المدة ساعتان .
العلامة 100.

الفيزياء النووية II
سنة 4 - فيزياء
امتحان الفصل الثاني 13.08.2024
سلم تصحيح

قسم الفيزياء
كلية العلوم
جامعة دمشق

QI . Nuclear modes (55 Mark).

$$M(A,Z)c^2 = Z m_p c^2 + (A-Z) m_n c^2 - a_v A + a_s A^{2/3} + a_c Z(Z-1) A^{-1/3} - a_a (A-2Z)^2 A^{-1} + \delta A$$

$$= [m_n c^2 - a_v + a_s A^{-1/3} + a_a] A + [(m_p - m_n)c^2 - 4a_a] Z + [4a_a A^{-1} + a_c A^{-1/3}] Z^2 + \delta A \quad -1 \quad \boxed{15}$$

$$= a_0 A + \alpha_1 Z + \alpha_2 Z^2 + \delta A$$

$$\left.\frac{\partial M}{\partial Z}\right|_{A=const} = 0 \Rightarrow -a_c A^{-1/3} - 4a_a + 2[a_c A^{-1/3} + 4 a_a A^{-1}] Z_{Stable} = 0$$

$$\Rightarrow Z_{Stable} = \frac{a_c A^{-1/3} + 4a_a}{2(4a_a A^{-1} + a_c A^{-1/3})} = \frac{\frac{a_c}{a_a} A^{2/3} + 4A}{2(4 + \frac{a_c}{a_a} A^{2/3})}$$

$$A = 91 \Rightarrow Z_{Stable} = \frac{\frac{0,71}{23,2}(91)^{2/3} + 4(91)}{2(4 + \frac{0,71}{23,2}(91)^{2/3})} = 39,5 \simeq 40$$

$-2 \ \boxed{16}$

$-i \ \boxed{3}$

$$N = N \Rightarrow N = \int_0^{E_F} g(E)\,dE = BA\int_0^{E_F} E^{1/2}\,dE = BA\,\frac{2}{3}\,E_F^{3/2}$$

$$\Rightarrow E_F = \left(\frac{3}{2}\frac{N}{BA}\right)^{2/3}$$

$-ii \ \boxed{3}$

$$N = Z = \frac{A}{2} \Rightarrow \bar{E}_F = \left(\frac{3}{2}\frac{A/2}{BA}\right)^{2/3} = \left(\frac{3}{4B}\right)^{2/3}$$

$$\Rightarrow \bar{E}_F = \left[\frac{3}{4}\,\frac{3\pi \hbar^3}{4\sqrt{2}\, m^{3/2} R_0^3}\right]^{2/3} = \left[\frac{9\pi}{16\sqrt{2}}\,\frac{\hbar^3}{m^{3/2} R_0^3}\right]^{2/3} = \left(\frac{9\pi}{16\sqrt{2}}\right)^{2/3}\frac{\hbar^2 c^2}{mc^2 R_0^2}$$

$$= \frac{(9\pi)^{2/3}}{(2^4\,2^{1/2})^{2/3}}\,\frac{\hbar^2 c^2}{mc^2 R_0^2} = \frac{(9\pi)^{2/3}}{2^3}\,\frac{\hbar^2 c^2}{mc^2 R_0^2} = \frac{9,28}{8}\,\frac{(197\,MeV\,fm)^2}{(940\,MeV)(1,2\,fm)^2}$$

$$\Rightarrow \bar{E}_F = 33,258\ MeV$$

iii $\boxed{5}$

$$\mathcal{E}(N) = \int_0^{E_F} E\, g(E)\, dE$$

$$= BA \int_0^{E_F} E^{3/2}\, dE = \frac{2}{5} BA E_F^{5/2}$$

$$\mathcal{N} = N \Rightarrow E_F = \left(\frac{3N}{2BA}\right)^{2/3} \Rightarrow$$

$$\mathcal{E}(N) = \frac{2}{5} BA \left[\left(\frac{3N}{2BA}\right)^{2/3}\right]^{5/2} = \frac{2}{5} BA \left(\frac{3N}{2BA}\right)^{5/3}$$

