# Elastodynamic Analysis of Wave Propagation in Layered Half-Space Media in Time Domain

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# Abstract

A time domain formulation in a cylindrical coordinate system is developed for the analysis of wave propagation in a layered half-space media. The formulation is based on Apsel & Luco's derivation of the problem in frequency domain by using Fourier synthesis. An integral equation is written for each layer as an independent domain, and they are assembled in to general equation taking into account the traction-free condition, the condition of continuity between the interfaces, and the radiation condition at infinity. The displacement and stress fields are expressed in a high efficiency and highly flexible way to be very convenient for applications of boundary-value problems. Examples of three-dimensional wave propagation in the layered half-space due to inner excitations are reported to demonstrate the accuracy of the method.

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#### Introduction

Solution of transient elastodynamic problems using the boundary element method (BEM) started with the work of Cruse and Rizzo (1968) who formulated the problem in Laplace domain. The frequency domain solution to equations of motion in cylindrical coordinates are written schematically as:

$$\sum_{n} \frac{\cos(n\theta)}{\sin(n\theta)} \cdots \int_{0}^{\infty} Fn(k,\omega,Z,Zs,L) Jm(kr) dk$$

In which the azimuthal dependence is represented by the Fourier series expansion, while the depth and radial dependence appear in the integrands, which correspond to solutions to the equations of motion in frequency domain, the arguments k,  $\omega$ , z, zs, L in the function F show the dependence on wavenumber, frequency, receiver and source depth, and layer properties, respectively. The argument kr of Bessel function Jm reveals the dependence on radial observation distance. The function F may be obtained as the solution to a set of linear algebraic equations together with boundary conditions of layered system. The first solutions was presented by Thomson (1950), who formulated the problem in terms of so-called layer matrices which transfer the components of motion from interface to interface in an elastic multilayered medium. Haskell (1962) evaluated the F functions at the free surface using Thomson's matrix formalism by finding the surface-wave poles of the function F. The Thomson-Haskell technique becomes unstable for short wave-length due to the computation of squares of large exponential terms.

Thrower & Dunkin (1965), and Watson (1970) modified Thomson-Hskell formulation using determinant matrix extensions. Hudson extended the work of Haskell, and Herrmann (1978) shows the truncation of the terms with poles leads to non-causal arrivals. They use contour integration to study the complete SH-Love wave propagation problem in layered media; however, this analysis is limited to large epicentral distances.

Fuchs and Muller (1971) introduce the attenuation into the layers in the form of complex velocities. This makes the medium more realistic (viscoelastic), and shifting all the singularities of the F integrands. However, Fuchs and Muller introduce several approximations, which limit the applicability of their method.

R. J. Apsel; J. E. Luco (1979) formulated and solved the problem of three-dimensional wave propagation in layered viscoelastic media in frequency domain. The integrands of these Hankel transform-type integral representations correspond to complete solutions of the equations of motion in frequency domain. The F integrands are given in terms of highly efficient factorizations of the upgoing and downgoing wave amplitude in each layer. So as to include all the reflection, conversion, transmission properties of the layered medium. The calculation of the three-dimensional Green's functions are conducted in the frequency domain by representing the complete response in terms of semi-infinite integral with respect to wavenumber so as to automatically include all types of waves. The complete displacement and stress fields at multiple receiver points anywhere in the layered viscoelastic medium are efficiently evaluated for different types of sources. Possible studies include the generation of nonreflecting boundary conditions for use in properly modeling the extended earth or structure with finite element method (FEM). It is also important that the method provide accurate solutions across the entire frequency band of interest. The final requirement is that the methodology remains cost-efficient and highly flexible.

