قسم الفيزياء
كلية العلوم
جامعة رشيد

امتحان الالكتروديناميك
السنة الثالثة فيزياء
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سلم تصحيح

العلامة 100
المدة: ساعتان.

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$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} \partial_x \phi(x,y,z) \\ \partial_y \phi(x,y,z) \\ \partial_z \phi(x,y,z) \end{pmatrix}$$

$$\phi(x,y,z) = x^2 + 3zx - yzx + \lambda(y,z)$$

$$\frac{\partial \phi}{\partial y} = -2y - xz \Rightarrow -zx + \frac{\partial \lambda}{\partial y} = -2y - xz \Rightarrow$$

$$\lambda(y,z) = -y^2 + \xi(z)$$

$$\phi(x,y,z) = x^2 + 3zx - yzx - y^2 + \xi(z)$$

$$\frac{\partial \phi}{\partial z} = 2 + 3x - xy \;,\quad 3x - yx + \frac{\partial \xi}{\partial z} = 2 + 3x - xy \Rightarrow \xi = 2z + C$$

$$\Rightarrow \phi(x,y,z) = x^2 + 3zx - yzx - y^2 + 2z + C$$

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$$\vec{v} = v_k \hat{e}_k \,,\quad \vec{f} = f_k \hat{e}_k \,,\quad \vec{v}\cdot f = v_k f_k \qquad k = 1,2,3$$

$$[\vec{v} \times \vec{f}\,]_i = \epsilon_{ijk} v_j f_k \,,\quad i,j,k = 1,2,3$$

$$\vec{\nabla} = \hat{e}_k \partial_k \equiv \hat{e}_k \nabla_k \,,\quad \vec{\nabla}\varphi = \hat{e}_k \partial_k \varphi = \hat{e}_k \nabla_k \varphi = \hat{e}_k \frac{\partial \varphi}{\partial r_k}$$

$$\vec{\nabla}\cdot\vec{D} = \nabla_k D_k = \partial_k D_k = \frac{\partial D_k}{\partial r_k}$$

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$$(\partial_\nu) = \left(\frac{\partial}{\partial x^\nu}\right) = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right)$$

$$(\partial^\nu) = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x_1}, -\frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_3}\right) \,,\quad (x^\nu) = (ct, x_1, x_2, x_3) = (ct, \vec{r}\,)$$

$$(x_\nu) = (ct, -\vec{r}\,) \,,\quad (J^\nu) = (c\rho, J_1, J_2, J_3) \,,\quad (J_\nu) = (c\rho, -\vec{J}\,)$$

$$a_{00} Y_{00} + a_{10} R Y_{10} + a_{11} R Y_{11} + a_{1-1} R Y_{1-1} \equiv \phi_0 \sin\theta \left( \frac{e^{i\varphi} + e^{-i\varphi}}{2} \right)$$

$$= \frac{\phi_0}{2} \left[ \sin\theta\, e^{i\varphi} + \sin\theta\, e^{-i\varphi} \right] \quad 3$$

$$= -\frac{\phi_0}{2} \sqrt{\frac{8\pi}{3}} \left[ Y_{11}(\theta,\varphi) - Y_{1-1}(\theta,\varphi) \right]$$

$$\Rightarrow a_{00} = 0, \quad a_{10} = 0, \quad a_{11} R = -\frac{\phi_0}{2}\sqrt{\frac{8\pi}{3}}, \quad a_{1-1} R = \frac{\phi_0}{2}\sqrt{8\pi/3} \quad 5$$

$$\Rightarrow \phi(r,\theta,\varphi) = -a_{11}\, r \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\varphi} + a_{1-1}\, r \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{-i\varphi}$$

$$= \frac{a_0}{2R} r \sin\theta \left( e^{i\varphi} + e^{-i\varphi} \right) = \frac{\phi_0}{R} r \sin\theta \cos\varphi \qquad r < R \quad 3$$

14 $\underline{r > R}$ $\quad r \to \infty \Rightarrow \vec{E} = E_0 \hat{z} \Rightarrow E_0 = -\frac{\partial \phi}{\partial z} \Rightarrow$

$$\phi(r \to \infty, \theta, \varphi) = -E_0 z = -E_0 r \sqrt{\frac{4\pi}{3}} Y_{10}(\theta,\varphi) \quad 6$$

$$a_{10} = -E_0 \sqrt{\frac{4\pi}{3}}, \quad a_{lm} = 0 \text{ for } (l,m) \neq (1,0)$$

