$$\sqrt{\langle \psi | \hat{P}_1 | \psi \rangle} = |\alpha|$$

نعوض في عبارة $|\psi_{after}\rangle$:

$$|\psi_{after}\rangle = \frac{\alpha}{|\alpha|} \cdot |\phi_1\rangle$$

14- نحصل على التحويلة الواحدية التي تقطر $[\hat{H}]$ انطلاقاً من الأشعة الذاتية لـ $[\hat{H}]$. نكتب المصفوفة $[U]$:

$$[U] = \begin{pmatrix} |\psi_1\rangle & |\psi_2\rangle \end{pmatrix}$$

$$[U] = \begin{pmatrix} \frac{1}{\sqrt{4-\sqrt{8}}} & \frac{1}{\sqrt{4+\sqrt{8}}} \\ \frac{-1+\sqrt{2}}{\sqrt{4-\sqrt{8}}} & \frac{-1-\sqrt{2}}{\sqrt{4+\sqrt{8}}} \end{pmatrix}$$

$$[U]^{+} = \begin{pmatrix} \frac{1}{\sqrt{4-\sqrt{8}}} & \frac{-1+\sqrt{2}}{\sqrt{4-\sqrt{8}}} \\ \frac{1}{\sqrt{4+\sqrt{8}}} & \frac{-1-\sqrt{2}}{\sqrt{4+\sqrt{8}}} \end{pmatrix}$$

$$[\tilde{H}] = [U]^{+} \cdot [H] \cdot [U]$$

$$[\tilde{H}] = \begin{pmatrix} \frac{1}{\sqrt{4-\sqrt{8}}} & \frac{-1+\sqrt{2}}{\sqrt{4-\sqrt{8}}} \\ \frac{1}{\sqrt{4+\sqrt{8}}} & \frac{-1-\sqrt{2}}{\sqrt{4+\sqrt{8}}} \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} \frac{1}{\sqrt{4-\sqrt{8}}} & \frac{1}{\sqrt{4+\sqrt{8}}} \\ \frac{-1+\sqrt{2}}{\sqrt{4-\sqrt{8}}} & \frac{-1-\sqrt{2}}{\sqrt{4+\sqrt{8}}} \end{pmatrix} = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

د. نبيل جودية

$$a = \frac{1}{\sqrt{4-\sqrt{8}}} \; ; \; b = \frac{\sqrt{2}-1}{\sqrt{4-\sqrt{8}}}$$

$$\lambda_1 = +1 \longrightarrow |\psi_1\rangle = \frac{1}{\sqrt{4-\sqrt{8}}} \cdot \begin{pmatrix} 1 \\ \sqrt{2}-1 \end{pmatrix}$$

$$[\hat{H}]|\psi_2\rangle = \lambda_2 \cdot |\psi_2\rangle$$

$$\frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = (-1) \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \cdot \begin{pmatrix} a+b \\ a-b \end{pmatrix} = \begin{pmatrix} -a \\ -b \end{pmatrix} \Rightarrow \begin{cases} \frac{1}{\sqrt{2}} \cdot (a+b) = -a \\ \frac{1}{\sqrt{2}} \cdot (a-b) = -b \end{cases} \xrightarrow{\text{Addition}}$$

$$b = -a - \sqrt{2} \cdot a$$

$$\frac{1}{\sqrt{2}} \cdot (2 \cdot a) = -a - b \Rightarrow b = -a \cdot (1+\sqrt{2})$$

$$\langle \psi_2 | \psi_2 \rangle = 1 \Rightarrow a^2 + a^2 \cdot (1+\sqrt{2})^2 = 1$$

$$a^2 \cdot [1 + 1 + 2 + 2 \cdot \sqrt{2}] = 1 \Rightarrow a^2 \cdot (4+\sqrt{8}) = 1$$

$$a = \frac{1}{\sqrt{4+\sqrt{8}}} \; ; \; b = -\frac{1+\sqrt{2}}{\sqrt{4+\sqrt{8}}}$$

$$\lambda_2 = -1 \longrightarrow |\psi_2\rangle = \frac{1}{\sqrt{4+\sqrt{8}}} \cdot \begin{pmatrix} 1 \\ -1-\sqrt{2} \end{pmatrix}$$

