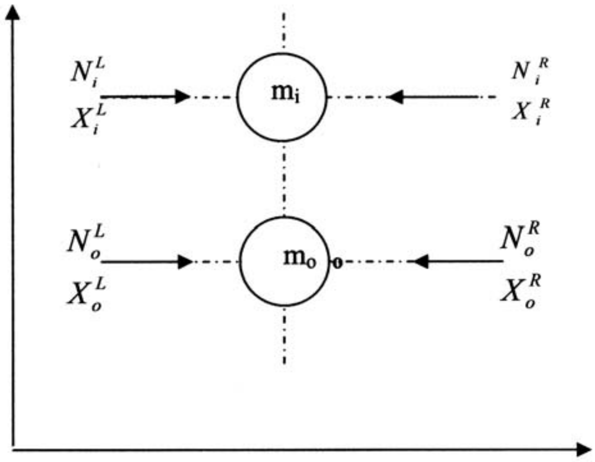


Fig (6) free-body diagram of mass (mi & mo) under thermal forces

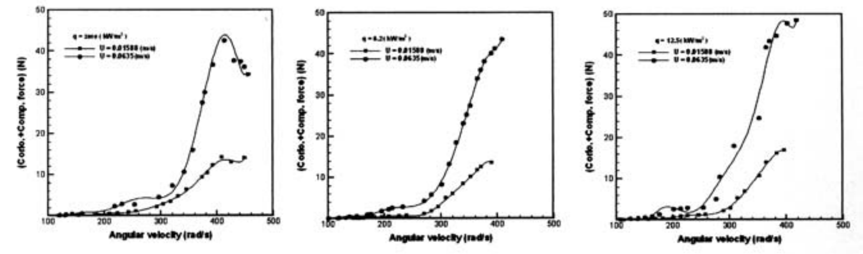


Fig (7) Coriolis and Compressive force of a simply support annulus pipe conveying fluid due to forced vibration at mid length with various velocities and heat flux

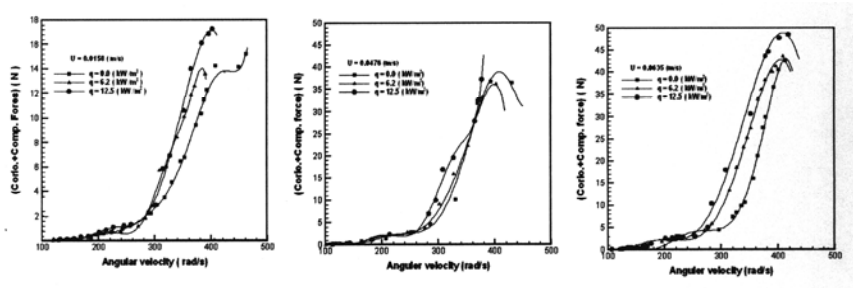


Fig (8) Coriolis force of simply supported annulus pipe conveying fluid due to forced vibration at mid length with various heat flux and various velocities

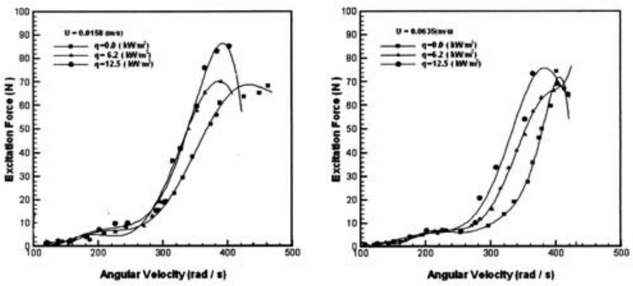


Fig (9) Excitation force for a simply support annular pipe conveying fluid with various excitation frequencies at mid length with various heat flux and velocities

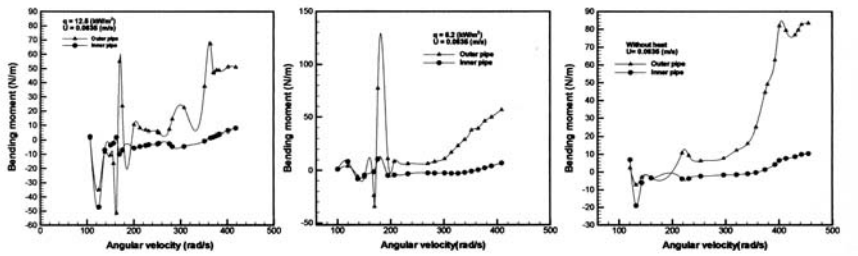


Fig (10) Theoretical bending moment for a simply supported annular pipe conveying fluid with various excitations frequencies (Angular velocity / 2π) at mid length with various heat flux and (u = 0.0635 m/s) for flexible support.

