26
30
15

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 4 \\ -1 & 3 & 5 & 2 \\ 1 & 4 & K^2-14 & K+2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 2 & 1 & -3 & 4 \\ 1 & 4 & K^2-14 & K+2 \end{array}\right] \xrightarrow[R_1+R_3]{2R_1+R_2}$$

10

$$\left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 7 & K^2-9 & K+4 \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 0 & K^2-16 & K-4 \end{array}\right]$$

6

$$\left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 0 & (K-4)(K+4) & K-4 \end{array}\right]$$

$K=4$
$$\left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
عدد لا نهائي من الحلول

3

$K=-4$
$$\left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 0 & 0 & -8 \end{array}\right]$$
مستحيل الحل

3

$K \neq 4, K \neq -4$
$$\left[\begin{array}{ccc|c} -1 & 3 & 5 & 2 \\ 0 & 7 & 7 & 8 \\ 0 & 0 & 1 & \frac{1}{K+4} \end{array}\right]$$
حل وحيد

3

2) $B^{-1} = \dfrac{adj(B)}{\det B}$

2

$$adj(B) = \begin{bmatrix} +15 & +27 & +(-2) \\ -6 & +(-18) & -4 \\ +(-3) & +9 & +(-2) \end{bmatrix}^T = \begin{bmatrix} 15 & -6 & -3 \\ 27 & -18 & 9 \\ -2 & -4 & -2 \end{bmatrix}$$

8

$$\det B = 2\begin{vmatrix} 4 & -6 \\ 2 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -4 & 2 \end{vmatrix}$$

$$= 2(3+12) - 3(2-4) = 30 + 6 = 36 \neq 0$$ $\underline{\underline{3}}$

$$B^{-1} = \frac{1}{36}\begin{bmatrix} 15 & -6 & -3 \\ 27 & -18 & 9 \\ -2 & -4 & -2 \end{bmatrix}$$ $\underline{\underline{3}}$

$$4I_3 - B^{-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} \frac{15}{36} & -\frac{1}{6} & -\frac{1}{12} \\ \frac{27}{36} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{18} & -\frac{1}{9} & -\frac{1}{18} \end{bmatrix} = \begin{bmatrix} \frac{43}{12} & \frac{1}{6} & \frac{1}{12} \\ -\frac{27}{36} & \frac{9}{2} & -\frac{1}{4} \\ \frac{1}{18} & \frac{1}{9} & \frac{73}{18} \end{bmatrix}$$

$$B^T \cdot A^T = \begin{bmatrix} 2 & 1 & -4 \\ 0 & -1 & 2 \\ -3 & -6 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 & 4 \\ 0 & 5 & -2 \\ 3 & 6 & 7 \end{bmatrix} = \begin{bmatrix} -8 & -21 & -32 \\ 6 & 7 & 16 \\ -15 & -45 & -21 \end{bmatrix}$$

3)
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix} = (b-a)(c-a)(c-b)$$
$$= (a-b)(b-c)(c-a)$$

1) $f(x,y,z,t) = (y-z,\ z-t-x,\ x-y+t)$

$\forall u, v \in \mathbb{R}^4,\ u(x,y,z,t)\ v(x_1,y_1,z_1,t_1),\ \forall \alpha \in \mathbb{R}$

$f(u+v) = f(x+x_1, y+y_1, z+z_1, t+t_1)$

$= (y+y_1-z-z_1,\ z+z_1-t-t_1-x-x_1,\ x+x_1-y-y_1+t+t_1)$ $\underline{\underline{5}}$

$= (y-z, z-t-x, x-y+t) + (y_1-z_1, z_1-t_1-x_1, x_1-y_1+t_1)$

$= f(u) + f(v)$

$f(\alpha u) = f(\alpha x, \alpha y, \alpha z, \alpha t) = (\alpha y - \alpha z,\ \alpha z - \alpha t - \alpha x,\ \alpha x - \alpha y + \alpha t)$

$= \alpha(y-z,\ z-t-x,\ x-y+t) = \alpha f(u)$

$\ker f = \{u : u \in R^4,\ f(u) = 0_{R^3}\}$

$f(u) = (y-z,\ z-t-x,\ x-y+t) = (0,0,0)$

$y - z = 0$

$z - t - x = 0$

$x - y + t = 0$

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ 5

$\ker f$ taban $\{(1,-1,0,1), (0,-1,1,0)\}$

$\dim \ker f = 2$

$e_1(1,0,0,0),\ e_2(0,1,0,0),\ e_3(0,0,1,0)\ e_4(0,0,0,1)$

$f(e_1) = (0,-1,1)\quad f(e_2) = (1,0,-1)\quad f(e_3) = (-1,1,0)\ f(e_4) = (0,-1,1)$

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5

$\operatorname{Im} f$ taban $\{(0,-1,1)(1,0,-1)\}\quad \dim \operatorname{Im} f = 2$

$\dim \operatorname{Im} f + \dim \ker f = 4 = \dim R^4$

$f: R^2 \longrightarrow R^3$

$f(x,y) = (x+4y,\ 0,\ 3x-2y)$

$A = \{a_1(0,2),\ a_2(1,1)\}\quad B = \{b_1(1,1,0), b_2(1,0,1), b_3(0,1,1)\}$

$f(a_1) = f(2,1) = (8,0,-4) = \alpha_1(1,1,0) + \alpha_2(1,0,1) + \alpha_3(0,1,1)$ 3

$f(a_2) = f(1,1) = (5,0,1) = \alpha_1(1,1,0) + \alpha_2(1,0,1) + \alpha_3(0,1,1)$

$\left.\begin{array}{l} 8 = \alpha_1 + \alpha_2 \\ 0 = \alpha_1 + \alpha_3 \\ -4 = \alpha_2 + \alpha_3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8 = \alpha_2 - \alpha_3 \\ -4 = \alpha_2 + \alpha_3 \end{array}\right\} \Rightarrow 4 = 2\alpha_2 \rightarrow \boxed{\alpha_2 = 2} \Rightarrow \boxed{\alpha_1 = 6}\ \boxed{\alpha_3 = -6}$ 5

$$\left.\begin{aligned} 5 &= \alpha_1 + \alpha_2 \\ 0 &= \alpha_1 + \alpha_3 \\ 1 &= \alpha_2 + \alpha_3 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 5 &= \alpha_2 - \alpha_3 \\ 1 &= \alpha_2 + \alpha_3 \end{aligned}\right\} \Rightarrow 6 = 2\alpha_2 \Rightarrow \alpha_2 = 3$$

$$\Rightarrow \alpha_1 = 2, \quad \alpha_3 = -2$$

$$M_A^B(f) = \begin{bmatrix} 6 & +2 \\ 2 & 3 \\ -6 & -2 \end{bmatrix}$$

$$f(a_1) = +6b_1 + 2b_2 - 6b_3$$

$$f(a_2) = 2b_1 + 3b_2 - 2b_3$$