13- ب

$$: \sqrt{\langle \psi | \hat{P}_1 | \psi \rangle}$$

$$\hat{P}_1 |\psi\rangle = \alpha \cdot |\phi_1\rangle \Rightarrow \langle \psi | \hat{P}_1 | \psi \rangle = \alpha \cdot \langle \psi | \phi_1 \rangle$$

$$\langle \psi | \hat{P}_1 | \psi \rangle = \alpha \cdot [\alpha^* \cdot \langle \phi_1 | + \beta^* \cdot \langle \phi_2 |] \cdot |\phi_1\rangle$$

$$\langle \psi | \hat{P} | \psi \rangle = |\alpha|^2 + 0$$

$$\langle \psi | \hat{P}_1 | \psi \rangle = |\alpha|^2$$

$$\hat{P}_n|\Psi\rangle = \alpha.|\phi_n\rangle.\langle\phi_n|\phi_1\rangle + \beta.|\phi_n\rangle.\langle\phi_n|\phi_2\rangle$$

$$\hat{P}_n|\Psi\rangle = \delta_{1n}.\alpha.|\phi_n\rangle + \delta_{2n}.\beta.|\phi_n\rangle \quad ; n = 1, 2$$

11- احتمال وجود الجملة على الترتيب في الحالة $|\phi_1\rangle$ و $|\phi_2\rangle$:

$$P(|\phi_1\rangle) = |\langle\phi_1|\Psi\rangle|^2 = |\alpha|^2$$

$$P(|\phi_2\rangle) = |\langle\phi_2|\Psi\rangle|^2 = |\beta|^2$$

12- القيم والأشعة الذاتية لـ $[\hat{H}]$:

$$P(\lambda) = \det\{[\hat{H}] - \lambda.[I]\}$$

$$P(\lambda) = \det\left\{\frac{1}{\sqrt{2}}.\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right\}$$

$$P(\lambda) = \det\begin{pmatrix} \frac{1}{\sqrt{2}} - \lambda & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} - \lambda \end{pmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} - \lambda & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} - \lambda \end{vmatrix}$$

$$P(\lambda) = -\left(\frac{1}{\sqrt{2}} - \lambda\right).\left(\frac{1}{\sqrt{2}} + \lambda\right) - \frac{1}{2} = -\left(\frac{1}{2} - \lambda^2\right) - \frac{1}{2}$$

$$P(\lambda) = 0 \Rightarrow \frac{1}{2} - \lambda^2 = -\frac{1}{2} \Rightarrow \lambda^2 = 1 \Rightarrow \lambda_1 = +1 \; ; \; \lambda_2 = -1$$

$$\lambda_1 = +1 \rightarrow |\Psi_1\rangle = \begin{vmatrix} a \\ b \end{vmatrix}$$

$$[\hat{H}]|\Psi_1\rangle = \lambda_1.|\Psi_1\rangle \Rightarrow \frac{1}{\sqrt{2}}.\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.\begin{pmatrix} a \\ b \end{pmatrix} = (+1).\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}.\begin{pmatrix} a+b \\ a-b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \begin{cases} \frac{1}{\sqrt{2}}.(a+b) = a \\ \frac{1}{\sqrt{2}}.(a-b) = b \end{cases} \Rightarrow \text{Addition}:$$

$$\frac{1}{\sqrt{2}}.(2.a) = a + b \Rightarrow \sqrt{2}.a = a + b \Rightarrow b = a.(\sqrt{2} - 1)$$

$$\langle\Psi_1|\Psi_1\rangle = 1 \Rightarrow \left(a \quad a.(\sqrt{2}-1)\right).\begin{pmatrix} a \\ a.(\sqrt{2}-1) \end{pmatrix} = 1 \Rightarrow$$

$$a^2 + a^2.(\sqrt{2}-1)^2 = 1 \Rightarrow a^2.[1 + 2 + 1 - 2.\sqrt{2}] = 1$$

$$a^2.(4 - 2.\sqrt{2}) = 1$$

$$[\hat{X}] = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{matrix} X_{11} = 0 \; ; \; X_{12} = 1 \\ X_{21} = 1 \; ; \; X_{22} = 0 \end{matrix} \Rightarrow$$ $\boxed{12}$ $\boxed{8}$

