

الموضوع مادة حساب التفاضل (2) لطلاب الرياضيات 10

السؤال الأول : 35 ، 30

$$\textcircled{1} \int \frac{x \, dx}{x^2 - 3x + 2} = \int \left(\frac{2}{9} \left(\frac{1}{x-1} \right) + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)} \right) dx$$

$$= \frac{2}{9} \ln|x-1| - \frac{1}{3(x-1)} - \frac{2}{9} \ln|x+2| + C \quad \textcircled{3}$$

$$\textcircled{2} \int \frac{(x+2) \, dx}{x^3 + 2x^2 - x - 2} = \int \frac{(x+2) \, dx}{(x-1)(x+1)(x+2)} = \int \frac{dx}{(x-1)(x+1)}$$

$$= \int \left(\frac{1}{2(x-1)} + \frac{-1}{2(x+1)} \right) dx$$

$$= \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \quad \textcircled{7}$$

$$\textcircled{3} \int \frac{dx}{\sqrt{x^2 + 6x + 4}} = \int \left(\frac{1}{1-t} + \frac{1}{1+t} \right) dt$$

$$\begin{aligned} \sqrt{x^2 + 6x + 4} &= x t + 2 \\ dx &= \frac{4t^2 - 12t + 4}{(1-t^2)^2} dt \end{aligned}$$

$$= \ln \left| \frac{1 + \frac{\sqrt{x^2 + 6x + 4} - 2}{x}}{1 - \frac{\sqrt{x^2 + 6x + 4} - 2}{x}} \right| + C \quad \textcircled{2}$$

$$\textcircled{4} \int \frac{dx}{\sin x} = \int \frac{dt}{t} \quad : t = \tan \frac{x}{2} \quad \textcircled{4}$$

$$= \ln t = \ln \left| \tan \frac{x}{2} \right| + C \quad \textcircled{3}$$

$$\textcircled{5} \int \sin^n x dx = -\sin^{n-1} x \cos x + (n-1) \int (1 - \sin^2 x) \sin^{n-2} x dx$$

$$\vdots$$

$$= -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2} \quad \textcircled{4}$$

المسألة 35 : المطلوب

$$\textcircled{1} \int_0^{\frac{\pi}{2}} e^{2x} \cos 3x dx = \left[\frac{1}{2} e^{2x} \cos 3x + \frac{6}{26} e^{2x} \sin 3x - \frac{18}{54} e^{2x} \cos 3x \right]_0^{\frac{\pi}{2}} = \textcircled{4}$$

$$= -\frac{6}{26} e^{\pi} + \frac{1}{2} - \frac{18}{54} = -\frac{3}{13} e^{\pi} + \frac{1}{6} \quad \textcircled{3}$$

$$\textcircled{2} \int_{-\infty}^{\infty} \frac{e^x dx}{1+e^{2x}} = [\arctan t]_0^1 = \textcircled{4}$$

$$= \frac{\pi}{2} \quad \textcircled{3}$$

$$\textcircled{3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2 \sin x \cos x - 3 \tan x) dx = 0$$

صحيح $\forall x$ \textcircled{7}

$$\textcircled{4} \int_0^1 \int_0^x (3-x-y) dx dy = \int_0^1 (3x - x^2 - \frac{x^2}{2}) dx \quad \textcircled{3}$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{2} \right]_0^1 = \textcircled{1}$$

$$\textcircled{5} \int_0^1 \int_0^1 \frac{dx dy}{1+x^2+y^2} = \int_0^1 \int_0^1 \frac{dx dy}{1+x^2+y^2} \quad \textcircled{2}$$

$x = r \cos \theta$
 $y = r \sin \theta$
 $J = r$

$$= \int_1^2 2r^2 dr \quad (2)$$

$$= \left[\frac{2}{3} r^3 \right]_1^2 \quad (2)$$

$$= \frac{14}{3} \quad (1)$$

بالطوال الثالث = 3 دائرة

(1) محيط الدائرة: $L = \int_a^b \sqrt{1+y'^2} dx$ (5)

$$L = 6\pi \quad (10)$$

مساحة الدائرة

(2) $S = 2\pi \int_a^b y \sqrt{1+y'^2} dx$ (5)

$$= 36\pi \quad (10)$$

انتهى

عبد الرحمن طالب