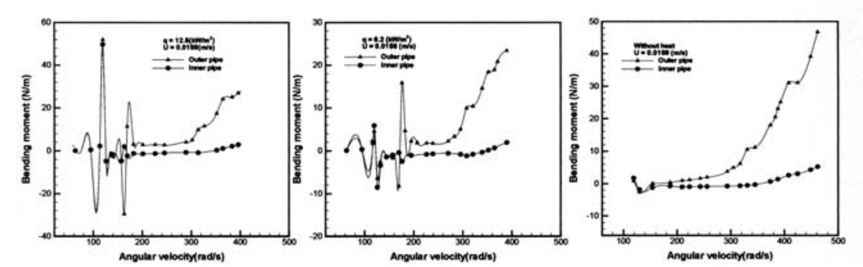
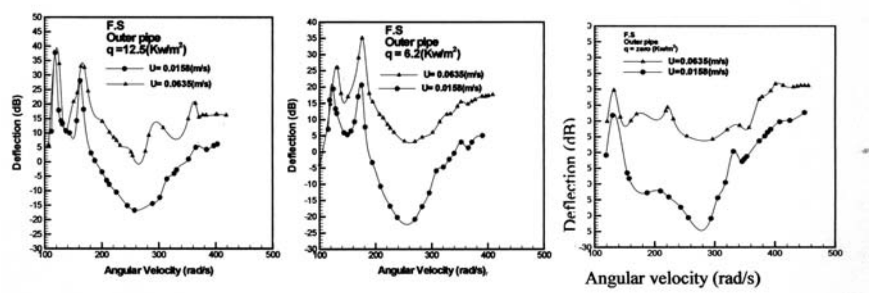


Fig (11) Theoretical bending moment for a simply supported annular pipe conveying fluid with various excitations frequencies (Angular velocity / 2π) at mid length with various heat flux and (u = 0.0158 m/s) for flexible support.



Fig(12)Deflection for simply support annular pipe conveying fluid with various excitation frequencies at mid length outer pipe for various velocities.

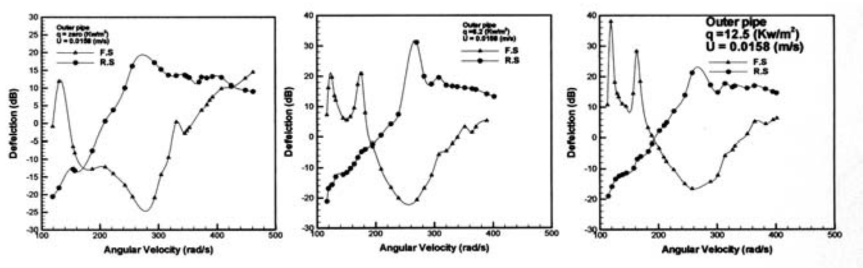


Fig (13) Outer pipe deflection of a simply supported annular pipe conveying fluid due to forced vibration at mid length for different type of support and heat flux with flow rate (u =0.0158 m/s)

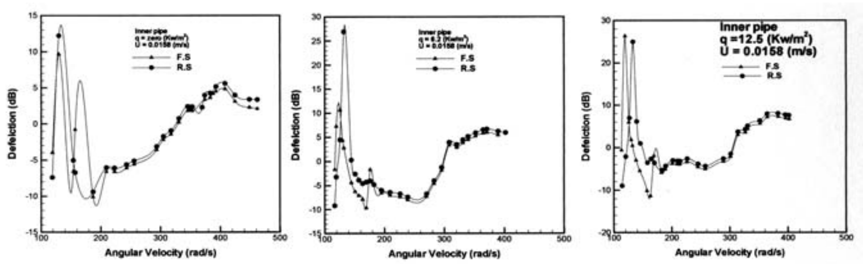


Fig (14) Inner pipe deflection of a simply supported annular pipe conveying fluid due to forced vibration at mid length for different type of support and heat flux with flow rate (u = 0.0158 m/s)

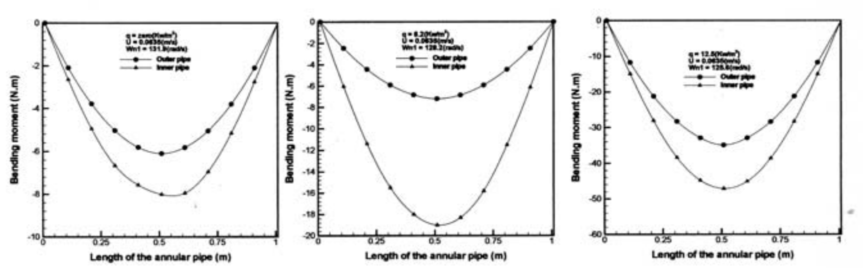


Fig (15) First bending mode of outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U= 0.0635 m/s)