Work directly in time domain poses the following aspects: (1) the transient response can be directly dealt without needs inverse transformation; (2) it can be dealt with non-linear problem. This problem has been analyzed numerically using finite difference method (FDM) and (FEM). These methods are insufficient to satisfy the radiation conditions at infinity for infinite or semi-infinite domains. Thus, the necessity arises to provide the conditional boundaries, such as the transmitting boundary Lysmer & Wass (1972), non-reflection boundary Smith, (1974) and viscous boundary Lysmer & Kuhlemeyer, (1969), thereby causing a loss of accuracy and consuming computation time. Besides, the discretization has a great effect on the accuracy. The (BEM) is relatively new method capable of making up for the above disadvantages. Y.K. Cheung, Lie, and Tham (1992) have developed a time domain (BEM) in a cylindrical coordinate for the wave propagation in a half-space. The integral formulation is based dynamic reciprocal theorem and stokes fundamental solutions. And they (1993) have developed a time domain (BEM) in a cylindrical coordinate for the wave propagation in a layered half-space.

# Formulation Of Wave Propagation In Layered Media In Time Domain

The present work have developed the methodology of (R. J. Apsel; J. E. Luco) formulating the problem of three-dimensional wave propagation in layered viscoelastic media in time domain, since it is efficient and highly flexible. The time domain formulations are generated through Fourier synthesis, which applied on the solution of inhomogeneous equations of motion for homogeneous viscoelastic medium. The layered viscoelastic half-space under consideration is assumed to be formed by N parallel horizontal layers overlying a uniform half-space as shown in figure (1).

Each of the N+1 viscoelastic media forming the layered half-space is characterized by compressional wave velocity Cp, shear wave velocity Cs, and density  $\rho_j$ . It is assumed that the buried source corresponds to a concentrated point load within the lth medium at the point of coordinates (0, 0, zs).



Figure (1): Model for layered viscoelastic half-space.

In the *jth* medium, the displacement vector in cylindrical coordinates must satisfy the homogeneous or inhomogeneous equations of motion depending on whether or not the source is located within the *jth* 

medium. In addition, the displacement and stress fields must satisfy the traction-free boundary condition on the surface, the condition of continuity of displacement and traction across each interface, and the radiation condition in the underlying half-space. It is possible to express the displacement and stress components in the *jth* medium in the form:

$$\begin{cases} u^{j}(r,\theta,z;t') \\ v^{j}(r,\theta,z;t') \\ w^{j}(r,\theta,z;t') \end{cases} = \frac{r_{o}}{4\pi \,\overline{G} \, r} \sum_{n} Q_{n} \begin{cases} U_{n}^{j}(r_{o},z_{o})\cos n(\theta-\theta_{o}) \\ V_{n}^{j}(r_{o},z_{o})\sin n(\theta-\theta_{o}) \\ W_{n}^{j}(r_{o},z_{o})\cos n(\theta-\theta_{o}) \end{cases}$$
(1a)

$$\begin{cases} \tau_{rz}^{j}(r,\theta,z;t') \\ \tau_{dz}^{j}(r,\theta,z;t') \\ \sigma_{z}^{j}(r,\theta,z;t') \end{cases} = \frac{r_{o}^{\prime 2}}{4\pi r^{\prime 2}} \sum_{n} Q_{n} \begin{cases} T_{rzn}^{j}(r_{o}^{\prime},z_{o}^{\prime})\cos n(\theta-\theta_{o}) \\ T_{den}^{j}(r_{o}^{\prime},z_{o}^{\prime})\sin n(\theta-\theta_{o}) \\ \Sigma_{zn}^{j}(r_{o}^{\prime},z_{o}^{\prime})\cos n(\theta-\theta_{o}) \end{cases}$$
(1b)

$$\begin{cases} \sigma_r^j(r,\theta,z;t') \\ \sigma_\theta^j(r,\theta,z;t') \\ \tau_{\theta^*}^j(r,\theta,z;t') \end{cases} = \frac{r_o^{'2}}{4\pi r^{'2}} \sum_n Q_n \begin{cases} \Sigma_{rn}^j(r_o^{'},z_o^{'}) \cos n(\theta-\theta_o) \\ \Sigma_{\theta n}^j(r_o^{'},z_o^{'}) \sin n(\theta-\theta_o) \\ T_{\theta n}^j(r_o^{'},z_o^{'}) \cos n(\theta-\theta_o) \end{cases}$$
(1c)

