ع / (20)

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -m_{21} & 1 & 0 & 0 \\ -m_{31} & 0 & 1 & 0 \\ -m_{41} & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_1[A \vdots B] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 2 & -5 \\ 1 & 2 & 4 & 4 & 5 \\ -1 & -3 & -9 & 8 & -27 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 3 & -5 \\ 0 & 2 & 4 & 3 & 5 \\ 0 & -3 & -9 & 9 & -27 \end{array}\right]$$

$$R_2 \leftrightarrow R_4 \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & -3 & -9 & 9 & -27 \\ 0 & 2 & 4 & 3 & 5 \\ 0 & -1 & -1 & 3 & -5 \end{array}\right] \qquad M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 \\ 0 & -\frac{1}{3} & 0 & 1 \end{bmatrix}$$

$$M_2(M_1[A \vdots B]) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 \\ 0 & -\frac{1}{3} & 0 & 1 \end{bmatrix} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & -3 & -9 & 9 & -27 \\ 0 & 2 & 4 & 3 & 5 \\ 0 & -1 & -1 & 3 & -5 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & -3 & -9 & 9 & -27 \\ 0 & 0 & -2 & 9 & -13 \\ 0 & 0 & 2 & 0 & 4 \end{array}\right]$$

$$M_3[M_2(M[A \vdots B])] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & -3 & -9 & 9 & -27 \\ 0 & 0 & -2 & 9 & -13 \\ 0 & 0 & 0 & 9 & -9 \end{array}\right] \Rightarrow$$

$$x_1 + x_4 = 0$$
$$-3x_2 - 9x_3 + 9x_4 = -27$$
$$-2x_3 + 9x_4 = -13$$
$$x_4 = -1 \Leftarrow 9x_4 = -9$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

ع (18)

متناظرة موجبة

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 13 & 13 \\ 1 & 13 & 27 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$l_{i1} = \frac{a_{i1}}{l_{11}} \longrightarrow l_{21} = \frac{3}{1} = 3 \qquad l_{31} = \frac{a_{31}}{l_{11}} = 1$$

$$l_{11} = \sqrt{a_{11}} = 1$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2} = \sqrt{13 - 9} = \sqrt{4} = 2$$

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$$

$$l_{32} = \frac{a_{32} - l_{31} l_{21}}{l_{22}} = \frac{13 - 3}{2} = 5$$

$$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2} = \sqrt{27 - 1 - 25} = \sqrt{1} = 1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LY = B \Leftrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ -10 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

$$L^T X = Y \Leftarrow \begin{bmatrix}1&3&1\\0&2&5\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\-2\\0\end{bmatrix} \Rightarrow$$

$$x_1 + 3x_2 + x_3 = 0$$
$$2x_2 + 5x_3 = -2$$
$$x_3 = 0$$

$$X = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}3\\-1\\0\end{bmatrix}$$

ج٣ : نغير ترتيب المعادلات من أجل تحقيق شروط التقارب

18

$$12x_1 + x_2 + x_3 + x_4 = 15$$
$$x_1 + 12x_2 + x_3 + x_4 = 15$$
$$x_1 + x_2 + 12x_3 + x_4 = 15$$
$$x_1 + x_2 + x_3 + 12x_4 = 15$$

$$\|\alpha\| < 1 \qquad \alpha = \begin{bmatrix}0&-\frac{1}{12}&-\frac{1}{12}&-\frac{1}{12}\\-\frac{1}{12}&0&-\frac{1}{12}&-\frac{1}{12}\\-\frac{1}{12}&-\frac{1}{12}&0&-\frac{1}{12}\\-\frac{1}{12}&-\frac{1}{12}&-\frac{1}{12}&0\end{bmatrix} \qquad \beta = \begin{bmatrix}\frac{15}{12}\\\frac{15}{12}\\\frac{15}{12}\\\frac{15}{12}\end{bmatrix}$$