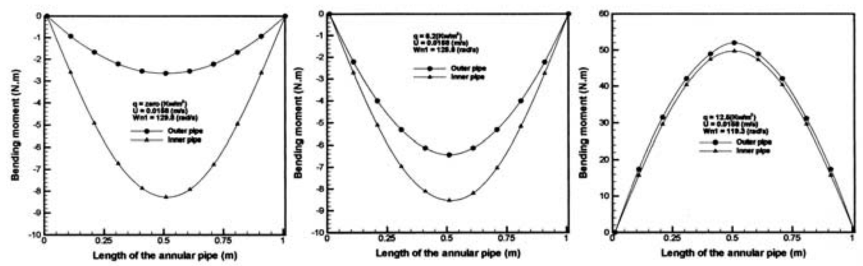


Fig (16) First bending mode of outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U= 0.0158 m/s)

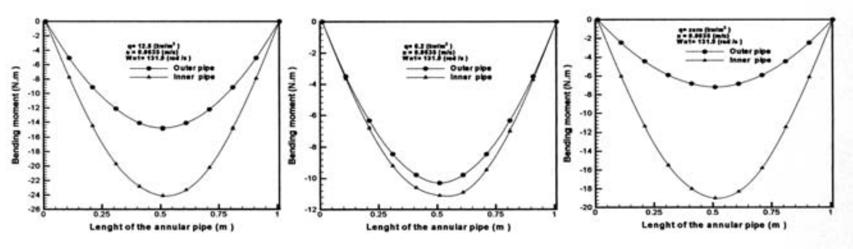


Fig (17) First bending mode of outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U= 0.0635 m/s) with first natural frequency (ω_{n1}=131.9 rad/s)

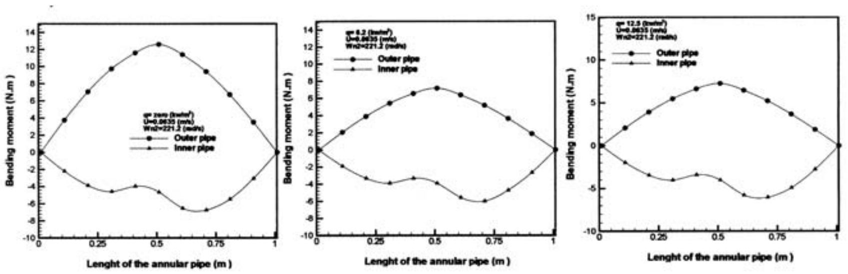


Fig (18) First bending mode of outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U= 0.0635 m/s) with first natural frequency (ω_{n2}=221.2 rad/s)

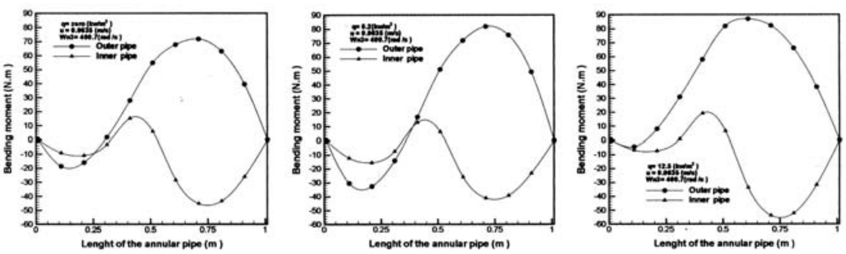


Fig (19) First bending mode of outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U= 0.0635 m/s) with first natural frequency (ω_{n3}=400.7 rad/s)

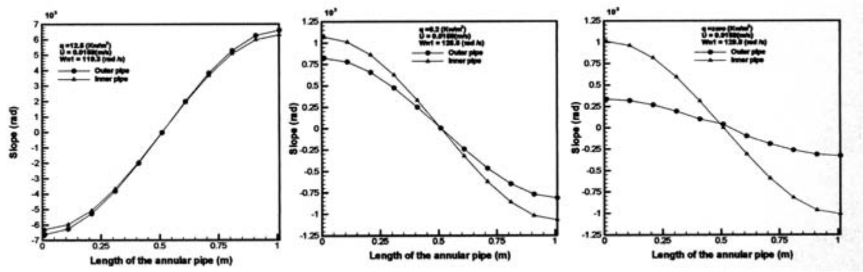


Fig (20) the slope outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U = 0.0158 m/s) with first natural frequencies