where:

$$(Z_{j-1} \le Z \le Z_j, Z_o = 0, Z_{n+1} = \infty, j = 1, n+1)$$
  
 $r'_o = \frac{2\pi r}{\bar{C}_s T}, Z'_o = \frac{2\pi z}{\bar{C}_s T}$ 

Where G correspond to a shear modulus, and superscript – refer to reference medium.  $Q\theta$  denotes to the vertical component, while Q1 represents the horizontal component along the  $\theta = \theta \theta$  of the point load. The terms  $\Sigma rn$ ,  $\Sigma \theta n$ ,  $\Sigma zn$ , Trzn,  $T\theta zn$ ,  $T\theta rn$ , Un, Vn, Wn are given by Hankel integrands:

$$\begin{cases} U_n^j + V_n^j \\ T_{zu}^j + T_{dun}^j \end{cases} = \int_0^\infty \begin{cases} u_n^j (z_0^j, t^i) + v_n^j (z_0^j, t^i) \\ \tau_{21n}^j (z_0^j, t^i) + \tau_{23n}^j (z_0^j, t^i) \end{cases} J_{n+1}(t^i r_o^j) dt^i$$
(2a)

$$\begin{cases} U_n^j - V_n^j \\ T_{ran}^j + T_{den}^j \end{cases} = \int_0^\infty \begin{cases} -u_n^j (z_0^i, t^i) + v_n^j (z_0^i, t^i) \\ -\tau_{21n}^j (z_0^i, t^i) + \tau_{23n}^j (z_0^i, t^i) \end{cases} J_{n-1}(t^i r_o^i) dt^i$$
(2b)

The terms *un*, *wn*,  $\tau 21n$ ,  $\sigma 2n$ ,  $\sigma 3n$ ,  $\sigma 1n$  are associated with waves whose particle motion is polarized in vertical planes (P, Sv, Rayleigh) waves are given by:

$$\begin{cases} u_{n}^{j}(z_{0}^{'},t') \\ w_{n}^{j}(z_{0}^{'},t') \\ \tau_{2\ln}^{j}(z_{0}^{'},t') \\ \sigma_{2n}^{j}(z_{0}^{'},t') \\ \sigma_{3n}^{j}(z_{0}^{'},t') \\ \sigma_{1n}^{j}(z_{0}^{'},t') \end{cases} = \begin{bmatrix} [I_{11}^{j}] & [I_{12}^{j}] \\ [I_{21}^{j}] & [I_{22}^{j}] \\ [I_{31}^{j}] & [I_{32}^{j}] \end{bmatrix} \begin{bmatrix} [E_{d}^{j}(z_{o})] & [0] \\ [0] & [E_{u}^{j}(z_{o})] \end{bmatrix} \begin{cases} \{\eta_{dn}^{j}(z_{0}^{'},t')\} \\ \{\eta_{un}^{j}(z_{0}^{'},t')\} \end{cases}$$
(3)

The terms *vn*,  $\tau 23n$ ,  $\tau 31n$  are associated with waves whose particle motion is polarized in horizontal planes (SH, Love) waves are given by:

$$\begin{cases} v_n^j(z_0^{'},t^{'}) \\ \tau_{23n}^j(z_0^{'},t^{'}) \\ \tau_{31n}^j(z_0^{'},t^{'}) \end{cases} = \begin{bmatrix} I_{11}^j & I_{12}^j \\ I_{21}^j & I_{22}^j \\ I_{31}^j & I_{32}^j \end{bmatrix} \begin{bmatrix} E_d^j(z_o) & 0 \\ 0 & E_u^j(z_o) \end{bmatrix} \begin{cases} \eta_{dn}^j(z_0^{'},t^{'}) \\ \eta_{un}^j(z_0^{'},t^{'}) \end{cases}$$
(4)

Where [I] matrix contains geometrical and mechanical properties of *Jth* layer, the diagonal [E] matrix represents the exponential variation of the corresponding functions across the thickness of *Jth* layer, and vector  $\{\eta\}$  represents the functions correspond respectively to downwardly and upwardly propagating (P, SV) waves for functions in equation (3), and propagating ( SH) waves in equation (4). These terms can be written in the form:

$$\eta_{1n}^{j}(z_{0}^{'},t') = A_{in}^{j}(t') + \delta_{jL}S_{in}^{L}(z_{o},t') \qquad (i = 1,6 ; j = 1, N+1)$$
(5)

In which, Ain(t') are undetermined coefficients,  $\delta jl$  is the delta function, and Sin (zo',t') are the source terms.