نختار $X^0 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

$$X^1 = \alpha X^0 + \beta = \begin{bmatrix}1.25\\1.25\\1.25\\1.25\end{bmatrix}$$

$$X^2 = \alpha X^1 + \beta = \begin{bmatrix}0.9375\\0.9375\\0.9375\\0.9375\end{bmatrix}$$

$$X = X^3 = \alpha X^2 + \beta = \begin{bmatrix}1.0156\\1.0156\\1.0156\\1.0156\end{bmatrix}$$

$\phi = 0$ ، ج٤

22

$$y_n^p = \frac{\phi}{\sum_{i=0} a_i} = \frac{2}{1-6+12-8} = \frac{2}{-1} = -2$$

المعادلة المميزة $\lambda^3 - 6\lambda^2 + 12\lambda - 8 = 0$

$\lambda = 2$ تحقق المعادلة

$$\lambda^3 - 6\lambda^2 + 12\lambda - 8 = (\lambda - 2)(\lambda^2 - 4\lambda + 4) = (\lambda-2)(\lambda-2)^2 = (\lambda-2)^3$$

$$y_n^c = (c_1 + c_2 n + c_3 n^2)(2)^n$$

$$y_n = y_n^c + y_n^p = (c_1 + c_2 n + c_3 n^2)(2)^n - 2$$

لتعيين الثوابت c_1, c_2, c_3

$$\Rightarrow \quad c_1 = 0$$

$$y_0 = c_1 - 2 = -2$$

$$y_1 = 2c_1 + 2c_2 + 2c_3 - 2 = 2 \qquad 2c_1 + 2c_2 + 2c_3 = 4$$

$$y_2 = 4c_1 + 8c_2 + 16c_3 - 2 = 22 \qquad 4c_1 + 8c_2 + 16c_3 = 24$$

بطرح السابقة من الأولى بجبر ا

$$c_2 + c_3 = 2$$
$$c_2 + 2c_3 = 3$$
$$-c_3 = -1 \Longrightarrow \boxed{c_3 = 1} \longrightarrow \boxed{c_2 = 1}$$

بالتعويض بجبر ا

$$y_n = (0 + n + n^2)(2)^n - 2 = n(n+1)(2)^n - 2$$

$$x_0 = 0 \qquad y_0 = 2 \qquad y_{n+1} = y_n + h f_n$$

$$y' = f_n = -x_n y_n^2 \qquad f_0 = -x_0 y_0^2 = 0$$

$$f_1 = -x_1 y_1^2 = -0.2(2)^2 = -0.8$$

$$y_1 = y_0 + h f_0 = 2$$

$$y_2 = y_1 + h f_1$$
$$y_2 = 2 + (0.2)(-0.8) = 2 - 0.16 = 1.84$$

$$f_2 = -x_2 y_2^2 = -0.4(1.84)^2 = -1.354241$$

$$y_3 = y_2 + h f_2 = 1.84 + 0.2(-1.35424) = 1.5692$$

$$f_3 = -x_3 y_3^2 = -0.6(1.5692)^2 = -1.4774$$

$$y_4 = y_3 + h f_3 = 1.5692 + 0.2(-1.4774) = 1.27372$$

$$y_5 = y_4 + h f_4 = 1.27372 + 0.2(-1.29781) = 1.01416$$

$$f_4 = -x_4 y_4^2 = -0.8(1.27372)^2 = -1.2978$$

الخطأ الحقيقي

$$y' = \frac{dy}{dx} = -xy^2 \Longrightarrow -y^{-1} = -\frac{1}{2}x^2 + c$$

نعين c بالاعتماد على الشرط الابتدائي

$$-\frac{1}{y} = -\frac{1}{2}x^2 + c \Longrightarrow -\frac{1}{2} = -\frac{1}{2}(0)^2 + c \Longrightarrow c = -\frac{1}{2}$$

ومنه

$$-\frac{1}{y} = -\frac{1}{2}x^2 - \frac{1}{2} = -\frac{x^2+1}{2} \Longrightarrow y = \frac{2}{x^2+1}$$

حل المعادلة الحقيقي

الخطأ الحقيقي
$$e_5 = y(x_5) - y_5 = \frac{2}{(1)^2+1} - y_5 = 1 - 1.01416 = -0.01416$$