$$= \frac{2}{5}\left(\frac{3}{2}\right)^{5/3} B \frac{1}{B^{5/3}} \frac{N^{5/3}}{A^{2/3}} = \left(\frac{2}{5}\right)\left(\frac{3}{2}\right)\left(\frac{3/2}{BA}\right)^{2/3} N^{5/3}$$

$$\Rightarrow \mathcal{E}(N) = \frac{3}{5}\left(\frac{3}{2BA}\right)^{2/3} N^{5/3} \Rightarrow$$

$$\mathcal{E}_{tot}(Z,N) = \frac{3}{5}\left(\frac{3}{2BA}\right)^{2/3}\left(N^{5/3} + Z^{5/3}\right)$$

$\boxed{5}$
iv

$$\mathcal{E}_{tot}(Z,N) = \frac{3}{5}\left(\frac{3}{2BA}\right)^{2/3}\left(\frac{A}{2}\right)^{5/3}\left[(1+\nu)^{5/3} + (1-\nu)^{5/3}\right]$$

$$= \frac{3}{5}\left(\frac{3}{2BA}\right)^{2/3}\left(\frac{A}{2}\right)^{5/3}\left[1 + \frac{5}{3}\nu + \frac{\frac{5}{3}\left(\frac{5}{3}-1\right)}{2}\nu^2 + \dots + 1 - \frac{5}{3}\nu + \frac{\frac{5}{3}\left(\frac{5}{3}-1\right)}{2}\nu^2 + \dots\right]$$

$$= \frac{3\times 2}{5}\left(\frac{3}{2BA}\right)^{2/3}\left(\frac{A}{2}\right)^{5/3}\left[1 + \frac{5}{9}\nu^2\right], \quad \nu = \frac{N-Z}{A} \Rightarrow$$

$$\mathcal{E}_{tot}(Z,N) = \frac{3\times 2}{5}\left(\frac{3}{2BA}\right)^{2/3} \frac{A^{2/3}}{2^{5/3}}\left[A + \frac{5}{9}\frac{(N-Z)^2}{A}\right]$$

$$= \frac{3\times 2}{5} \frac{3^{2/3}}{2^{2/3} B^{2/3} A^{2/3}} \frac{A^{2/3}}{2^{5/3}}\left[A + \frac{5}{9}\frac{(N-Z)^2}{A}\right]$$

$$= \frac{3}{5}\left(\frac{3}{4B}\right)^{2/3}\left[A + \frac{5}{9}\frac{(N-Z)^2}{A}\right]$$

$$= \frac{3}{5}\bar{E}_F A + \frac{1}{3}\bar{E}_F \frac{(N-Z)^2}{A}$$

where $a_a = \frac{1}{3}\bar{E}_F$

$$^{41}_{20}Ca_{21}$$

-2 $\boxed{12}$ -3 $\boxed{18}$

20 بروتون تملأ حتى الطبقة $1d_{3/2}$ وهي طبقة مغلقة.

21 نيوترون تملأ حتى الطبقة المغلقة $1d_{3/2}$ ونيوترون مفرد في $1f_{7/2}$

وهو يحدد spin للنواة لذلك $I = 7/2$ ويحدد parity

$$f \Rightarrow \ell = 3 \Rightarrow \xi = (-1)^{\ell} = (-1)^3 = -1$$

$$I^{\xi}(^{30}_{14}Si) = 0^+ ,\qquad I^{\xi}(^{14}_{7}N_{7}) : \begin{cases} \pi \to p_{1/2} \Rightarrow j_\pi = 1/2 \\ \nu \to p_{1/2} \Rightarrow j_\nu = 1/2 \end{cases}$$

even-neutron.

$$|j_\pi - j_\nu| \le j \le |j_\pi + j_\nu| \Rightarrow$$

$$0 \le j \le 1 \Rightarrow j = 1$$

$$\xi = (-1)^{\ell_\pi}(-1)^{\ell_\nu} = (-1)(-1) = +1$$

$$\Rightarrow I^{\xi}(^{14}_{7}N_{7}) = 1^+$$

$^{41}_{20}Ca_{21}$ $\pi[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^4)]$