Undetermined coefficients Ain(t') or equivalently the unknown functions  $\eta$  in (zo',t'), (i=1,6; j=1,N+1) are determined by imposing the boundary, continuity, and radiation conditions. The free-boundary and continuity conditions can be expressed as:

$$\left\{\eta_{dn}^{1}(0,t')\right\} = \left[R\right]_{o}^{u} \left\{\eta_{un}^{1}(0)\right\}$$
(6a)

$$\{\eta_{dn}^{j+1}(z_o^{'j},t')\} = [T]_j^d \{\eta_{dn}^j(z_o^{'j},t')\} + [R]_j^u \{\eta_{un}^{j+1}(z_o^{'j},t')\}$$
(6b)

$$\left\{\eta_{un}^{j+1}(z_{o}^{'j},t')\right\} = \left[R\right]_{j}^{d} \left\{\eta_{dn}^{j}(z_{o}^{'j},t')\right\} + \left[T\right]_{j}^{u} \left\{\eta_{un}^{j+1}(z_{o}^{'j},t')\right\}$$

$$(6c)$$

$$(i-1,N)$$

$$(j-1, N)$$

In which [R], [T] matrices represent modified reflection coefficients and modified transmission coefficients for the waves impinging on the Jth interface from below and above respectively. The modified reflection and transmission matrices are given by:

$$\begin{bmatrix} R \end{bmatrix}_{o}^{u} = -([I_{21}^{1}])^{-1}[I_{22}^{1}][E_{u}^{1}(0)] \\ \begin{bmatrix} T \end{bmatrix}_{j}^{d} \quad [R]_{j}^{u} \\ [R]_{j}^{d} \quad [T]_{j}^{u} \end{bmatrix} = \begin{bmatrix} -[I_{11}^{j+1}] \quad [I_{12}^{j}] \\ -[I_{21}^{j+1}] \quad [I_{22}^{j}] \end{bmatrix}^{-1} \begin{bmatrix} -[I_{11}^{j}] \quad [I_{12}^{j+1}] \\ -[I_{21}^{j}] \quad [I_{22}^{j+1}] \end{bmatrix} \begin{bmatrix} E_{d}^{j}(z_{o}^{'j})] & 0 \\ 0 & [E_{u}^{j+1}(z_{o}^{'j})] \end{bmatrix}$$

$$(1 \le j \le N)$$

$$(7)$$

The radiation conditions in the bottom half-space (J=N+1) can be expressed by:

$$A_{in}^{N+1}(t') = 0, \quad (i = 3, 4, 6)$$
 (8)

The unknown functions  $\eta$  in (zo',t') in Jth layer can be obtained by the following factorization including the effects of other layers:

$$\begin{cases} \{\eta_{xv}^{j}(\vec{x_{v}},t')\} = [\hat{T}]_{j}^{v}[\hat{T}]_{j+1}^{v} \bullet \bullet [\hat{T}]_{l-1}^{v}\{\eta_{xv}^{j}(\vec{x_{o}^{\prime l-1}},t')\} & (j=1,l-1) \\ (9a) \\ \{\eta_{dv}^{j}(\vec{x_{v}},t')\} = [\hat{R}]_{j-1}^{u}\{\eta_{dv}^{j}(\vec{x_{o}^{\prime l-1}},t')\} & (j=1,l-1) \end{cases}$$