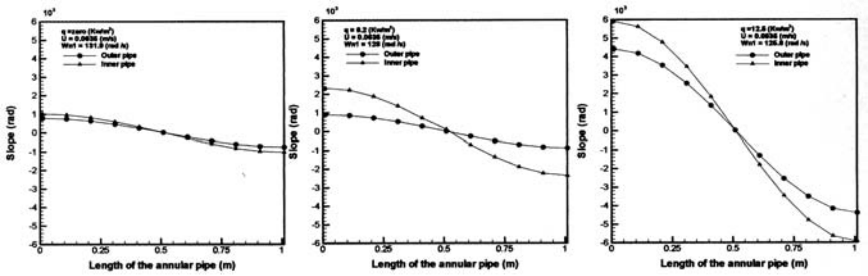


Fig (21) the slope outer and inner from annulus pipe conveying fluid with various heat flux and fluid velocity (U = 0.0635 m/s) with first natural frequencies

APPENDIX (1)

The forces and deflection are illustrated as shown in fig (2). The equilibrium of the massless pipe element length (L_e) requires that the sum of the vertical forces be zero also the sum of the moment about point (n-1) be zero, Hence:

$$\left(\overset{L}{V}_n+\overset{L}{N}_n.\sin\overset{L}{\theta}_n\right)+W_e/2-\left(\overset{R}{V}_{n-1}+\overset{R}{N}_{n-1}.\sin\overset{R}{\theta}_{n-1}\right)=0 \qquad (1\text{-}1)$$

$$\overset{L}{M}_n=\overset{R}{M}_{(n-1)}+\overset{R}{V}_{(n-1)}.L_e+\overset{R}{N}_{(n-1)}.\overset{R}{\theta}_{(n-1)}.L_e-\frac{W_e.L_e}{4} \qquad (1\text{-}2)$$

For simple beam theory, the deflection and slope of a cantilever of flexural stiffness (EI) subjected to a bending moment and shear force applied at its free end as shown in fig (3) are given by [10].

$$Y = -\frac{M.L^2}{2.EI} + \frac{V.L^3}{3EI}, \qquad \theta = \frac{M.L}{EI} - \frac{V.L^2}{2EI}$$

Hence, it may be written in the form,:

$$\overset{L}{\overline{Y}}_{n0} = \overset{R}{\overline{Y}}_{(n-1)0} - \frac{\overset{R}{\overline{\theta}}_{(n-1)0}}{L_m}\left[L_e + \overset{R}{\overline{N}}_{(n-1)0}\left(\frac{L_e^3}{6E_0I_0} - \frac{x_0L_e}{G_0A_{p0}}\right)\right] - \overset{R}{\overline{M}}_{(n-1)0}\left(\frac{L_e^{\,2}.(E_0I_0)_m}{2L_m^{\,2}(E_0I_0)}\right) -$$
$$\overset{R}{\overline{V}}_{(n-1)0}.\frac{(E_0I_0)_m}{L_m^{\,3}}\left[\frac{L_e^3}{6E_0I_0} - \frac{x_0L_e}{G_0A_{p0}}\right] + \frac{W_e}{2L_m}\left[\frac{L_e^3}{48E_0I_0} - \frac{x_0L_e}{G_0A_{p0}}\right] \qquad (1\text{–}3)$$

$$\overset{L}{\overline{\theta}}_{n0} = \overset{R}{\overline{\theta}}_{(n-1)0}\left[1 + \overset{R}{\overline{N}}_{(n-1)0}.\frac{L_e^2}{2E_0I_0}\right] + \overset{R}{\overline{M}}_{(n-1)0}.\frac{L_e.(E_0I_0)_m}{L_m.(E_0I_0)} +$$
$$\overset{R}{\overline{V}}_{(n-1)0}.\frac{L_e^{\,2}.(E_0I_0)_m}{L_m^{\,2}.2(E_0I_0)} - W_e\frac{L_e^{\,2}}{16E_0I_0} \qquad (1-4)$$

$$\overset{L}{\overline{M}}_{n0} = \overset{R}{\overline{\theta}}_{(n-1)0}\left[\overset{R}{\overline{N}}_{(n-1)0}.\frac{L_e.L_m}{(E_oI_o)_m}\right] + \overset{R}{\overline{M}}_{(n-1)0} + \overset{R}{\overline{V}}_{(n-1)_0}.\left(\frac{L_e}{L_m}\right) - W_e\left(\frac{L_e.L_m}{4\,(E_oI_o)_m}\right) \qquad (1\text{-}5)$$

$$\overset{L}{\overline{V}}_n = \overset{R}{\overline{\theta}}(n-1)_o.\frac{L_m^2}{(E_oI_o)_m}.\left[\overset{R}{\overline{N}}(n-1)_o\left(1 - \overset{L}{\overline{N}}no.\frac{L_e^2}{2E_oI_o}\right) - \overset{L}{\overline{N}}no\right] - \overset{R}{\overline{M}}(n-1)o.\left(\frac{L_e.L_m\overset{L}{\overline{N}}no}{(E_oI_o)}\right) +$$
$$\overset{R}{\overline{V}}(n-1)o\left[1 - \overset{L}{\overline{N}}no.\frac{L_e^{\,2}}{2(E_oI_o)}\right] - \frac{W_e}{2}\left(\frac{L_m^2}{(E_o\,I_o\,)_m}\right)\left[1 - \overset{L}{\overline{N}}no.\frac{L_e^2}{8E_oI_o}\right] \qquad (1-6)$$

