كلية العلوم - قسم الفيزياء

مقرر التفاضل والمتتاليات — الفصل الثاني 17.08/2025 — السنة الأولى فيزياء — سلم التصحيح

العلامة 100 — المدة: $14^{00}-16^{00}$

$\boxed{60} = Q\,I$

20 -1 20
25
15
$\boxed{60}$

a) 5

$$\sum_{k=2}^{\infty} 2^k 3^{-2k} = \sum_{k=2}^{\infty} 2^k \frac{1}{(3^2)^k} = \sum_{k=2}^{\infty} \left(\frac{2}{9}\right)^k$$

وهي سلسلة هندسية متقاربة حيث $q=\frac{2}{9}<1$

$$\Rightarrow \sum_{k=2}^{\infty} 2^k 3^{-2k} = -1 - \frac{2}{9} + \frac{1}{1-\frac{2}{9}} = \frac{4}{63}$$

b) 5

$$\sum_{k=3}^{\infty}\left(1-\frac{1}{k}\right), \quad a_k = \frac{k-1}{k}, \quad a_{k+1} = \frac{k}{k+1}$$

نعلم أن $\sum_{k=1}^{\infty} \frac{1}{k}$ سلسلة متباعدة

$$a_k = 1-\frac{1}{k}, \quad b_k = \frac{1}{k}$$

$$\forall k \geq 3 \in \mathbb{N}:\ 1-\frac{1}{k} \geq \frac{1}{k} \Rightarrow \sum_{k=3}^{\infty}\left(1-\frac{1}{k}\right)$$ سلسلة متباعدة

$$a_k \geq b_k$$

c) 5

$$\sum_{k=0}^{\infty} \frac{3^k k!}{k^k}, \quad a_k = \frac{3^k k!}{k^k}, \quad a_{k+1} = \frac{3^{k+1}(k+1)!}{(k+1)^{k+1}} = \frac{3^k 3 (k+1) k!}{(k+1)^k (k+1)}$$

$$\Rightarrow \frac{a_{k+1}}{a_k} = \frac{3^k 3 (k+1) k!}{(k+1)^k (k+1)} \frac{k^k}{3^k k!} = 3 \frac{k^k}{(k+1)^k} = 3\left(\frac{k}{k+1}\right)^k = 3\left(\frac{1}{1+\frac{1}{k}}\right)^k$$

$$= 3 \frac{1}{\left(1+\frac{1}{k}\right)^k}$$

$$\Rightarrow \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 3 \lim_{k\to\infty} \frac{1}{\left(1+\frac{1}{k}\right)^k} = 3\frac{1}{e} > 1$$ متباعدة

d) 5

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}, \quad a_k = \frac{(k!)^2}{(2k)!}, \quad a_{k+1} = \frac{[(k+1)!]^2}{(2k+2)!}$$

$$\lim_{k\to\infty}\left|\frac{[(k+1)!]^2}{(2k+2)!}\,\frac{(2k)!}{(k!)^2}\right| = \lim\left|\frac{(k+1)^2(k!)^2}{2(k+1)(2k+1)(2k)!}\,\frac{(2k)!}{(k!)^2}\right|$$

$$=\lim_{k\to\infty}\left|\frac{k+1}{4k+2}\right| = \lim_{k\to\infty}\left|\frac{1+\frac{1}{k}}{4+\frac{2}{k}}\right| = \frac{1}{4} < 1$$

متقاربة ومجموعها $\frac{1}{4}$

- 2 (a $f(z)=\sin(z)$

10

5

$f(z)=\sin z \Rightarrow f(0)=0$

$f'(z)=\cos z \Rightarrow f^{(1)}(0)=1$

$f''(z)=-\sin z \Rightarrow f^{(2)}(0)=0$

$f^{(3)}(z)=-\cos z \quad f^{(3)}(0)=-1$

نشر Malaurin حول $x=0$

$$Tf(z,0)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}(z-0)^k$$

$$\sin(z)=z-\frac{z^3}{3!}+\frac{z^5}{5!}-\frac{z^7}{7!}+\dots=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+1}$$

(b $f(z)=e^{-2z^2}$ 5

الطريقة الأولى: استخدام نشر Maclaurin للتابع الأسي $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$

$x \to -2x$

$$e^{-2z^2}=\sum_{n=0}^{\infty}\frac{(-2z^2)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-2)^n x^{2n}}{n!}=\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{n!}x^{2n}$$

$$\Rightarrow Tf(z,0)=\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{n!}x^{2n}$$

