سلم تصحيح تحليل (4) السنة الثالثة رياضيات [illegible]

السؤال الأول 35 درجة $\frac{20}{20}$

(1) بما أن f قابلة للاشتقاق في c يوجد $\delta_1 > 0$ بحيث أياً كان $h=(h_1,h_2,\dots,h_n)\in\mathbb{R}^n$

يحقق $\|h\|<\delta_1$ فهنالك أشعة $A=(A_1,A_2,\dots,A_n)$ في $\mathbb{R}^n$

بحيث يتحقق

$$f(c+h)-f(c)=\sum_{i=1}^{n}A_i h_i+\eta\|h\|$$

حيث η دالة في h تحقق $\lim_{\|h\|\to 0}\eta=0$

نستنتج من هذا أنه أياً كان $\|h\|<\delta_1$

$$|f(c+h)-f(c)|=\left|\sum_{i=1}^{n}A_i h_i+\eta\|h\|\right|$$

$$\le\sum_{i=1}^{n}|A_i||h_i|+|\eta|\,\|h\|$$

$$\le\sum_{i=1}^{n}|A_i|\,\|h\|+|\eta|\,\|h\|$$ (لأن $|h_i|\le\|h\|$, $i=1,\dots,n$)

$$\le\left(\sum_{i=1}^{n}|A_i|+|\eta|\right)\|h\| \quad (*)$$

وبما أن $\lim_{\|h\|\to 0}\eta(h)=0$ فإنه يقابل العدد الموجب 1 عدد موجب δ_2 بحيث

أياً كان $0<\|h\|<\delta_2$ فإن $|\eta(h)|<1$

إذن

$$|f(c+h)-f(c)|\le\left(\sum_{i=1}^{n}|A_i|+1\right)\|h\|$$

نضع $c+h=x$ و $\delta=\min(\delta_1,\delta_2)$ و $K=\sum_{i=1}^{n}|A_i|+1>0$

عندئذ $\|h\|<\delta$

$$|f(x)-f(c)|\le K\,\|x-c\|$$

وهو المطلوب

(2) [illegible] إذا كان $y=0_X$ فإن

$$\ell_1=\langle x,y\rangle^2=\langle x,0_X\rangle^2=\langle x,0_{\mathbb{R}}y\rangle^2=0_{\mathbb{R}}^2\langle x,y\rangle=0_{\mathbb{R}}$$

$$\ell_2=\langle x,x\rangle\langle y,y\rangle=\langle x,x\rangle\langle 0_X,0_X\rangle=\langle x,x\rangle\langle 0_{\mathbb{R}}y,0_{\mathbb{R}}y\rangle$$

$$=0_{\mathbb{R}}^2\langle x,x\rangle\langle y,y\rangle$$

$$=0_{\mathbb{R}}$$

$\ell_1=\ell_2\Rightarrow$ فالعلاقة صحيحة

[illegible] $y\neq 0_X$ [illegible] a

[illegible] $\langle x,x\rangle+\langle x,\dots\rangle$ [illegible]

$=\langle x,x\rangle-a\langle x,y\rangle$ [illegible]

الحد الأول $f(1,1) = \ln(3)$

الحد الثاني $\frac{1}{1!}\left(h \frac{\partial f}{\partial x}(1,1) + k \frac{\partial f}{\partial y}(1,1)\right) = \frac{2h}{3} + \frac{k}{3}$

الحد الثالث $\frac{1}{2!}\left(h^2 \frac{\partial^2 f}{\partial x^2}(1,1) + 2hk \frac{\partial^2 f}{\partial x \partial y}(1,1) + k^2 \frac{\partial^2 f}{\partial y^2}(1,1)\right) = \frac{1}{9} h^2 - \frac{hk}{9}$

$$f(h+1, k+1) = \ln(3) + \frac{2h}{3} + \frac{k}{3} + \frac{h^2}{9} - \frac{hk}{9} - \frac{k^2}{18}$$

$$f(x,y) = \ln 3 + \frac{2(x-1)}{3} + \frac{y-1}{3} + \frac{(x-1)^2}{9} - \frac{(x-1)(y-1)}{9} - \frac{(y-1)^2}{18}$$

$0 \le \langle x, x\rangle - 2a\langle x, y\rangle + a^2\langle y, y\rangle$

$a = \dfrac{\langle x, y\rangle}{\langle y, y\rangle}$

$0 \le \langle x, x\rangle - 2\dfrac{\langle x, y\rangle^2}{\langle y, y\rangle} + \dfrac{\langle x, y\rangle^2}{\langle y, y\rangle^2}\langle y, y\rangle$