نتبع نفس الطريقة نجد أن

$$\hat{X} = |\phi_1\rangle . \langle\phi_2| + |\phi_2\rangle . \langle\phi_1|$$

$$\hat{Y} = -i . |\phi_1\rangle . \langle\phi_2| + i . |\phi_2\rangle . \langle\phi_1|$$

$$\hat{Z} = |\phi_1\rangle . \langle\phi_1| - |\phi_2\rangle . \langle\phi_2|$$

c- مؤثرات الإسقاط الحالة الكمومية لبعد الإسقاط والقيم والأشعة الذاتية لمؤثر Hadamard (ص 37)

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9-

$$\hat{P}_n = |\phi_n\rangle . \langle\phi_n| \quad , n = 1, 2$$

$$\hat{P}_1 |\phi_1\rangle = |\phi_1\rangle . \langle\phi_1|\phi_1\rangle = |\phi_1\rangle$$

$$\langle\phi_i|\hat{P}_1|\phi_1\rangle = \delta_{i1} \Rightarrow P_{11} = 1 \; ; \; P_{21} = 0$$

$$\hat{P}_1 . |\phi_1\rangle = |\phi_1\rangle \Rightarrow$$

$$\hat{P}_1 |\phi_2\rangle = |\phi_1\rangle . \langle\phi_1|\phi_2\rangle = 0 . |\phi_1\rangle$$

$$\langle\phi_i|\hat{P}_1|\phi_2\rangle = 0 \Rightarrow P_{12} = 0 \; ; \; P_{22} = 0 \Rightarrow$$

$$[\hat{P}_1] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{P}_2 |\phi_1\rangle = |\phi_2\rangle . \langle\phi_2|\phi_1\rangle = 0 . |\phi_2\rangle$$

$$\langle\phi_i|\hat{P}_2|\phi_1\rangle = 0 \quad \forall i = 1, 2 \Rightarrow P_{11} = 0 \; ; \; P_{21} = 0$$

$$\hat{P}_2 |\phi_2\rangle = |\phi_2\rangle . \langle\phi_2|\phi_2\rangle = |\phi_2\rangle \Rightarrow P_{i2} = \delta_{i2} \Rightarrow$$

$$P_{12} = 0 \; ; \; P_{22} = 1 \Rightarrow$$

$$[\hat{P}_2] = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\hat{P}_n |\psi\rangle = |\phi_n\rangle . \langle\phi_n| . [\alpha . |\phi_1\rangle + \beta . |\phi_2\rangle] \quad n = 1, 2$$

$$P_1(a_1) = \left| \frac{1}{\sqrt{34}} \cdot \begin{pmatrix} 4 & -\sqrt{17} & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right|^2 = \left| \frac{1}{\sqrt{34}} \cdot (1) \right|^2 = \frac{1}{34}$$

تصبح الحالة بعد القياس :

$$|\phi_2\rangle \equiv |\Psi_{Before}\rangle \xrightarrow[P_1(a_1)]{mes(\hat{A}) = a_1} |\Psi_{After}\rangle$$

$$a_1 = -\sqrt{17} \cdot a \longrightarrow |a_1\rangle \longrightarrow \hat{P}_1 = |a_1\rangle \cdot \langle a_1|$$

$$\hat{P}_1 |\Psi_{Before}\rangle = \hat{P}_1 |\phi_2\rangle = |a_1\rangle \cdot \langle a_1 | \phi_2\rangle$$

$$\langle a_1 | \phi_2\rangle = \frac{1}{\sqrt{34}} \cdot \begin{pmatrix} 4 & -\sqrt{17} & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{34}} \cdot (1) = \frac{1}{\sqrt{34}}$$