$$\begin{cases} \{\eta_{dv}^{j}(\vec{x_{v}},t')\} = [\hat{T}]_{j-1}^{d}[\hat{T}]_{j-2}^{d} \bullet \bullet [\hat{T}]_{l}^{d}\{\eta_{dv}^{j}(\vec{x_{o}^{\prime l}},t')\} & (j=l+1,N+1) \\ \{\eta_{uv}^{j}(\vec{x_{v}},t')\} = [\hat{R}]_{j}^{d}\{\eta_{dv}^{j}(\vec{x_{o}^{\prime l}},t')\} & (j=l+1,N+1) \end{cases}$$

$$\{\eta_{xv}^{i}(z_{0}^{i},t')\} = ([I] - [\hat{R}]_{I}^{d}[\hat{R}]_{I-1}^{v})^{-1} (\{S\}_{xv}^{i} + [\hat{R}]_{I}^{d}\{S\}_{dv}^{i} \quad (z_{o}^{i-1} \leq z_{o} \leq z_{o}^{i})$$

$$\{\eta_{dv}^{i}(z_{0}^{i},t')\} = [\hat{R}]_{I-1}^{v} \{\eta_{xv}^{i}(z_{o}^{i'-1},t')\} \quad (z_{o}^{i-1} \leq z_{o} \leq z_{o}^{s})$$

$$\{\eta_{dv}^{i}(z_{0}^{i},t')\} = ([I] - [\hat{R}]_{I-1}^{o}[\hat{R}]_{I}^{d})^{-1} (\{S\}_{dv}^{i} + [\hat{R}]_{I-1}^{v}\{S\}_{uv}^{i} \quad (z_{o}^{s} \leq z_{o} \leq z_{o}^{i})$$

$$\{\eta_{xv}^{i}(z_{0}^{i},t')\} = [\hat{R}]_{I}^{d} \{\eta_{dv}^{i}(z_{o}^{i'},t')\} \quad (z_{o}^{i-1} \leq z_{o} \leq z_{o}^{i})$$

$$\{\eta_{xv}^{i}(z_{0}^{i},t')\} = [\hat{R}]_{I}^{d} \{\eta_{dv}^{i}(z_{o}^{i'},t')\} \quad (z_{o}^{i-1} \leq z_{o} \leq z_{o}^{i})$$

In which; the superscript  $\ ^{\wedge}$  denotes to generalized transmission and reflection matrices.

The free-boundary and continuity conditions above the source, as well as the continuity and radiation conditions below the source are satisfied if the generalized transmission and reflection matrices obey respectively the recurrence relations:

$$\begin{split} &[\hat{R}]_{o}^{u} = R_{o}^{u} \\ &[\hat{T}]_{j}^{u} = ([I] - [R]_{j}^{d} [\hat{R}]_{j-1}^{s})^{-1} [T]_{j}^{s} \quad (j \ge 1) \\ &[\hat{R}]_{j}^{u} = [R]_{j}^{u} + [T]_{j}^{d} [\hat{R}]_{j-1}^{u} [\hat{T}]_{j}^{s} \quad (j \ge 1) \\ &[\hat{R}]_{N+1}^{d} = [0] \\ &[\hat{T}]_{j}^{d} = ([I] - [R]_{j}^{u} [\hat{R}]_{j+1}^{d})^{-1} [T]_{j}^{d} \quad (j \le N) \\ &[\hat{R}]_{j}^{d} = [R]_{j}^{d} + [T]_{j}^{u} [\hat{R}]_{j+1}^{d} [\hat{T}]_{j}^{d} \quad (j \le N) \end{split}$$

The factorization given by equations (9) provides the means to determine the displacement and stress fields within each layer above or below the source once the fields in the medium containing the source is known. The fields in the medium containing the source (j = l) can be obtained the use of the following equations:



Figure (2): representation of generalized transmission and reflection coefficients.



