تصحيح محاضرات تفاضلية (2) 2025

السؤال الأول $x_0=0$

$$y''-2x^2y'+4xy=x^2+2x+2$$

$p(x)=-2x^2 \qquad q(x)=4x$

نقطة عادية ونحن نفرض الحل من الشكل $x_0=0$

$$y=\sum_{n=0}^{\infty}c_nx^n$$

$$y'=\sum_{n=1}^{\infty}nc_nx^{n-1} \qquad y''=\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}$$

نعوضها في المعادلة التفاضلية

$$\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}-2\sum_{n=1}^{\infty}nc_nx^{n+1}+4\sum_{n=0}^{\infty}c_nx^{n+1}=x^2+2x+2$$

$n\to n-3 \qquad n\to n-3$

$$\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}-2\sum_{n=4}^{\infty}(n-3)c_{n-3}x^{n-2}+4\sum_{n=3}^{\infty}c_{n-3}x^{n-2}=x^2+2x+2$$

$$2(1)c_2x^0+3(2)c_3x^1+\sum_{n=4}^{\infty}n(n-1)c_nx^{n-2}-2\sum_{n=4}^{\infty}(n-3)c_{n-3}x^{n-2}+4c_0x+4\sum_{n=4}^{\infty}c_{n-3}x^{n-2}=x^2+2x+2$$

$$2c_2+[6c_3+4c_0]x+\sum_{n=4}^{\infty}\left[n(n-1)c_n+[-2(n-3)+4]c_{n-3}\right]x^{n-2}=0$$

$-2n+6$

$$2c_2+[6c_3+4c_0]x+(4(3)c_4+2c_1]x^2+\sum_{n=5}^{\infty}\left[n(n-1)c_n+(-2n+10)c_{n-3}\right]x^{n-2}=x^2+2x+1$$

بالمطابقة

$12c_4+2c_1=1 \Rightarrow c_4=\frac{1-2c_1}{12}$

$6c_3+4c_0=2 \Rightarrow c_3=\frac{2-4c_0}{6}$

$2c_2=2 \Rightarrow c_2=1$

$$c_n=\frac{[-2n+10]c_{n-3}}{n(n-1)} \qquad n\geq 4$$

قيم الثوابت

$c_0=\frac12 \qquad c_1=\frac12$

$c_3=0 \qquad c_4=0 \qquad c_5=0 \quad \dots$

ونعوض الحل الخاص بالشكل

$$y=\sum_{n=0}^{\infty}c_nx^n=c_0+c_1x+c_2x^2+c_3x^3 \dots$$

$$y=\frac12+\frac12x+x^2+0+0 \dots$$

نوجد الحل العام للمعادلة المتجانسة بالاستفادة من الخطوات السابقة نجد

$$2c_2+[6c_3+4c_0]x+\sum_{n=4}^{\infty}\left[n(n-1)c_n+(-2n+10)c_{n-3}\right]x^{n-2}=0$$

بالمطابقة نجد

$2c_2=0 \Rightarrow c_2=0$

$6c_3+4c_0=0 \Rightarrow c_3=-\frac23c_0$

$$c_n=\frac{(-2n+10)}{n(n-1)}c_{n-3} \qquad n\geq 3$$

ونعوض الحل

$$y=c_0+c_1x+c_2x^2+c_3x^3+\dots$$

$$y=c_0+c_1x+0+-\frac23c_0+\dots\frac{(-2n+10)}{n(n-1)}c_{n-3}$$

السؤال الثاني

$$y''+4y=\cos(2t)$$

بأخذ تحويل لابلاس للطرفين

$$s^2Y(s)-sy(0)-y'(0)+4Y(s)=\frac{s}{s^2+4}$$

نعوض الشروط الابتدائية

$$s^2Y(s)+4Y(s)=\frac{s}{s^2+4}$$

$$Y(s)\{s^2+4\}=\frac{s}{s^2+4}$$

$$Y(s)=\frac{s}{(s^2+4)^2}$$

$$L^{-1}\{Y(s)\}=L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$$

$$y(t)=\frac14t\sin(2t)$$

CamScanner

السؤال الثالث:

نفرض $u = 2x^2 + 2z$

$v = y^2 + 2z$

نشتق F بالنسبة ل x, y

بالنسبة ل x

$$\frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial x}$$

$$\frac{\partial F}{\partial u}\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial x}\right] = 0$$

$$\frac{\partial f}{\partial u}[4x + 2p] + \frac{\partial f}{\partial v}[0 + 2p] = 0 \quad (1)$$