$$\hat{P}_1 |\Psi_{Before}\rangle \equiv \hat{P}_1 |\phi_2\rangle = \frac{1}{\sqrt{34}} \cdot |a_1\rangle$$

$$\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle = \frac{1}{\sqrt{34}} \cdot \langle \phi_2 | a_1\rangle = \frac{1}{\sqrt{34}} \cdot \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{34}} \cdot \begin{pmatrix} 4 \\ -\sqrt{17} \\ 1 \end{pmatrix}$$

$$\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle = \frac{1}{34} \cdot (1) = \frac{1}{34} \Rightarrow \sqrt{\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle} = \frac{1}{\sqrt{34}}$$

$$|\Psi_{After}\rangle = \frac{\hat{P}_1 |\Psi_{Before}\rangle}{\sqrt{\langle \Psi_{Before} | \hat{P}_1 | \Psi_{Before}\rangle}} = \frac{\hat{P}_1 |\phi_2\rangle}{\sqrt{\langle \phi_2 | \hat{P}_1 | \phi_2\rangle}}$$

$$|\Psi_{After}\rangle = \frac{1}{\sqrt{34}} \cdot \frac{|a_1\rangle}{\frac{1}{\sqrt{34}}} = \frac{1}{\sqrt{34}} \cdot |a_1\rangle \cdot \sqrt{34}$$

$$|\Psi_{After}\rangle = |a_1\rangle$$

الاحتمال $P_2(a_2)$: $|\langle a_2 | \phi_2\rangle|^2$

$$a_2 = 0 \longrightarrow |a_2\rangle \longrightarrow P_2(a_2) = |\langle a_2 | \phi_2 \rangle|^2$$

$$P_2(a_2) = \left| \frac{1}{\sqrt{17}} \cdot \begin{pmatrix} 1 & 0 & -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right|^2 = \frac{16}{17}$$

$$E_2 = -E_0 \longrightarrow |\phi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$[\hat{H}]_u \cdot |\phi_3\rangle = E_3 \cdot |\phi_3\rangle$$

$$E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (2 \cdot E_0) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} 2\cdot x \\ 2\cdot y \\ 2\cdot z \end{pmatrix} \Rightarrow$$

$$\left.\begin{array}{l} y = -x \\ y = -x \\ z = 0 \end{array}\right\} \Rightarrow |\phi_3\rangle = \begin{pmatrix} x \\ -x \\ 0 \end{pmatrix} \Rightarrow \langle\phi_3|\phi_3\rangle = \overline{x^* \; -x^* \; 0} \cdot \begin{pmatrix} x \\ -x \\ 0 \end{pmatrix}$$

$$\langle\phi_3|\phi_3\rangle = 2 \cdot |x|^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$E_3 = 2 \cdot E_0 \longrightarrow |\phi_3\rangle = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

6- النشر الطيفي للهاملتوني في $\mathbb{B}_\phi$: الهاملتوني قطري في القاعدة $\mathbb{B}_\phi$. إذن نكتب :

$$\hat{H} = \hat{I} \cdot \hat{H} \cdot \hat{I}$$

$$\hat{H} = \left(\sum_{i=1}^{3} |\phi_i\rangle \cdot \langle\phi_i|\right) \cdot \hat{H} \cdot \left(\sum_{i=1}^{3} |\phi_i\rangle \cdot \langle\phi_i|\right)$$

$$\hat{H} = \sum_i \sum_j |\phi_i\rangle \cdot \langle\phi_i|\hat{H}|\phi_j\rangle \cdot \langle\phi_j|$$

$$\langle\phi_i|\hat{H}|\phi_j\rangle = E_j \cdot \langle\phi_i|\phi_j\rangle = \delta_{ij} \cdot E_j$$

$$\hat{H} = \sum_{i=1}^{3} E_i \cdot |\phi_i\rangle \cdot \langle\phi_i|$$

$$\hat{H} = -E_0 \cdot |\phi_2\rangle \cdot \langle\phi_2| + 2 \cdot E_0 \cdot |\phi_3\rangle \cdot \langle\phi_3| + (0) \cdot |\phi_1\rangle \cdot \langle\phi_1|$$