Figure (3): representation of recurrence relations for the generalized transmission and reflection coefficients.

$$\left\{\eta_{as}^{\prime}(z_{o}^{\prime},t')\right\} = \left[\left[I\right] - \left[\hat{R}\right]_{t}^{\prime}\left[\hat{R}\right]_{t-1}^{\prime}\right]^{-1} \left(\left\{\left\{S(t')\right\}_{tat}^{\prime} + \left[\hat{R}\right]_{t}^{\prime}\left\{S(t')\right\}_{tat}^{\prime}\right\}\right)$$
(11.a)

$$\left\{ \eta_{ab}^{I}(z_{o}^{i}, t^{i}) \right\} = \left[ \hat{R} \right]_{I=1}^{\sigma} \left\{ \eta_{ab}^{I}(z_{o}^{I-1}, t^{i}) \right\}, \qquad (z_{o}^{I-1} \leq z_{o}^{i} \leq z_{o}^{i})$$
(11b)

$$\left\{ g_{aa}^{l} \left( \boldsymbol{z}_{a}^{l}, t^{\prime} \right) \right\} = \left( \left[ I \right] - \left[ \hat{R} \right]_{l}^{d} \left[ \hat{R} \right]_{l-1}^{n} \right]^{-1} \left( \left\{ \left\{ S(t^{\prime}) \right\}_{aa}^{l} + \left[ \hat{R} \right]_{l}^{d} \left\{ S(t^{\prime}) \right\}_{ab}^{L} \right\} \right)$$
(11c)

$$\left\{ \eta_{ab}^{i}(z_{o}^{'},t') \right\} = \left[ \hat{R} \right]_{i=1}^{r} \left\{ \eta_{ab}^{i}(z_{o}^{\gamma-1},t') \right\}, \qquad (z_{a}^{\gamma-1} \le z_{o}^{'} \le z_{o}^{\gamma})$$
(11d)

The complete description of the displacement and stress fields associated with propagating waves in layered half-space system has been obtained. The particular form of these equations makes possible the simultaneous evaluation of the response at a number of observation locations for a number of different source locations. An accurate and effective developed method for studying wave propagation in a layered viscoelastic half-space has been presented. The three-dimensional wave propagation problem is reformulated and resolved in time domain with azimuthal dependence represented by a Fourier series expansion. This method is based on the generalized reflection and transmission coefficient matrices, the kernel functions F are evaluated in terms of highly efficient factorizations for the upgoing and downgoing wave amplitudes in each layer. The appearance of common factors is taken advantage of when computing the displacement and stress components for multiple source-receiver depth pairs. A realistic attenuation has been introduced for shear and compressional waves in each layer, which shifts the singularities of the F integrands. A numerical integration method is implemented for the F integrands, which are sampled at discrete t' points in each time interval  $\Delta t$ ' satisfying the requirement that quartic polynomials accurately interpolate the amplitudes of the F integrands over each 5-point integration interval. Thereby, the numerical integration with Bessel functions is performed analytically over each integration interval, thus avoiding the oscillation hazards of the Bessel functions. Since the radial dependence appears only the Bessel functions, it is expedient to calculate the integrals for multiple epicentral distances simultaneously.

# The Motion Due To A Buried Dislocation

The displacement field as a function of time at a point resulting from the action of a buried dislocation is derived in terms of the stress tensor solution in time domain evaluated at the depth of the dislocation resulting from the action of an impulse force at that point. As shown in figure (3), the idealized dislocation occurs at a point Y on surface S in volume V.

The receiver is located at point x on the free surface of volume V. Volume V may represents the layered vescoelastic half-space consistent with present work. The slip vector is constrained to have a rake of  $\gamma$  degrees in the plane defining surface S:





#### Figure (4): Source-receiver geometry for buried point dislocation

$$\vec{s}(\vec{y}) = s_{\infty}(\vec{y})[\vec{e}_{1}\cos\gamma + \vec{e}_{2}\sin\gamma\cos\delta + \vec{e}_{3}\sin\gamma\sin\delta]$$
(12)