Where, $(V.x.L_e / G.A_p)$, is the deflection caused by the action of the shearing forces, is given by [11], for the annular cross section pipe as :

$$\frac{4.V.L_e}{3.G.A_p}.\left(1 + \frac{R.r}{R^2 + r^2}\right)$$

X, is the numerical factor by which the average shear stress must be multiplied in order to allow for its distribution over the transverse section. Referring to fig (2), the dimensionless forms for the deflection, slope, moment and shear force of the inner pipe may be written as follows:

$$\overset{L}{Y}_{ni}=\overset{R}{Y}_{(n-1)i}-\frac{\overset{R}{\theta}_{(n-1)i}}{L_m}\cdot\left[L_e+\overset{R}{N}_{(n-1)i}\left(\frac{L_e^3}{6E_iI_i}-\frac{x_iL_e}{G_i.A_{pi}}\right)\right]-\overset{R}{M}_{(n-1)i}\left[\frac{L_e^{\ 2}.(E_iI_i)_m}{2.L_m^{\ 2}E_iI_i}\right]-$$
$$\overset{R}{V}_{(n-1)i}\cdot\frac{(E_iI_i)_m}{L_m^3}\left[\frac{L_e^3}{6E_iI_i}-\frac{x_iL_e}{G_i.A_{pi}}\right]+\frac{W_e}{2L_m}\left[\frac{L_m^{\ 3}}{48E_iI_i}-\frac{x_iL_e}{G_i.A_{pi}}\right] \qquad (1-7)$$

$$\overset{L}{\theta}_{ni}=\overset{R}{\theta}_{(n-1)i}\left[1+\overset{R}{N}_{(n-1)i}\cdot\frac{L_e^{\ 2}}{2E_iI_i}\right]+\overset{R}{M}_{(n-1)i}\left[\frac{L_e.(E_iI_i)_m}{L_m.E_iI_i}\right]+$$
$$\overset{R}{V}_{(n-1)i}\cdot\left[\frac{L_e^{\ 2}(E_iI_i)_m}{L_m^2\,2.E_iI_i}\right]-W_e\left[\frac{L_e^{\ 2}}{16\ E_iI_i}\right] \qquad (1-8)$$

$$\overset{L}{M}_{ni}=\overset{R}{\theta}_{(n-1)i}\left[\overset{R}{N}_{(n-1)i}\cdot\left(\frac{L_eL_m}{(E_iI_i)_m}\right)\right]+\overset{R}{M}_{(n-1)i}+\overset{R}{V}_{(n-1)i}\cdot\left[\frac{L_e}{L_m}\right]-W_e\left[\frac{L_e.L_m}{4(E_iI_i)_m}\right] \qquad (1-9)$$

$$\overset{L}{V}_n=\overset{R}{\theta}_{(n-1)i}\left(\frac{L_m^{\ 2}}{(E_iI_i)_m}\right)\left[\overset{R}{N}_{(n-1)i}\cdot\left(1-\overset{L}{N}_{ni}\cdot\frac{L_e^{\ 2}}{2E_iI_i}\right)-\overset{L}{N}_{ni}\right]-\overset{R}{M}_{(n-1)i}\left[\frac{L_eL_m\cdot\overset{L}{N}_{ni}}{E_iI_i}\right]+$$
$$\overset{R}{V}_{(n-1)i}\left[1-\overset{L}{N}_{ni}\cdot\frac{L_e^{\ 2}}{2E_iI_i}\right]-\frac{W_e}{2}\left(\frac{L_m^{\ 2}}{(E_iI_i)_m}\right)\left[1-\overset{L}{N}_{ni}\cdot\frac{L_e^{\ 2}}{8E_iI_i}\right] \qquad (1-20)$$