$\nu[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^4\, 1f_{7/2}^1)]$

$^{30}_{14}Si_{16}$ $\pi[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^4)]$, $\nu[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6)]$

$^{14}_{7}N_{7}$: $\pi[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^1)]$, $\nu[(1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^1)]$

$$\vec{\mu}_j = \vec{\mu}_\ell + \vec{\mu}_s \Rightarrow g_j \vec{j} = g_\ell \vec{\ell} + g_s \vec{s}$$ -11 $\boxed{6}$

$$\Rightarrow g_j\, \vec{j}\cdot\vec{j} = g_\ell\, \vec{\ell}\cdot\vec{j} + g_s\, \vec{s}\cdot\vec{j} \quad , \quad \vec{j}\cdot\vec{j} = j^2 = j(j+1)$$

$$\vec{\ell}\cdot\vec{j} = \frac{1}{2}(j^2 + \ell^2 - s^2) = \frac{1}{2}\left[j(j+1) + \ell(\ell+1) - s(s+1)\right] \Rightarrow$$

$$\vec{s}\cdot\vec{j} = \frac{1}{2}(j^2 + s^2 - \ell^2) = \frac{1}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1)\right]$$

$$g_j = g_\ell \frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$$

For proton: $g_\ell = 1$, $g_s = g_\pi$, $\ell = 0$, $s = \frac{1}{2} \Rightarrow$

the gyromagnetic ratio for free proton:

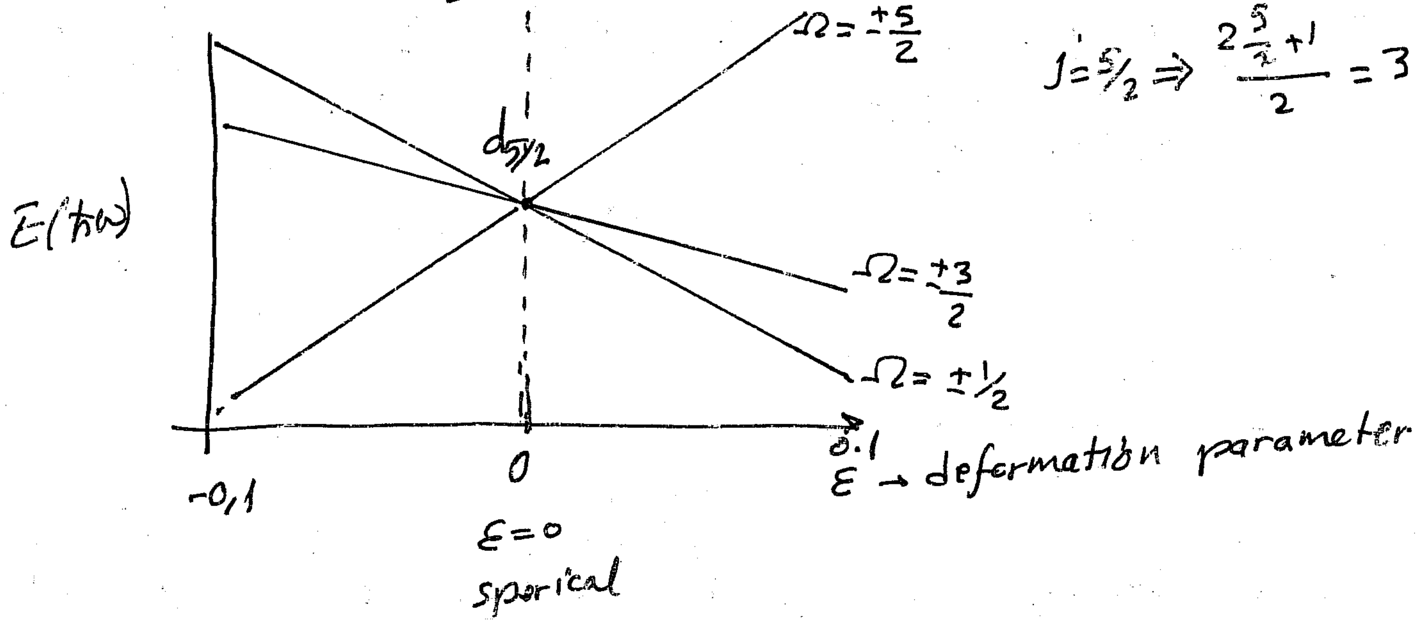
$$g_j = \begin{cases} \frac{2j-1}{2j} + \frac{g_\pi}{2j} & \text{for } j = \ell + 1/2 \\ \frac{1}{j+1}\left(j + \frac{3}{2} - \frac{g_\pi}{2}\right) & \text{for } j = \ell - 1/2 . \end{cases}$$