الشرط الحدي

$$\phi(R,\theta,\varphi) = a_{10} R Y_{10} + \frac{b_{10}}{R^2} Y_{10} + \frac{b_{11}}{R^2} Y_{11} + \frac{b_{1,-1}}{R^2} Y_{1-1} \equiv -\frac{\phi_0}{2}\sqrt{\frac{8\pi}{3}} (Y_{11} - Y_{1-1})$$

5 $$\Rightarrow a_{10} R + \frac{b_{10}}{R^2} = 0 \Rightarrow b_{11} = -\frac{\phi_0}{2}\sqrt{\frac{8\pi}{3}} R^2, \quad b_{1-1} = \frac{\phi_0}{2}\sqrt{\frac{8\pi}{3}} R^2, \quad b_{10} = E_0 \sqrt{\frac{4\pi}{3}} R^3$$

$$\phi(r,\theta,\varphi) = -E_0 \sqrt{\frac{4\pi}{3}}\, r \sqrt{\frac{3}{4\pi}} \cos\theta + \frac{R^3}{r_0} E_0 \sqrt{\frac{4\pi}{3}} \cos\theta$$
$$- \frac{R^2}{r^2} \frac{\phi_0}{2} \sqrt{\frac{8\pi}{3}} \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\varphi} + \frac{R^2}{r^2} \frac{\phi_0}{2} \sqrt{\frac{8\pi}{3}} \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{-i\varphi}$$

3 $$\Rightarrow \phi(r,\theta,\varphi) = E_0 \left( \frac{R^3}{r^3} - 1 \right) r \cos\theta + \phi_0 R^2 \frac{1}{r^2} \sin\theta \cos\varphi. \quad \text{for } r > R.$$

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$r < R$ :

$$E(r,\theta,\varphi) = \frac{\phi_0}{R} \left[ (-\sin\theta \cos\varphi)\, \hat{e}_r - \left( \frac{1}{r} \cos\theta \cos\varphi \right) \hat{e}_\theta + \left( \frac{1}{r} \sin\varphi \right) \hat{e}_\varphi \right] \quad 4$$

$r > R$ :

$$\vec{E}(r,\theta,\varphi) = \left[ -2 E_0 R^3 \frac{\cos\theta}{r^3} - E_0 \cos\theta - 2\phi_0 R^2 \frac{1}{r^3} \sin\theta \cos\varphi \right] \hat{e}_r +$$
$$\left[ -E_0 \left( \frac{R^3}{r^3} - 1 \right) \frac{\sin\theta}{r} + \phi_0 R^2 \frac{1}{r^3} \cos\theta \cos\varphi \right] \hat{e}_\theta + \quad 4$$
$$\left[ -\phi_0 R^2 \frac{1}{r^3} \sin\varphi \right] \hat{e}_\varphi.$$

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1- بسبب التناظر الاسطواني فإن $\Phi = \Phi(\rho)$ وعندها $\vec{E} = E(\rho)\,\hat{e}_\rho$ 1

25 معادلة Poisson.

$$\nabla^2 \Phi(\vec{r}) = -4\pi \varrho(\vec{r}) \quad 1$$

$$1\ \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) = -4\pi\varrho_0 \quad \text{for } \rho \le R \quad (1)$$

$$1\ \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) = 0 \quad \text{for } \rho > R \quad (2)$$

(1) $\rho \le R$: $\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) = -4\pi\varrho_0\rho \Rightarrow \rho\frac{\partial\Phi}{\partial\rho} = -4\pi\varrho_0\frac{\rho^2}{2} + C_1$ 1

$$\Rightarrow \frac{\partial\Phi}{\partial\rho} = -2\pi\varrho_0\rho + \frac{C_1}{\rho} \Rightarrow \Phi(\rho) = -\pi\varrho_0\rho^2 + C_1\ln\frac{\rho}{R} + d_1 \quad 1$$