For dimensionless fluid velocity and pressure, let:

$$\overline{U}_n=\left[\frac{m_f}{E_0I_0+E_iI_i}\right]^{1/2}.U_n.L_m \qquad (1-21)$$

$$\overline{P}_n=\left[P_n/P_{(inlet)}\right] \qquad (1-22)$$

The field matrix [F] for a pipe element may be written in matrix notation as follows in dimensionless form as,

$$\begin{bmatrix} -\overline{Y}_0 \\ \overline{\theta}_0 \\ \overline{M}_0 \\ \overline{V}_0 \\ -\overline{Y}_i \\ \overline{\theta}_i \\ \overline{M}_i \\ \overline{V}_i \\ \overline{u} \\ \overline{P} \\ 1 \end{bmatrix}_n^L = \begin{bmatrix} 1 & F_{12} & F_{13} & F_{14} & 0 & 0 & 0 & 0 & 0 & 0 & F_{111} \\ 0 & F_{22} & F_{23} & F_{24} & 0 & 0 & 0 & 0 & 0 & 0 & F_{211} \\ 0 & F_{32} & F_{33} & F_{34} & 0 & 0 & 0 & 0 & 0 & 0 & F_{311} \\ 0 & F_{42} & F_{43} & F_{44} & 0 & 0 & 0 & 0 & 0 & 0 & F_{411} \\ 0 & 0 & 0 & 0 & 1 & F_{56} & F_{57} & F_{58} & 0 & 0 & F_{511} \\ 0 & 0 & 0 & 0 & 0 & F_{66} & F_{67} & F_{68} & 0 & 0 & F_{611} \\ 0 & 0 & 0 & 0 & 0 & F_{76} & F_{77} & F_{78} & 0 & 0 & F_{711} \\ 0 & 0 & 0 & 0 & 0 & F_{86} & F_{87} & F_{88} & 0 & 0 & F_{811} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\overline{Y}_0 \\ \overline{\theta}_0 \\ \overline{M}_0 \\ \overline{V}_0 \\ -\overline{Y}_i \\ \overline{\theta}_i \\ \overline{M}_i \\ \overline{V}_i \\ \overline{u} \\ \overline{P} \\ 1 \end{bmatrix}_{n-1}^R$$

OR

$$\left(\overline{Z}_n\right)^L = \left(F_n\right).\left(\overline{Z}_{n-1}\right)^R \qquad (1\text{-}23)$$

Hence (F_n) is the field transfer matrix which relates the state vectors $\left(\overline{Z}_{n-1}\right)^R$ and $\left(\overline{Z}_n\right)^L$ at the beginning and the end of the span (Le).

APPENDIX(2)

The displacement, slope, and moment at the mass (m_{no} and m_{ni}) can be written in dimensionless forms as :

$$\left.\begin{aligned} \overset{L}{\overline{Y}}_{n0} &= \overset{R}{\overline{Y}}_{n0} \;\&\; \overset{L}{\overline{Y}}_{ni} = \overset{R}{\overline{Y}}_{ni} \\ \overset{L}{\overline{\theta}}_{n0} &= \overset{R}{\overline{\theta}}_{n0} \;\&\; \overset{L}{\overline{\theta}}_{ni} = \overset{R}{\overline{\theta}}_{ni} \\ \overset{L}{\overline{M}}_{n0} &= \overset{R}{\overline{M}}_{n0} \;\&\; \overset{L}{\overline{M}}_{ni} = \overset{R}{\overline{M}}_{ni} \\ \overset{L}{\overline{U}}_{n} &= \overset{R}{\overline{U}}_{n} \;\&\; \overset{L}{\overline{P}}n = \overset{R}{\overline{P}}n \end{aligned}\right\} \qquad (2\text{-}1)$$

The force equation of the mass is,

$\sum$Forces= Mass *Acceleration

$$\overset{R}{V}_{ni} - \overset{L}{V}_{ni} = (m_{pi})_n \ddot{Y}_i$$

$$\overset{R}{V}_{no} = \left(m_{po} + m_f\right)_n . \ddot{Y}_o + \overset{L}{V}_{no} - F_o * \sin \omega t$$

$$\overset{R}{\overline{V}}_{ni} = -\omega^2 \, (m_{pi})_n . \frac{L_m^3}{(EI)_m} . \overline{Y}_{ni} + \overset{L}{\overline{V}}_{ni} \qquad (2\text{-}2)$$

$$\overset{R}{\overline{V}}_{no} = -\left(m_{po} + m_f\right)_n . \omega^2 . \frac{L_m^3}{(EI)_m} . \overline{Y}_{no} - F_o . \frac{L_m^2}{(EI)_m} + \overset{L}{\overline{V}}_{no} \qquad (2\text{-}3)$$

Where, $\left(m_{po} + m_f\right)_n . \omega^2 . Y_{no}$ is the inertia force introduced due to vibrating mass for harmonic motion at the excitation frequency (ω).

$$\overset{L}{U}_n = \overset{R}{U}_n \qquad \& \qquad \overset{L}{P}_n = \overset{R}{P}_n \qquad (2\text{-}4)$$