الطريقة الثانية:

$$Tf(z,0)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}z^{2k+1}$$

$f^{(0)}(z)=e^{-2x^2} \Rightarrow f^{(0)}(0)=1$

$f^{(1)}(z)=-4xe^{-2x^2} \Rightarrow f^{(1)}(0)=0$

$$\overset{(2)}{f}(z) = -4e^{-2x^2} + 16x^2 e^{-2x^2} \Rightarrow \overset{(2)}{f}(0) = -4$$

$$\overset{(3)}{f}(z) = 16x e^{-2x^2} + 32x e^{-2x^2} - 64x^3 e^{-2x^2} \Rightarrow \overset{(3)}{f}(0) = 0$$

$$\overset{(4)}{f}(z) = 16 e^{-2x^2} - 64x^2 e^{-2x^2} + 32 e^{-2x^2} - 128x^2 e^{-2x^2} - 192x^2 e^{-2x^2} + 256x^4 e^{-2x^2}$$

$$\Rightarrow \overset{(4)}{f}(0) = 48$$

$$Tf(z,0) = 1 + 0 + (-1)\frac{4}{2!}z^2 + 0 + \frac{48}{4!}z^4 + \cdots$$

$$= 1 + (-1)^1 \frac{2}{1!}z^2 + \frac{48}{\underbrace{4!}_{4\times3\times2!}} z^4 + \cdots$$

$$\Rightarrow Tf(z,0) = \sum_{k=0}^{\infty} (-1)^k \frac{2^k}{k!} z^{2k}.$$

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$= \frac{2}{2\pi}\left[\int_{-\pi}^{0} -t\,dt + \int_0^{\pi} t\,dt\right] = \frac{1}{\pi}\left[\left[-\frac{t^2}{2}\right]_{-\pi}^{0} + \left.\frac{t^2}{2}\right|_0^{\pi}\right] = \pi$$

4 $\quad$ 3 $\quad$ 25 $\quad$ 10 $\quad$ 7 $\quad$ 4 $\quad$ 25

$$a_k = \frac{2}{T}\int_0^T f(t)\cos(k\omega t)\,dt$$

$$= \frac{2}{2\pi}\left[\int_{-\pi}^{0} -t\cos(k\omega t)\,dt + \int_0^{\pi} t\cos(k\omega t)\,dt\right]$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$I_1 = -\int_{-\pi}^{0} t\cos(k\omega t)\,dt \qquad , u = t \Rightarrow du = dt$$

$$dv = \cos(k\omega t)\,dt \Rightarrow v = \frac{1}{k\omega}\sin(k\omega t)$$

$$\Rightarrow I_1 = -\left[\frac{1}{k\omega} t\sin(k\omega t)\Big|_{-\pi}^{0} - \frac{1}{k\omega}\int_0^{\pi} \sin(k\omega t)\,dt\right]$$

$$= -\left[\frac{1}{k\omega} 0 \sin(k\omega 0) - \frac{1}{k\omega}(-\pi)\sin(k\,1(-\pi))\right] + \frac{1}{k^2\omega^2}\cos(k\omega t)\Big|_{-\pi}^{0}$$

$$= -\frac{1}{k^2\omega^2}\Big[\underbrace{\cos(k\,0)}_{=+1} - \underbrace{\cos(k\pi)}_{=(-1)^k}\Big] = -\frac{1}{k^2\omega^2}\left[1 - (-1)^k\right]$$

$$I_2 = \int_0^{\pi} t\cos(k\omega t)dt$$

$$= \left[\frac{1}{k\omega} t\sin(k\omega t)\Big|_0^{\pi} - \frac{1}{k\omega}\int_0^{\pi}\sin(k\omega t)dt\right]$$

$$= \frac{1}{k^2\omega^2}\left[(-1)^k - 1\right] \Rightarrow$$

$$a_k = \frac{1}{\pi}\left[-\frac{1}{k^2}\left(1-(-1)^k\right) + \frac{1}{k^2}\left((-1)^k - 1\right)\right]$$

$$\Rightarrow \boxed{a_k = \frac{2}{\pi k^2}\left[(-1)^k - 1\right]} \quad 10$$

$$b_k = \frac{2}{2\pi}\left\{\int_{-\pi}^{0} -t\sin(k\omega t)dt + \int_0^{\pi} t\sin(k\omega t)dt\right\}$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} t\sin(k\omega t)dt + \int_0^{\pi} t\sin(k\omega t)dt\right\}$$

$$u = t \Rightarrow du = dt, \quad dv = \sin(k\omega t) \Rightarrow v = -\frac{1}{k\omega}\cos(k\omega t)$$

$$\Rightarrow b_k = \frac{1}{\pi}\left\{-\left[-\frac{1}{k\omega} t\cos(k\omega t)\Big|_{-\pi}^{0} + \frac{1}{k\omega}\int_{-\pi}^{0}\cos(k\omega t)\right] + \right.$$

$$\left. \left(-\frac{1}{k\omega}\right) t\cos(k\omega t)\Big|_0^{\pi} + \frac{1}{k\omega}\int_0^{\pi}\cos(k\omega t)\right\}$$

$$= \frac{1}{\pi}\left\{-\left[-\frac{1}{k\omega}\underset{0}{\underset{\|}{0\cos(k0)}} + \frac{1}{k\omega}(-\pi)\overbrace{\cos(-k\pi)}^{(-1)^k}\right] + \right. \qquad \omega = 1$$

$$\left. \left(-\frac{1}{k\omega}\right)\pi\underbrace{\cos(k\pi)}_{(-1)^k} + \frac{1}{k\omega}\underset{0}{\underset{\|}{0\cos(k0)}}\right\} \quad 7$$