(2) $\rho > R$

$$\Rightarrow \rho\frac{\partial\Phi}{\partial\rho} = C_2 \Rightarrow \frac{\partial\Phi}{\partial\rho} = \frac{C_2}{\rho} \Rightarrow \Phi(\rho) = C_2\ln\frac{\rho}{R} + d_2 \quad 1$$

$$\rho \to 0 \Rightarrow C_1 = 0 \ \ \&\ \ d_2 = 0 \quad 1$$

$$-\pi\varrho_0R^2 + d_1 = C_2\ln\frac{R}{R} \Rightarrow d_1 = \pi\varrho_0R^2 = \frac{q}{\ell} \quad 1$$

$$\Phi'_{in}\Big|_{\rho=R} = \Phi'_{out}\Big|_{\rho=R} \Rightarrow -2\pi\varrho_0R = C_2\frac{1}{R} \Rightarrow C_2 = -2\pi\varrho_0R^2 = -\frac{2q}{\ell} \quad 1$$

$$\Rightarrow \Phi(\rho) = \begin{cases} \frac{q}{\ell}\left(1 - \frac{\rho^2}{R^2}\right) & 1 \quad \rho \le R \\ -\frac{2q}{\ell}\ln\frac{\rho}{R} & 1 \quad \rho > R \end{cases}$$

$$\vec{E} = -\Phi'(\rho)\,\hat{e}_\rho = \frac{2q}{\ell}\begin{cases} \frac{\rho}{R^2} & \rho \le R \quad 1 \\ \frac{1}{\rho} & \rho > R \quad 1 \end{cases}$$

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13 for $r < R$: $b_{\ell m} = 0$

$$\Phi(r,\theta,\varphi) = \sum_{\ell m} a_{\ell m} r^\ell Y_{\ell m}(\theta,\varphi)$$

$$= a_{00}Y_{00} + a_{10}Y_{10} + a_{11}rY_{11} + a_{1,-1}rY_{1,-1} \quad 1$$

الشرط الحدي عند سطح الكرة

$$\Phi(R,\theta,\varphi) = \sum_{\ell m} R^\ell Y_{\ell m}(\theta,\varphi) \equiv \Phi_0 \sin\theta\cos\varphi \quad 1$$

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Q III

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$$\vec{\mu} = \frac{1}{2c}\int d^3r\, \vec{r} \times \vec{J}(\vec{r})$$

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$$= \frac{1}{2c}\int_0^R \rho\, d\rho \int_0^{2\pi} d\varphi \int_{-\infty}^{+\infty} dz\, (\rho\hat{e}_\rho + z\hat{e}_z)\times\hat{e}_\varphi\, I\, \delta(\rho-R)\,\delta(z)$$

$$= \frac{I}{2c}\int_0^R \rho\, d\rho \int_0^{2\pi} d\varphi \int_{-\infty}^{+\infty} dz\, (\rho\hat{e}_z - z\hat{e}_\rho)\,\delta(\rho-R)\,\delta z$$

$$= \frac{2\pi I}{2c}\left[\int_0^R \rho\, d\rho\, \delta(\rho-R)\int_{-\infty}^{+\infty} dz\, \delta z\, (\rho\hat{e}_z - z\hat{e}_\rho)\right]$$

$$= \frac{2\pi I}{2c}\int_0^\infty \delta(\rho-R)\,\rho^2\, d\rho = \frac{\pi I R^2}{c}\,\hat{e}_z$$

$$\vec{\nabla}\times\vec{A} = \frac{\pi R^2 I}{c}\left[\frac{3z\rho}{(\rho^2+z^2)^{5/2}}\,\hat{e}_\varphi + \frac{1}{\rho}\left(\frac{\rho}{(\rho^2+z^2)^{3/2}} - \frac{3\rho^3}{(\rho^2+z^2)^{5/2}}\right)\hat{e}_z\right]$$

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$$= \frac{\pi R^2 I}{c}\left[\frac{3z\rho}{(\rho^2+z^2)^{5/2}}\,\hat{e}_\varphi + \left(\frac{\rho^2+z^2}{(\rho^2+z^2)^{5/2}} - \frac{3\rho^2}{(\rho^2+z^2)^{5/2}}\right)\hat{e}_z\right]$$

$$= \frac{\pi R^2 I}{c}\,\frac{1}{(\rho^2+z^2)^{5/2}}\left[3z\rho\,\hat{e}_\varphi + (\rho^2+z^2-3\rho^2)\,\hat{e}_z\right]$$

$$\Rightarrow \vec{B} = \frac{\pi R^2 I}{c}\,\frac{1}{(\rho^2+z^2)^{5/2}}\left[3z\rho\,\hat{e}_\varphi + (z^2-2\rho^2)\,\hat{e}_z\right]$$

الدكتور علي قطب