Hence, combining Eqs (2-2), (2-3), (2-4) in matrix notation gives the following point transfer matrix.

$$\begin{bmatrix} -\overline{Y}_o \\ \overline{\theta}_o \\ \overline{M}_o \\ \overline{V}_o \\ \\ -\overline{Y}_i \\ \overline{\theta}_i \\ \overline{M}_i \\ \overline{V}_i \\ \overline{U} \\ \overline{P} \\ 1 \end{bmatrix}_n^R = \left[\begin{array}{cccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & 0 \\ -(m_{p_0} + m_f).\omega^2 L^3 m/(EI)om & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & & -F_{no} L^2 m/(EI)om \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & -m_{pi}.\omega^2 L^3 m/(EI)im & 0 & 0 & 1 & 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \end{array}\right] \begin{bmatrix} -\overline{Y}_o \\ \overline{\theta}_o \\ \overline{M}_o \\ \overline{V}_o \\ \\ -\overline{Y}_i \\ \overline{\theta}_i \\ \overline{M}_i \\ \overline{V}_i \\ \overline{U} \\ \overline{P} \\ 1 \end{bmatrix}_n^L$$

$$\text{OR} \qquad \left(\overline{Z}_n\right)^R = \left(\overline{P}_n\right).\left(\overline{Z}_{n-1}\right)^L \qquad (2\text{-}5)$$

(i.e.) the point transfer matrix at station (n) relates the vectors $\left(\overline{Z}_n\right)^L$ and $\left(\overline{Z}_n\right)^R$ at the left and the right side of the mass $(m)_n$.

APPENDIX (3)

Considering Newton's second law for the mass supported at the station (n), gives,

$$m_{pi}\ddot{Y}_i + K_2\left(Y_i - Y_o\right) + \overset{L}{V_i} = \overset{R}{V_i} \qquad (3\text{-}1)$$

$$\left(m_f + m_{po}\right).\ddot{Y}_o + K_2\left(Y_o - Y_i\right) + K_1.Y_o + \overset{L}{V_o} = \overset{R}{V_o} \qquad (3\text{-}2)$$

Considering the harmonic vibration then:

$$Y = \overline{Y}.\sin\omega t \qquad ,\ddot{Y} = -\omega^2.\overline{Y}.\sin\omega t$$
$$Also$$
$$V = \overline{V}\sin\omega t \qquad ,M = \overline{M}\sin\omega t$$

Eqs (3-1) and (3-2) may be written in dimensionless form as follows,

$$\overset{R}{\overline{V}_i} = -\left(m_{pi}.\omega^2 - K_2\right).\frac{L_m^3}{(EI)_m}.\overline{Y}_i - K_2.\frac{L_m^3}{(EI)_m}.\overline{Y}_o + \overset{L}{\overline{V}_i} \qquad (3\text{-}3)$$

$$\overset{R}{\overline{V}_o} = -\left(\left(m_f + m_{po}\right).\omega^2 - K_1 - K_2\right).\frac{L_m^3}{(EI)_m}.\overline{Y}_o - K_2.\frac{L_m^3}{(EI)_m}.\overline{Y}_i + \overset{L}{\overline{V}_o} \qquad (3\text{-}4)$$

$$\left.\begin{matrix} \overset{R}{\overline{M}_0} = \overset{L}{\overline{M}_0} & \overset{R}{\overline{M}_i} = \overset{L}{\overline{M}_i} & \overset{R}{\overline{U}_n} = \overset{L}{\overline{U}_n} \\ \overset{R}{\overline{\theta}_0} = \overset{L}{\overline{\theta}_0} & \overset{R}{\overline{\theta}_i} = \overset{L}{\overline{\theta}_i} & \overset{R}{\overline{P}_n} = \overset{L}{\overline{P}_n} \end{matrix}\right\} \qquad (3\text{-}5)$$

The point matrix for a node with a flexible support may be written as:

$$
\begin{bmatrix} -\bar{Y}_o \\ \bar{\theta}_o \\ \bar{M}_o \\ \bar{V}_o \\ -\bar{Y}_i \\ \bar{\theta}_i \\ \bar{M}_i \\ \bar{V}_i \\ \bar{U} \\ \bar{P} \\ 1 \end{bmatrix}_n^R
=
\left[\begin{array}{cccc|cccc|ccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\left(m_f - m_{p0}\right)\omega^2\left(k_1 - k_2\right)L^3 m/(EI)m & 0 & 0 & 1 & -k_2.L^2 m/(EI)m & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-k_2 L^3 m/(EI)m & 0 & 0 & 0 & -\left(\omega^2 m_{pi} - k_2\right)L^3 m/(EI)m & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\begin{bmatrix} -\bar{Y}_o \\ \bar{\theta}_o \\ \bar{M}_o \\ \bar{V}_o \\ -\bar{Y}_i \\ \bar{\theta}_i \\ \bar{M}_i \\ \bar{V}_i \\ \bar{U} \\ \bar{P} \\ 1 \end{bmatrix}_n^L
$$