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$$|\psi_{Before}\rangle \xrightarrow{mes(\hat{H}) = -E_0} |\psi_{After}\rangle$$

$$|\Psi_{After}\rangle = \frac{\hat{P}_n|\Psi_{Before}\rangle}{\sqrt{\langle\Psi_{Before}|\hat{P}_n|\Psi_{Before}\rangle}}$$

$$E_2 = -E_0 \longrightarrow |\phi_2\rangle \longrightarrow \hat{P}_2 = |\phi_2\rangle\cdot\langle\phi_2|$$

تشكل الأشعة الذاتية $\{|\phi_i\rangle\}$ مجموعة منتظمة تامة متعامدة يمكن أن ننشر بها $|\Psi_{Before}\rangle$:

$$|\Psi_{Before}\rangle = \sum_i c_i\cdot|\phi_i\rangle$$

$$\hat{P}_2|\Psi_{Before}\rangle = \left(|\phi_2\rangle\cdot\langle\phi_2|\right)\cdot\left(\sum_i c_i\cdot|\phi_i\rangle\right)$$

$$\hat{P}_2|\Psi_{Before}\rangle = \sum_i c_i\cdot|\phi_2\rangle\cdot\langle\phi_2|\phi_i\rangle = \sum_i \delta_{2i}\cdot c_i\cdot|\phi_2\rangle$$

$$\hat{P}_2|\Psi_{Before}\rangle = c_2\cdot|\phi_2\rangle$$

$$\langle\Psi_{Before}|\hat{P}_2|\Psi_{Before}\rangle = \left(\sum_i c_i^*\cdot\langle\phi_i|\right)\cdot\left(c_2|\phi_2\rangle\right)$$

$$= \sum_i c_2\cdot c_i^*\cdot\langle\phi_i|\phi_2\rangle = |c_2|^2$$

$$|\Psi_{After}\rangle = \frac{\hat{P}_2|\Psi_{Before}\rangle}{\sqrt{\langle\Psi_{Before}|\hat{P}_2|\Psi_{Before}\rangle}}$$

$$|\Psi_{After}\rangle = \frac{c_2\cdot|\phi_2\rangle}{\sqrt{|c_2|^2}} = \frac{c_2\cdot|\phi_2\rangle}{c_2} = |\phi_2\rangle$$

$$|\Psi_{After}\rangle = |\phi_2\rangle$$

8- الاحتمال : $P_1(a_1)$

$$a_1 = -\sqrt{17}\cdot a \longrightarrow |a_1\rangle \longrightarrow P_1(a_1) = |\langle a_1|\phi_2\rangle|^2$$

$$P(\lambda) = (E_0+\lambda)\cdot\{[E_0-(E_0-\lambda)]\cdot[E_0+(E_0-\lambda)]\}$$

$$P(\lambda) = (E_0+\lambda)\cdot(\lambda)\cdot(2.E_0-\lambda)$$

$$P(\lambda)=0 \Rightarrow \lambda = 0 \; ; \; \lambda = -E_0 \; ; \; \lambda = 2.E_0$$

نتحصل على الأشعة الذاتية:

$$E_1 = 0 \; ; \; E_2 = -E_0 \; ; \; E_3 = 2.E_0$$

$$[\hat{H}]_u \cdot |\phi_1\rangle = (E_1)\cdot|\phi_1\rangle$$

$$E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (0)\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$y = x \; ; \; y = x \; ; \; z = 0 \Rightarrow |\phi_1\rangle = \begin{pmatrix} x \\ x \\ 0 \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle\phi_1|\phi_1\rangle = \begin{pmatrix} x^* & x^* & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ x \\ 0 \end{pmatrix} = 2.|x|^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$E_1 = 0 \longrightarrow |\phi_1\rangle = \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$[\hat{H}]_u \cdot |\phi_2\rangle = E_2 \cdot |\phi_2\rangle$$

$$E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (-E_0)\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \Rightarrow$$

$$\left.\begin{array}{l} x-y = -x \\ -x+y = -y \\ -z = -z \end{array}\right\} \Rightarrow \left.\begin{array}{l} y = 2.x \\ 2.y = x \\ z = z \end{array}\right\} \Rightarrow 2.(2.x) = x \Rightarrow 4.x = x$$