Nilsson نموذج $V(r) = \frac{1}{2} m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2) - C\,\vec{l}\cdot\vec{s} - D\,l^2$ $\quad -4 \; \boxed{6}$

axial symmetry تناظر محوري $\omega_1 = \omega_2 \neq \omega_3$

$\Omega = \pm j$, $\frac{2j+1}{2}$ spliting at $\varepsilon = 0$

$j = 5/2 \Rightarrow \frac{2\frac{5}{2}+1}{2} = 3$



$\boxed{45}$ QII

$S = 0$ (singlet), $S = 1$ (triplet)

$\hat{S}^2 = \hbar\, s(s+1)$ $\quad -1 \; \boxed{10}$

نكتبه بدلالة $\hat{S}_n$ و $\hat{S}_p$

$\hat{S}^2 = \hat{S}_p^2 + \hat{S}_n^2 + 2\hat{S}_p\cdot\hat{S}_n$

$\Rightarrow \hat{S}_p\cdot\hat{S}_n = \frac{1}{2}(\hat{S}^2 - \hat{S}_p^2 - \hat{S}_n^2)$, $\quad \hat{S}_n^2, \hat{S}_p^2, \hat{S}^2 \Rightarrow$ commutated متبادلة

$\Rightarrow \langle \hat{S}_p\cdot\hat{S}_n \rangle = \langle S, S_p, S_n, S_z | \hat{S}_p\cdot\hat{S}_n | S, S_p, S_n, S_z \rangle$

$= \frac{\hbar^2}{2}[S(S+1) - S_p(S_p+1) - S_n(S_n+1)]$, $\quad S_p = S_n = 1/2$

$$\Rightarrow \langle \hat{S}_p\cdot\hat{S}_n \rangle = \begin{cases} \frac{\hbar^2}{4} & \text{for triple state } |S=1, \frac{1}{2}, \frac{1}{2}, m_z\rangle \\ -\frac{3}{4}\hbar^2 & \text{" single state } |S=0, \frac{1}{2}, \frac{1}{2}, 0\rangle \end{cases}$$

$V_1(r) < 0 \Rightarrow$

$V_{nuc,\, S=1} = V_T = V_0 + \frac{1}{4} V_1$

$V_{nuc,\, S=0} = V_S = V_0 - \frac{3}{4} V_1$

-2 $\boxed{20}$

-2 $\boxed{12}$

① $\frac{d^2u(r)}{dr^2} + k^2 u(r) = 0$, $r < b$, $k = \sqrt{\frac{M(V_0 - \epsilon)}{\hbar^2}}$

② $\frac{d^2u(r)}{dr^2} - k_1^2 u(r) = 0$, $r > b$, $k_1 = \sqrt{\frac{M\epsilon}{\hbar^2}}$

③ $\Rightarrow u_<(r) = \lambda \sin(kr) + \beta \cos(kr)$

④ $\Rightarrow u_>(r) = \lambda_1 e^{-k_1 r} + \beta_1 e^{k_1 r}$

$\psi(r \to 0) \to 0 \Rightarrow \beta = 0$ in ③, $\psi(r \to \infty) \to$ finite $\Rightarrow \beta_1 = 0$