In equation (12),  $\delta$  is the dip of the fault plane,  $\hat{e}1$ ,  $\hat{e}2$ ,  $\hat{e}3$  are the unit vectors in the Cartesian coordinate system, and  $s\infty(y)$  is the amplitude of the dislocation. Assuming that the displacement and stress components have harmonic time dependence, it is convenient to apply the representation theorem to volume *V* of figure (3). In the absence of sources in *V* and assuming continuity of traction on *S*, the representation theorem is written in time domain to represent  $D\alpha(x, t) \alpha$ -component of displacement at receiver point *x* as:

$$\left\{ D_{\alpha}(\vec{x},t) \right\} = -\iint_{t's} \left[ A(x) \right] \left[ H_{ij}(\vec{x},t;\vec{y},t') \right] \left[ A(y) \right] \left\{ D_{\alpha}(\vec{y},t') \right\} ds dt',$$

$$(i,j = X_1, X_2, X_3), \quad (\alpha = r, \theta, z)$$

$$(13)$$

In which; Hij(x,t;y,t') denotes Green functions corresponding *i*component of the traction vector at point  $y \in S$  at proceeding time interval t' due to unit impulse point load in the *j*-component at xpoint on free surface of the volume V at time interval t, and [A(x)], [A(y)] represent transform matrices between Cartesian and cylindrical coordinates.  $D\alpha$  (y, t') represents the  $\alpha$ -component which it is equivalent to the slip vector component.

The displacements for an extended source can be obtained by spatially integrating over surface S. Green functions Hij(x,t;y,t') consistent with formulation of present work.

# Validation Of The Present Work

Comparison studies of present work are performed to demonstrate the utilizable of the developed method directly in time domain for seismological applications, which represent the determination of exciting force vector as it is considered the first step of soilstructure interaction problem. The particular studies in the validation include comparisons the exact Cagniard-dehoop approach for uniform half-space (Johnson, 1974); complete finite element approach for a layered half-space (Day, 1977). Dash curve refers to present work and continuos curve refers to reference study.

# Comparison With Cagniard- Dehoop Approach:

The representation theorem in time domain is used in conjunction with the present developed method to reciprocally generate the surface motion due to a buried dislocation. The source time dependence is represented by eight second ramp function and Poisson's ratio is taken to be 0.25. The depth of the point dislocation is 5.0 km and the epicentral distance is 20 km. The surface displacements are evaluated at an observation azimuth of 22.5 degrees from strike of the fault, and are normalized by suitable factor.

#### Strike-slip

#### dislocation



Strike=22.5 = 
$$\beta$$
  
Dip = 90 =  $\delta$   
Rake = 0 =  $\gamma$ 







Dip-slip dislocation









# • Comparison With Finite Element Apraoch:

The model consists of two layers overlying a semi-infinite half-space as shown in figure (7), where the individual parameters characterizing the layers are defined. Source depth of 5.0 km and 1.0 km are considered and the source time-dependence is represented by a ramp of one-second duration in both cases. The source is equivalent to the vertical strike slip dislocation with receivers located at epicentral distances of 5, 15, 25, and 35 km at an azimuth of 22.5 degrees from the strike of the fault. The ground motion is normalized by suitable factor.



Figure (7): Source-receiver geometry model of two layers overlying half-space.







#### Conclusions

An accurate and effective developed method for studying wave propagation in layered viscoelastic half-space has been presented. The three-dimensional wave propagation problem reformulated in time domain with the azimuthal dependence represented by a Fourier series expansion. The complete response of quantities field include all type of waves. Based on the generalized reflection and transmission coefficient matrices, the kernel functions are evaluated in terms of highly efficient factorizations for the upgoing and downgoing wave amplitudes in each layer. The integrals are evaluated by direct integration, basically, the kernel functions are sequentially sampled fine enough to allow piecewise polynomials to interpolate the amplitudes of the kernels between the integration points. Thereby, the numerical integration over each time interval is performed analytically, thus avoiding the oscillation hazard of the Bessel functions. The comparison studies found in the validation section include comparisons to particular methods. The results of these studies demonstrate the validation and accuracy of the developed method. In addition, it shows utilizable flexibility of the method for determination of the exciting vector in the soil-structure problem due to various far field excitations.

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