$0 \le \langle x, x\rangle - 2\dfrac{\langle x, y\rangle^2}{\langle y, y\rangle} + \dfrac{\langle x, y\rangle^2}{\langle y, y\rangle}$

$0 \le \langle x, x\rangle - \dfrac{\langle x, y\rangle^2}{\langle y, y\rangle}$

$0 \le \langle x, x\rangle\langle y, y\rangle - \langle x, y\rangle^2$

$\Rightarrow \langle x, y\rangle^2 \le \langle x, x\rangle\langle y, y\rangle$

$\forall x, y \in X$

$I) 20$, $II) 20$, $III) 20$ $\Rightarrow$ (60)

4) $f(x,y) = 4x^2 - 2x^2y + y^2$

$f_x(x,y) = 8x - 4xy = 0 \; (1)$
$f_y(x,y) = -2x^2 + 2y^2 = 0 \; (2)$ $\Rightarrow 4x(2 - y) = 0 \Rightarrow x = 0, \; y = 2$

$y = 0 \Leftarrow 2y^2 = 0 \Leftarrow x = 0$

$(0,0)$ $\Leftarrow$ $y = 2$

$x^2 = 4 \Leftarrow -2x^2 + 8 = 0$

$x = \mp 2 \Rightarrow (-2, 2), \; (+2, +2)$

$(0,0), (2,2), (-2,2)$

$f_{xx} = 8 - 4y$
$f_{xy} = -4x$
$f_{yy} = 2y$

$f_{xx}(0,0) = 8 > 0, \; f_{xy}(0,0) = 0, \; f_{yy}(0,0) = 0$

$D_1 = 8 > 0$

$D_2 = \begin{vmatrix} 8 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$N((0,0), \delta) \ni (\frac{\delta}{2}, 0)$

$f(\frac{\delta}{2}, 0) = \frac{4\delta^2}{4} = \delta^2 > 0 = f(0,0)$

$N((0,0), \delta) \ni (\frac{\delta}{2}, \frac{\delta}{2})$

[illegible]

$\Delta_1 = 16 - 2(4)(2) + 4 = 4 > 0$ $\qquad (2, 2)$

$\Delta_2 = \begin{vmatrix} 0 & -8 \\ -8 & 8 \end{vmatrix} = -64 < 0 \qquad (2, 2)$

$(-2, 2)$

$\Delta_1 = 4 > 0$

$\Delta_2 = \begin{vmatrix} 0 & 8 \\ 8 & 8 \end{vmatrix} = -64 < 0 \qquad (-2, 2)$

$f(h,k) - f(0,0) = h f_x(0,0) + k f_y(0,0) + \eta\sqrt{h^2 + k^2}$

$$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

$$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

$$\frac{h^2 k}{h^4 + k^2} - 0 = \eta\sqrt{h^2 + k^2} \Longrightarrow \eta = \frac{h^2 k}{(h^4 + k^2)(h^2 + k^2)^{\frac{1}{2}}}$$

$(h = k)$

$$\eta = \frac{h^3}{(h^4 + h^2)(2h^2)^{\frac{1}{2}}} = \frac{h^3}{h^2(h^2+1)(2)^{\frac{1}{2}} h}$$

$$= \frac{h^3}{h^3(h^2+1)\, 2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}(h^2+1)} \xrightarrow[h \to 0]{} \frac{1}{\sqrt{2}} \neq 0$$

⇐ الدالة f غير قابلة للاشتقاق في $(0,0)$

$$f(a+h, b+k) - f(a,b) = \sum_{n=1} \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^n f(a,b) + R_{m+1}$$

$f(1,1) = \ln(3)$

$f_x(x,y) = \frac{2x}{x^2+y+1} \rightarrow f_x(1,1) = \frac{2}{3}$ $\qquad f_y(x,y) = \frac{1}{x^2+y+1} \rightarrow f_y(1,1) = \frac{1}{3}$

$f_{xx}(x,y) = \frac{2(x^2+y+1) - 4x^2}{(x^2+y+1)^2} \rightarrow f_{xx}(1,1) = \frac{2}{9}$ $\qquad f_{yy}(x,y) = \frac{-1}{(x^2+y+1)^2} \rightarrow f_{yy}(1,1) = -\frac{1}{9}$

$f_{xy}(x,y) = \ldots \rightarrow f_{xy}(1,1) = -\frac{2}{9}$

(3)