$\psi(r \to b) = e^{br}$

$\Rightarrow u_<(r) = \lambda \sin(kr)$, $u_>(r) = \lambda_1 e^{-k_1(r-b)}$

$-ii$ $\boxed{8}$

$P = 4\pi \lambda^2 \int_0^b \sin^2(kr)\, dr$

$= 4\pi \frac{1}{2\pi b(1 + \frac{1}{bk_1})} \int_0^b \sin^2(kr)\, dr$

4

$I = \frac{1}{2}\int_0^b (1 - \cos(2kr))\, dr = \frac{b}{2} - \frac{1}{4k}\sin(2kr)\Big|_0^{\pi} = \frac{b}{2}$

$\Rightarrow P = \left(1 + \frac{1}{k_1 b}\right)^{-1}$

4

$k = \sqrt{\frac{M(V_0 - \epsilon)}{\hbar}} \simeq \frac{\pi}{2b} \Rightarrow \epsilon \simeq V_0 - \frac{1}{Mc^2}\left(\frac{\pi \hbar c}{2b}\right)^2$

$\Rightarrow \epsilon = 40 - \frac{1}{940\,\text{MeV}}\left[\frac{\pi \times 197\,\text{MeV fm}}{2 \times 1{,}9\,\text{fm}}\right]^2 = 11{,}8\,\text{MeV}$

$k_1 = \frac{\sqrt{Mc^2 \epsilon}}{197} = \frac{\sqrt{940 \times 11{,}8}}{197} = 53\,\text{fm} \Rightarrow \boxed{P = 0{,}5}$

$V(r)$ متناظر كروياً ⇐ الإحداثيات الكروية .

-3 $\boxed{15}$

$$\vec{q}\cdot\vec{r_s} = q r \cos\theta_s$$

$$f_B(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\int_0^{2\pi} d\phi_s \int_0^{\pi} d\theta_s \sin\theta_s \int_0^{R} dr_s\, r_s^2\, e^{i q r_s \cos\theta_s}\, V(r_s)$$

$$\int_0^{2\pi} d\phi_s = 2\pi$$

$$\int_0^{\pi} d\theta_s \sin\theta_s\, e^{i q r_s \cos\theta_s} = -\frac{1}{i q r_s} e^{i q r_s \cos\theta_s}\Big|_0^{\pi} = -\frac{1}{i q r_s}\left(e^{-i q r_s} - e^{i q r_s}\right)$$

$$= \frac{1}{i q r_s}\left(e^{i q r_s} - e^{-i q r_s}\right) = \frac{2}{q r_s}\sin(q r_s)$$

10

$$\Rightarrow f_B(\theta) = -\frac{\mu}{2\pi\hbar^2}(2\pi)\left(\frac{2}{q}\right)\int_0^{R} r_s \sin(q r_s)\, V(r_s)\, dr_s$$

$$\Rightarrow f_B(\theta) = -\frac{2\mu}{\hbar^2 q}\int_0^{R} r_s \sin(q r_s)\, dr_s\, V(r_s)$$

$$V(r_s) = -V_0 \Rightarrow$$

$$f_B(\theta) = \frac{2\mu V_0}{\hbar^2 q}\int_0^{R} r_s \sin(q r_s)\, dr_s$$

integration by part :

5

$$\int_0^{R} r_s \sin(q r_s)\, dr_s = -\frac{r_s}{q}\cos(q r_s)\Big|_0^{R} + \frac{1}{q}\int_0^{R}\cos(q r_s)\, dr_s$$

$$= -\frac{R}{q}\cos(qR) + \frac{1}{q^2}\sin(qR)$$

$$\Rightarrow f_B(\theta) = \frac{2\mu V_0}{\hbar^2 q^3}\left[\sin(qR) - Rq\cos(qR)\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_B = |f_B(\theta)|^2 \Rightarrow$$

$$\left(\frac{d\sigma}{d\Omega}\right)_B = \frac{4\mu^2 V_0^2}{\hbar^4 q^6}\left[\sin(qR) - Rq\cos(qR)\right]^2$$

الدكتور علي خطيب