$$\Rightarrow b_k = \frac{1}{\pi}\left[\frac{\pi}{k}(-1)^k - \frac{\pi}{k}(-1)^k\right] = 0 \Rightarrow b_k = 0$$

$$\Rightarrow S f(t) = \frac{\pi}{2} + \sum_{k=1}^{\infty}\left[\frac{2}{\pi k^2}\left[(-1)^k - 1\right]\cos(kt)\right]$$

$$S_0 f(t) = \frac{\pi}{2}, \quad S_1 f(t) = \frac{\pi}{2} - \frac{4}{\pi}\cos t, \quad S_3 f(t) = \frac{\pi}{2} - \frac{4}{\pi}\left[\cos t + \frac{1}{32}\cos 3t\right]$$

$$S_2 f(t) = \frac{\pi}{2} - \frac{4}{\pi}\cos t = S_1 f(t)$$

$$f(x) = \ln x \,, \quad x_0 = 2$$

$$Tf(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

$$= f^{(0)}(x_0) + \frac{f^{(1)}(x_0)}{1!}(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \dots$$

$$f^{(0)}(x) = \ln x, \quad f^{(1)}(x) = \frac{1}{x} \Rightarrow f^{(1)}(2) = \frac{1}{2}$$

$$\Rightarrow f^{(0)}(2) = \ln 2 \quad f^{(2)}(x) = -\frac{1}{x^2} \Rightarrow f^{(2)}(2) = -\frac{1}{2^2} = -\frac{1}{4}$$

$$f^{(3)}(x) = \frac{2}{x^3} \Rightarrow f^{(3)}(2) = \frac{2}{2^3} = \frac{1}{4}$$

$$f^{(4)}(x) = -\frac{6}{x^4} \Rightarrow f^{(4)}(2) = -\frac{6}{2\times2\times2\times2} = -\frac{3}{8}$$

$$T\ln(x) = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{4}\frac{(x-2)^2}{2!} + \frac{1}{4}\frac{(x-2)^3}{3!} - \frac{3}{8}\frac{(x-2)^4}{4!} + \dots$$

15 + 15 + 10 = 40

QII الحساب التفاضلي. $\boxed{40}$

a) $y = x^{\sqrt{x}}$

7 $\ln y = \sqrt{x}\ln x = x^{\frac{1}{2}}\ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}x^{-1/2}\ln x + x^{\frac{1}{2}}\frac{1}{x} \Rightarrow$ $-1/5$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\frac{\times\sqrt{x}}{\times\sqrt{x}} \Rightarrow \boxed{\frac{dy}{dx} = y\left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right) = \frac{x^{\sqrt{x}}(\ln x + 2)}{2\sqrt{x}}}$$

b) $y = x^{\cos^2 x} \Rightarrow$

8 $\ln y = \cos^2 x \ln x \Rightarrow \frac{dy}{dx}\frac{1}{y} = -\cos x \sin x \ln x + \cos^2 x \frac{1}{x}$

$$\Rightarrow \frac{dy}{dx} = y\left[-\frac{1}{2}\sin 2x \ln x + \frac{\cos^2 x}{x}\right] \Rightarrow \boxed{\frac{dy}{dx} = x^{\cos^2 x}\left[-\frac{1}{2}\sin 2x \ln x + \frac{\cos^2 x}{x}\right]}$$

2 - التفاضل الضمني : $\quad 2/5$

a) $(3x - y)(2x + 3y) = 8$

7 $6x^2 + 9xy - 2yx - 3y^2 = 8 \Rightarrow 6x^2 + 7xy - 3y^2 = 8$

$$\Rightarrow 12x + 7y + 7x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0 \Rightarrow \boxed{\frac{dy}{dx} = \frac{12x + 7y}{6y - 7x}}$$

$$y = \sin(3x+4y)$$

$$\frac{dy}{dx} = \left[\frac{d}{dx}\,\cancel{\sin}(3x+4y)\right]\cos(3x+4y)$$

$$= \left(3+4\frac{dy}{dx}\right)\cos(3x+4y)$$

$$= 3\cos(3x+4y) + 4\cos(3x+4y)\frac{dy}{dx}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{3\cos(3x+4y)}{1-4\cos(3x+4y)}}$$

$$y = \frac{u^2-1}{u^2+1}, \quad u = \sqrt[3]{x^2+1}, \quad \frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$\frac{du}{dx} = \frac{d}{dx}(x^2+1)^{\frac{1}{3}} = \frac{1}{3}(2x)(x^2+1)^{-2/3}$$

$$\frac{dy}{du} = \frac{2u(u^2+1) - 2u(u^2-1)}{(u^2+1)^2} = \frac{4u}{(u^2+1)^2} = \frac{4(x^2+1)^{1/3}}{\left[(x^2+1)^{2/3}+1\right]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4(x^2+1)^{1/3}}{\left[(x^2+1)^{2/3}+1\right]^2}\cdot\frac{1}{3}(2x)(x^2+1)^{-2/3}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{4}{3}\,2x\,\frac{(x^2+1)^{-1/3}}{\left[(x^2+1)^{2/3}+1\right]^2}}$$