$z = z$

علاقة محققة فقط من أجل

$\Leftarrow x = 0$

$$|\phi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \langle\phi_2|\phi_2\rangle = \begin{pmatrix} 0 & 0 & z^* \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = |z|^2 = 1$$

$$\Rightarrow z = 1$$

الطريقة الثانية:

$$\hat{H}\cdot\hat{A} = \left(\sum_i \sum_j H_{ij}\cdot|i\rangle\cdot\langle j|\right)\cdot\left(\sum_{ij} A_{ij}\,|i\rangle\cdot\langle j|\right)$$

$$\hat{H}\cdot\hat{A} = \sum_{ijkl} H_{ij}\cdot A_{kl}\cdot|i\rangle\cdot\langle j|k\rangle\cdot\langle l|$$

$$\langle j|k\rangle = \delta_{jk}$$

$$\hat{H}\cdot\hat{A} = \sum_{ijl} H_{ij}\cdot A_{jl}\cdot|i\rangle\cdot\langle l|$$

$$\hat{A}\cdot\hat{H} = \sum_{ijkl} A_{ij}\cdot H_{kl}\cdot|i\rangle\cdot\langle j|k\rangle\langle l|$$

$$\hat{A}\cdot\hat{H} = \sum_{ijl} H_{jl}\cdot A_{ij}\cdot|i\rangle\cdot\langle l|$$

يمكن تبديل $i \leftrightarrow j$

$$\hat{A}\cdot\hat{H} = \sum_{ijl} H_{il}\cdot A_{ji}\cdot|j\rangle\cdot\langle l|$$

$$l \leftrightarrow j$$

$$\hat{A}\cdot\hat{H} = \sum_{jil} H_{ij}\cdot A_{li}\cdot|l\rangle\cdot\langle j|$$

نجد أنه من غير الممكن الحصول على $\hat{H}\cdot\hat{A} = \hat{A}\cdot\hat{H}$. بالمقابل $[\hat{H},\hat{A}] \neq 0$. لا يمكن اذن للمؤثرين مجموعة مشتركة التي تكون بآن معاً أشعة خاصة لكلا المؤثرين.

5- القيم الممكن الحصول عليها لدى إجراء قياس للطاقة هي القيم الخاصة للمؤثر الممثل للطاقة أي الهاملتوني. نقوم بتقطير الهاملتوني:

$$P(\lambda) = \det\left\{[\hat{H}]_n - \lambda\cdot I\right\}$$

$$P(\lambda) = \begin{vmatrix} E_0-\lambda & -E_0 & 0 \\ -E_0 & E_0-\lambda & 0 \\ 0 & 0 & -E_0-\lambda \end{vmatrix} = -(E_0-\lambda)\cdot(E_0-\lambda)\cdot(E_0+\lambda) + E_0\cdot E_0\cdot(E_0+\lambda)$$

$$\hat{H} = E_0 \cdot \big[|u_1\rangle \cdot \langle u_1| - |u_1\rangle \cdot \langle u_2| - |u_2\rangle \cdot \langle u_1| + |u_2\rangle \cdot \langle u_2| - |u_3\rangle \cdot \langle u_3| \big]$$

باتباع نفس الأسلوب:

$$\hat{A} = \sum_{i=1}^{3} \sum_{j=1}^{3} A_{ij} \cdot |u_i\rangle \cdot \langle u_j|$$

$A_{11} = 0 \quad ; \quad A_{12} = 4.a \; ; \quad A_{13} = 0$

$A_{21} = 4.a \quad ; \quad A_{22} = 0 \quad ; \quad A_{23} = a$

$A_{31} = 0 \quad ; \quad A_{32} = a \quad ; \quad A_{33} = 0$

$$\hat{A} = a.\big[4.|u_1\rangle \cdot \langle u_2| + 4.|u_2\rangle \cdot \langle u_1| + |u_2\rangle \cdot \langle u_3| + |u_3\rangle \cdot \langle u_2| \big]$$