نشتق بالنسبة ل y

$$\frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial y} = 0$$

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial y}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial y}\right] = 0 \quad (2)$$

$$\frac{\partial f}{\partial u}[0 + 2q] + \frac{\partial f}{\partial v}[2y + 2q] = 0$$

ونحله باستخدام المحدد

$$\begin{vmatrix} 4x+2p & 2p \\ 2q & 2y+2q \end{vmatrix} = 0$$

$(4x+2p)(2y+2q) - 4pq = 0$

$8xy + 8xq + 4py + 4pq - 4pq = 0$

$$\boxed{8xq + 4py + 8xy = 0}$$

السؤال الرابع:

$* \; q^2 - pz^2 + qz^2 = 0$

$\frac{\partial f}{\partial x} = 0 \,/\, \frac{\partial f}{\partial y} = 0 \,/\, \frac{\partial f}{\partial z} = -2zp + 2qz \,/$

$\frac{\partial f}{\partial p} = -z^2 \,/\, \frac{\partial f}{\partial q} = 2q + z^2$

$$\frac{-dy}{\frac{\partial f}{\partial q}} = \frac{-dz}{p\frac{\partial f}{\partial p} + q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}}$$

$$\frac{-dx}{-z^2} = \frac{-dy}{2q + z^2} = \frac{-dz}{-pz^2 + q(2q + z^2)} = \frac{dp}{p(-2zp + 2qz)} = \frac{dq}{0 + q[-2zp + 2qz]}$$

$$\frac{+dx}{z^2} = \frac{-dy}{2q + z^2} = \frac{-dz}{z^2(-p+q) + 2q^2} = \frac{dp}{p2z(-p+q)} = \frac{dq}{2zq[-p+q]}$$

ومن

$$\frac{dp}{p2z\cancel{(-p+q)}} = \frac{dq}{2zq\cancel{[-p+q]}} \Rightarrow \frac{dp}{p\cancel{2z}} = \frac{dq}{\cancel{2z}q}$$

$\ln(p) = \ln(q) + \ln(a)$

$\ln\left(\frac{p}{q}\right) = \ln(a) \Rightarrow \frac{p}{q} = a \Rightarrow p = aq$

نعوض *

$q^2 - aqz^2 + qz^2 = 0 \Rightarrow q[q - az^2 + z^2] = 0$

مرفوض $q = 0 \,/\, q = z^2[a-1]$

ومن

$p = az^2[a-1]$

$dz = p\,dx + q\,dy \Rightarrow dz = az^2[a-1]\,dx + z^2[a-1]\,dy$

$\frac{dz}{z^2} = a[a-1]\,dx + (a-1)\,dy$

$-\frac{1}{z} = a[a-1]x + (a-1)y + b$

$$z = -\frac{1}{a[a-1]x + (a-1)y}$$

$$\frac{dy}{dx} = 3y - 2Z \quad (1)$$

$$\frac{dZ}{dx} = 2y - Z \quad (2)$$

$$y' = 3y - 2Z \quad (1)$$

$$Z' = 2y - Z \quad (2)$$

نشتق (2)

$$Z'' = 2y' - Z' \quad (3)$$

نعوض (1) في (3)

$$Z'' = 2(3y - 2Z) - Z'$$

$$Z'' = 6y - 4Z - Z' \quad ٤$$

من (2) نجد

$$y = \frac{Z' + Z}{2}$$

$$Z'' = 3[Z' + Z] - 4Z - Z' \quad \text{*} \text{ نعوض في ٤}$$

$$Z'' = 2Z' - 1Z$$

$$Z'' - 2Z' + 1Z = 0$$

$$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \cancel{\lambda = 3 \pm \ldots}$$

$$(\lambda - 1)^2 = 0 \qquad \lambda = 1$$ جذر متساوي

نحل

$$Z = c_1 x e^{x} + c_2 e^{x}$$

نشتق

$$Z' = c_1 e^{x} + c_1 x e^{x} + c_2 e^{x}$$

نعوض

$$y = \frac{c_1 e^{x} + c_1 x e^{x} + c_2 e^{x} + c_1 x e^{x} + c_2 e^{x}}{2}$$

$$y = \frac{1}{2}\left[2c_1 x e^{x} + 2c_2 e^{x} + c_1 e^{x}\right]$$

$$y = c_1 x e^{x} + c_2 e^{x} + \frac{1}{2} c_1 e^{x}$$