OR $$\left(\bar{Z}_n\right)^R = \left(\bar{P}_n\right)\left(\bar{Z}_n\right)^L \qquad (3\text{-}6)$$

APPENDIX (4)

The displacement equations for the outer and inner pipes are:

$$
\left.\begin{aligned}
\overset{L}{X}_{(n)0} &= \overset{R}{X}_{(n-1)0} + \delta t - \overset{L}{N}_{(n)0}.\left(L_e / E_0 A_0\right) \\
\overset{L}{X}_{(n)i} &= \overset{R}{X}_{(n-1)i} + \delta t - \overset{L}{N}_{(n)i}.\left(L_e / E_i A_i\right)
\end{aligned}\right\} \qquad (4\text{-}1)
$$

From fig (5):

$$\delta t = \alpha . L_e . \Delta T$$

$$\overset{L}{N}_{(n)0} = \overset{R}{N}_{(n-1)0} \quad \& \quad \overset{L}{N}_{(n)i} = \overset{R}{N}_{(n-1)i}$$

Substituting the above relations in Eq (4-1) gives:

$$
\left.\begin{aligned}
\overset{L}{X}_{(n)i} &= \overset{R}{X}_{(n-1)o} - \overset{R}{N}_{(n-1)o}.\left(L_e / E_o A_o\right) + \alpha_o . L_e . \Delta T_o \\
X^L_{(n)i} &= X^R_{(n-1)i} - \overset{R}{N}_{(n-1)i}.\left(L_e / E_i A_i\right) + \alpha_i . L_e . \Delta T_i
\end{aligned}\right\} \qquad (4\text{-}2)
$$

The above equation can be written in dimensionless forms as:

$$
\left.\begin{aligned}
\overset{L}{\bar{X}}_{(n)0} &= \overset{R}{\bar{X}}_{(n-1)0} - \overset{R}{\bar{N}}_{(n-1)0}.\left(\frac{L_e}{L_m E_0 A_0}\right).\frac{\left(E_0 I_0\right)_m}{L^2{}_m} + \left(\alpha_0 . \Delta T_0 L_e / L_m\right) \\
\overset{L}{\bar{X}}_{(n)i} &= \overset{R}{\bar{X}}_{(n-1)i} - \overset{R}{\bar{N}}_{(n-1)i}.\left(\frac{L_e}{L_m E_i A_i}\right).\frac{\left(E_i I_i\right)_m}{L^2{}_m} + \left(\alpha_i . \Delta T_i L_e / L_m\right)
\end{aligned}\right\} \qquad (4\text{-}3)
$$

Hence, combining Eqs (4-3) in matrix notation gives the following field transfer matrix.

$$\begin{bmatrix} \bar{x}_0 \\ \bar{N}_0 \\ \bar{x}_i \\ \bar{N}_i \\ 1 \end{bmatrix}^L_{(n)} = \left[\begin{array}{cc|cc|c} 1 & \left(\frac{-L_e}{L_m E_0 A_0}\right)\cdot\left(\frac{(E_0 I_0)_m}{L^2{}_m}\right) & 0 & 0 & \alpha_0 \Delta T_0 \left(\frac{L_e}{L^2{}_m}\right) \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & \left(\frac{-L_e}{L_m E_i A_i}\right)\cdot\left(\frac{(E_0 I_0)_m}{L^2{}_m}\right) & \alpha_i \Delta T_i \left(\frac{L_e}{L^2{}_m}\right) \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} \bar{x}_0 \\ \bar{N}_0 \\ \bar{x}_i \\ \bar{N}_i \\ 1 \end{bmatrix}^R_{(n-1)}$$

OR $$\left(\overline{Z}_n\right)^L = \left(F_n\right).\left(\overline{Z}_{(n-1)}\right)^R \qquad (4\text{-}4)$$

As well as, from fig (6) the point matrix for thermal force and displacement may be written as follows:

$$\begin{bmatrix} \bar{x}_0 \\ \bar{N}_0 \\ \bar{x}_i \\ \bar{N}_i \\ 1 \end{bmatrix}^R_{(n)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_0 \\ \bar{N}_0 \\ \bar{x}_i \\ \bar{N}_i \\ 1 \end{bmatrix}^L_{(n)}$$

OR $$\left(\overline{Z}_n\right)^R = \left(P_n\right).\left(\overline{Z}_{(n-1)}\right)^L \qquad (4\text{-}5)$$

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