4- يملك المؤثرين $\hat{H}$ و $\hat{A}$ مجموعة مشتركة من الأشعة التي تكون بآن معاً أشعة ذاتية لكلا المؤثرين إذا كان المبدل معدوماً أي

$$[\hat{H}, \hat{A}] = \hat{0} \iff [[\hat{H}]_u, [\hat{A}]_u] = 0$$

إذن إما أن نحسب قيمة المبدل باستعمال النشر الطيفي للمؤثرين وإما أن نحسب قيمته بحساب جداء المصفوفات الممثلة أي $[\hat{H}]_u \cdot [\hat{A}]_u$ و $[\hat{A}]_u \cdot [\hat{H}]_u$ وإذا كان الجداء متساوياً في الحالتين ينعدم المبدل.

الطريقة الأولى:

$$[\hat{H}]_u \cdot [\hat{A}]_u = a.E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = a.E_0 \cdot \begin{pmatrix} -4 & 4 & -1 \\ 4 & -4 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$[\hat{A}]_u \cdot [\hat{H}]_u = a.E_0 \cdot \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = a.E_0 \cdot \begin{pmatrix} -4 & 4 & 0 \\ 4 & -4 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$[\hat{H}, \hat{A}] = [\hat{H}]_u \cdot [\hat{A}]_u - [\hat{A}]_u \cdot [\hat{H}]_u = a.E_0 \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 2 \\ 1 & -2 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1 سلم تصحيح كم ١ - س٣ ف - ف 2 - 2024

I - المراصيد: قياس، نشر طيفي، حالات خاصة، احتمالات، تبادل (06 ن)

1- القاعدة $\mathbb{B}_u$ ليست قاعدة خاصة للمرصودين $\hat{H}$ ، $\hat{A}$ لأن المصفوفات الممثلة لهما على هذه القاعدة ليست قطرية.

2- حتى يكون مؤثراً هرميتياً يجب أن يتبادل مع مرافقه أي يجب أن يكون:

$$\hat{A}^+ = \hat{A} \quad ; \quad \hat{H}^+ = \hat{H} \quad ;$$

$$[\hat{A}]_u^+ = [\hat{A}]_u^{T*} = a^* \cdot \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_u$$

نلاحظ بالمقارنة مع $[\hat{A}]_u$ أنه يكون $[\hat{A}]_u^+ = [\hat{A}]_u$ إذا كان $a \in \mathbb{R}$

$$[\hat{H}]_u^+ = [\hat{H}]_u^{T*} = E_0^* \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}_u$$

هنا أيضاً يكون $[\hat{H}]_u^+ = [\hat{H}]_u$ إذا كان $E_0 \in \mathbb{R}$.

3- النشر الطيفي:

$$\hat{H} = \hat{I} \cdot \hat{H} \cdot \hat{I} = \left(\sum_{i=1}^{3} |u_i\rangle \cdot \langle u_i|\right) \cdot \hat{H} \cdot \left(\sum_{i=1}^{3} |u_i\rangle \cdot \langle u_i|\right)$$

$$\hat{H} = \sum_{i=1}^{3} \sum_{j=1}^{3} |u_i\rangle \cdot \langle u_i|\hat{H}|u_j\rangle \cdot \langle u_j|$$

$$\hat{H} = \sum_{i=1}^{3} \sum_{j=1}^{3} H_{ij} \cdot |u_i\rangle \cdot \langle u_j|$$

من عبارة $[\hat{H}]_u$:

$H_{11} = E_0 \; ; \; H_{12} = -E_0 \; ; \; H_{13} = 0$

$H_{21} = -E_0 \; ; \; H_{22} = +E_0 \; ; \; H_{23} = 0$

$H_{31} = 0 \; ; \; H_{32} = 0 \; ; \; H_{33} = -E_0$

بالتعويض نحصل على: