2025-2-15

I- المواضيع: قياس، نشر طيفي، حيث خاص، احتمالات، تبادل (54 د 10)

1- القاعدة B_u ليست قاعدة خاصة للمؤثرين $\hat{H}$ ، $\hat{A}$ لأن المصفوفات الممثلة لهما على هذه القاعدة ليست قطرية.

2- حتى يكون مؤثرا هرميتيا يجب أن يتساوى مع مرافقه أي يجب أن يكون:

$$\hat{A}^+ = \hat{A} \quad ; \quad \hat{H}^+ = \hat{H} \quad ;$$

$$[\hat{A}]_u^+ = [\hat{A}]_u^{T*} = a^* \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_u$$

نلاحظ بالمقارنة مع $[\hat{A}]_u$ أنه يكون $[\hat{A}]_u^+ = [\hat{A}]_u$ إذا كان $a \in \mathbb{R}$

$$[\hat{H}]_u^+ = [\hat{H}]_u^{T*} = E_0^* \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}_u$$

هنا أيضا يكون $[\hat{H}]_u^+ = [\hat{H}]_u$ إذا كان $E_0 \in \mathbb{R}$.

3- أ- النشر الطيفي:

$$\hat{H} = \hat{I} \cdot \hat{H} \cdot \hat{I} = \left(\sum_{i=1}^{3} |u_i\rangle \cdot \langle u_i| \right) \cdot \hat{H} \cdot \left(\sum_{i=1}^{3} |u_i\rangle \cdot \langle u_i| \right)$$

$$\hat{H} = \sum_{i=1}^{3} \sum_{j=1}^{3} |u_i\rangle \cdot \langle u_i| \hat{H} |u_j\rangle \cdot \langle u_j|$$

$$\hat{H} = \sum_{i=1}^{3} \sum_{j=1}^{3} H_{ij} \cdot |u_i\rangle \cdot \langle u_j|$$

من عبارة $[\hat{H}]_u$:

$$H_{11} = E_0 \; ; \; H_{12} = -E_0 \; ; \; H_{13} = 0$$

$$H_{21} = -E_0 \; ; \; H_{22} = +E_0 \; ; \; H_{23} = 0$$

$$H_{31} = 0 \; ; \; H_{32} = 0 \; ; \; H_{33} = -E_0$$

بالتعويض نحصل على:

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$$\hat{H} = E_0 \cdot [\,|u_1\rangle \cdot \langle u_1| - |u_1\rangle \cdot \langle u_2| - |u_2\rangle \cdot \langle u_1| + |u_2\rangle \cdot \langle u_2| - |u_3\rangle \cdot \langle u_3|\,]$$ 2

باتباع نفس الأسلوب:

$$\hat{A} = \sum_{i=1}^{3} \sum_{j=1}^{3} A_{ij} \cdot |u_i\rangle \cdot \langle u_j|$$

$A_{11} = 0 \quad ; \quad A_{12} = 4.a \; ; \quad A_{13} = 0$

$A_{21} = 4.a \quad ; \quad A_{22} = 0 \quad ; \quad A_{23} = a$

$A_{31} = 0 \quad ; \quad A_{32} = a \quad ; \quad A_{33} = 0$

$$\hat{A} = a.[\,4.|u_1\rangle . \langle u_2| + 4.|u_2\rangle . \langle u_1| + |u_2\rangle . \langle u_3| + |u_3\rangle . \langle u_2|\,]$$

4- يملك المؤثرين $\hat{H}$ و $\hat{A}$ مجموعة مشتركة من الأشعة التي تكون بآن معاً أشعة خاصة لكلا المؤثرين إذا كان المبدل معدوماً أي

$$[\hat{H}, \hat{A}] = \hat{0} \iff [\,[\hat{H}]_u , [\hat{A}]_u\,] = 0$$

إذن إما أن نحسب قيمة المبدل باستعمال النشر الطيفي للمؤثرين وإما أن نحسب قيمته بحساب جداء المصفوفات الممثلة أي $[\hat{H}]_u \cdot [\hat{A}]_u$ و $[\hat{A}]_u \cdot [\hat{H}]_u$ وإذا كان الجداء متساوياً في الحالتين ينعدم المبدل.

الطريقة الأولى:

$$[\hat{H}]_u \cdot [\hat{A}]_u = a \cdot E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = a.E_0 \cdot \begin{pmatrix} -4 & 4 & -1 \\ 4 & -4 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$[\hat{A}]_u \cdot [\hat{H}]_u = a.E_0 \cdot \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = a.E_0 \cdot \begin{pmatrix} -4 & 4 & 0 \\ 4 & -4 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$[\hat{H}, \hat{A}] = [\hat{H}]_u \cdot [\hat{A}]_u - [\hat{A}]_u \cdot [\hat{H}]_u = a.E_0 \cdot \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 2 \\ 1 & -2 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3 الطريقة الثانية:

$$\hat{H}\cdot\hat{A} = \left(\sum_i \sum_j H_{ij}\cdot|i\rangle\cdot\langle j|\right)\cdot\left(\sum_{ij} A_{ij}|i\rangle\cdot\langle j|\right)$$

$$\hat{H}\cdot\hat{A} = \sum_{ijkl} H_{ij}\cdot A_{kl}\cdot|i\rangle\cdot\langle j|k\rangle\cdot\langle l|$$

$$\langle j|k\rangle = \delta_{jk}$$

$$\hat{H}\cdot\hat{A} = \sum_{ijl} H_{ij}\cdot A_{jl}\cdot|i\rangle\cdot\langle l|$$

$$\hat{A}\cdot\hat{H} = \sum_{ijkl} A_{ij}\cdot H_{kl}\,|i\rangle\cdot\langle j|k\rangle\langle l|$$

$$\hat{A}\cdot\hat{H} = \sum_{ijl} H_{jl}\cdot A_{ij}\cdot|i\rangle\cdot\langle l|$$

يمكن تبديل $i \leftrightarrow j$

$$\hat{A}\cdot\hat{H} = \sum_{ijl} H_{il}\cdot A_{ji}\cdot|j\rangle\cdot\langle l|$$

$$l \leftrightarrow j$$

$$\hat{A}\cdot\hat{H} = \sum_{jil} H_{ij}\cdot A_{li}\cdot|l\rangle\cdot\langle j|$$

نجد أنه من غير الممكن الحصول على $\hat{H}\cdot\hat{A} = \hat{A}\cdot\hat{H}$.

المبدل $[\hat{H},\hat{A}] \neq 0$. لا يملك إذن المؤثرين مجموعة مشتركة التي تكون بآن معاً أشعة خاصة لكلا المؤثرين.

5- القيم الممكن الحصول عليها لدى إجراء قياس للطاقة هي القيم الخاصة للمؤثر الممثل للطاقة أي الهاملتوني. نقوم بتقطير الهاملتوني:

$$P(\lambda) = \det\{[\hat{H}]_n - \lambda\cdot I\}$$

$$P(\lambda) = \begin{vmatrix} E_0-\lambda & -E_0 & 0 \\ -E_0 & E_0-\lambda & 0 \\ 0 & 0 & -E_0-\lambda \end{vmatrix} = -(E_0-\lambda)\cdot(E_0-\lambda)\cdot(E_0+\lambda) + E_0\cdot E_0\cdot(E_0+\lambda)$$

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$$P(\lambda) = (E_0+\lambda)\cdot\{[E_0-(E_0-\lambda)]\cdot[E_0+(E_0-\lambda)]\}$$

$$P(\lambda) = (E_0+\lambda)\cdot(\lambda)\cdot(2\cdot E_0-\lambda)$$

$$P(\lambda) = 0 \Rightarrow \lambda = 0 \; ; \; \lambda = -E_0 \; ; \; \lambda = 2\cdot E_0$$

نسمي الإصطلاح:

$$E_1 = 0 \; ; \; E_2 = -E_0 \; ; \; E_3 = 2\cdot E_0$$

الأشعة الذاتية:

$$[\hat{H}]_u \cdot |\phi_1\rangle = (E_1)\cdot|\phi_1\rangle$$

$$E_0\cdot\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (0)\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$y = x \; ; \; y = x \; ; \; z = 0 \Rightarrow |\phi_1\rangle = \begin{pmatrix} x \\ x \\ 0 \end{pmatrix} = x\cdot\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle\phi_1|\phi_1\rangle = \overline{\underline{x^* \quad x^* \quad 0}}\cdot\begin{pmatrix} x \\ x \\ 0 \end{pmatrix} = 2\cdot|x|^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$E_1 = 0 \longrightarrow |\phi_1\rangle = \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$[\hat{H}]_u\cdot|\phi_2\rangle = E_2\cdot|\phi_2\rangle$$

$$E_0\cdot\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (-E_0)\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \Rightarrow$$

$$\left.\begin{matrix} x-y = -x \\ -x+y = -y \\ -z = -z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 2\cdot x \\ 2\cdot y = x \\ z = z \end{matrix}\right\} \begin{matrix} z = z \\ \Rightarrow 2\cdot(2\cdot x) = x \Rightarrow 4\cdot x = x \end{matrix}$$

علاقة محققة فقط من أجل

$$\Leftarrow x = 0$$

$$|\phi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = z\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \langle\phi_2|\phi_2\rangle = \overline{\underline{0 \quad 0 \quad z^*}}\cdot\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = |z|^2 = 1$$

$$\Rightarrow z = 1$$

...تابع العددي المعرف بالمعادلة الآتية :

$$E_2 = -E_0 \longrightarrow |\phi_2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$[\hat{H}]_u \cdot |\phi_3\rangle = E_3 \cdot |\phi_3\rangle$$

$$E_0 \cdot \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = (2 \cdot E_0) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} x-y \\ -x+y \\ -z \end{pmatrix} = \begin{pmatrix} 2 \cdot x \\ 2 \cdot y \\ 2 \cdot z \end{pmatrix} \Rightarrow$$

$$\left.\begin{aligned} y &= -x \\ y &= -x \\ z &= 0 \end{aligned}\right\} \Rightarrow |\phi_3\rangle = \begin{pmatrix} x \\ -x \\ 0 \end{pmatrix} \Rightarrow \langle\phi_3|\phi_3\rangle = \overline{x^* \;\; -x^* \;\; 0} \cdot \begin{pmatrix} x \\ -x \\ 0 \end{pmatrix}$$

$$\langle\phi_3|\phi_3\rangle = 2 \cdot |x|^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$E_3 = 2 \cdot E_0 \longrightarrow |\phi_3\rangle = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

6-1 النشر الطيفي للهاملتوني على B_ϕ : الهاملتوني قطري على القاعدة B_ϕ . إذن نكتب :

$$\hat{H} = \hat{I} \cdot \hat{H} \cdot \hat{I}$$

$$\hat{H} = \left(\sum_{i=1}^{3} |\phi_i\rangle \cdot \langle\phi_i|\right) \cdot \hat{H} \cdot \left(\sum_{i=1}^{3} |\phi_i\rangle \cdot \langle\phi_i|\right)$$

$$\hat{H} = \sum_i \sum_j |\phi_i\rangle \cdot \langle\phi_i|\hat{H}|\phi_j\rangle \cdot \langle\phi_j|$$

$$\langle\phi_i|\hat{H}|\phi_j\rangle = E_j \cdot \langle\phi_i|\phi_j\rangle = \delta_{ij} \cdot E_j$$

$$\hat{H} = \sum_{i=1}^{3} E_i \cdot |\phi_i\rangle \cdot \langle\phi_i|$$

$$\hat{H} = -E_0 \cdot |\phi_2\rangle \cdot \langle\phi_2| + 2 \cdot E_0 \cdot |\phi_3\rangle \cdot \langle\phi_3| + (0) \cdot |\phi_1\rangle \cdot \langle\phi_1|$$

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$$|\Psi_{Before}\rangle \xrightarrow{mes(\hat{H}) = -E_0} |\Psi_{After}\rangle$$

لدينا التابع العمودي المعرف بالمعادلة الآتية:

$\boxed{6}$

$$|\Psi_{After}> = \frac{\hat{P}_n |\Psi_{Before}>}{\sqrt{<\Psi_{Before}|\hat{P}_n|\Psi_{Before}>}}$$

$$E_2 = -E_0 \longrightarrow |\phi_2> \longrightarrow \hat{P}_2 = |\phi_2><\phi_2|$$

تشكل الأشعة الذاتية $\{|\phi_i>\}$ مجموعة منتظمة تامة متعامدة يمكن أن ننشر بها $|\Psi_{Before}>$:

$$|\Psi_{Before}> = \sum_i c_i \cdot |\phi_i>$$

$$\hat{P}_2 |\Psi_{Before}> = \left(|\phi_2> \cdot <\phi_2|\right) \cdot \left(\sum_i c_i \cdot |\phi_i>\right)$$

$$\hat{P}_2 |\Psi_{Before}> = \sum_i c_i \cdot |\phi_2> \cdot <\phi_2|\phi_i> = \sum_i \delta_{2i} \cdot c_i \cdot |\phi_2>$$

$$\hat{P}_2 |\Psi_{Before}> = c_2 \cdot |\phi_2>$$

$$<\Psi_{Before}|\hat{P}_2|\Psi_{Before}> = \left(\sum_i c_i^* <\phi_i|\right) \cdot \left(c_2 |\phi_2>\right)$$

$$= \sum_i c_2 \cdot c_i^* \cdot <\phi_i|\phi_2> = |c_2|^2$$

$$|\Psi_{After}> = \frac{\hat{P}_2 |\Psi_{Before}>}{\sqrt{<\Psi_{Before}|\hat{P}_2|\Psi_{Before}>}}$$

$$|\Psi_{After}> = \frac{c_2 \cdot |\phi_2>}{\sqrt{|c_2|^2}} = \frac{c_2 \cdot |\phi_2>}{c_2} = |\phi_2>$$

$$|\Psi_{After}> = |\phi_2>$$

8- الاحتمال $P_1(a_1)$:

$$a_1 = -\sqrt{17} \cdot a \rightarrow |a_1> \rightarrow P_1(a_1) = |<a_1|\phi_2>|^2$$

$$P_1(a_1) = \left| \frac{1}{\sqrt{34}} \cdot \overline{4 \quad -\sqrt{17} \quad 1} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} \right|^2 = \left| \frac{1}{\sqrt{34}} \cdot (1) \right|^2 = \frac{1}{34}$$

شعاع الحالة بعد القياس:

$$|\phi_2\rangle \equiv |\Psi_{Before}\rangle \xrightarrow[P_1(a_1)]{mes(\hat{A}) = a_1} |\Psi_{After}\rangle$$

$$a_1 = -\sqrt{17} \cdot a \longrightarrow |a_1\rangle \longrightarrow \hat{P}_1 = |a_1\rangle \cdot \langle a_1|$$

$$\hat{P}_1 |\Psi_{Before}\rangle = \hat{P}_1 |\phi_2\rangle = |a_1\rangle \cdot \langle a_1 | \phi_2\rangle$$

$$\langle a_1 | \phi_2 \rangle = \frac{1}{\sqrt{34}} \cdot \overline{4 \quad -\sqrt{17} \quad 1} \cdot \begin{vmatrix}0\\0\\1\end{vmatrix} = \frac{1}{\sqrt{34}} \cdot (1) = \frac{1}{\sqrt{34}}$$

$$\hat{P}_1 |\Psi_{Before}\rangle \equiv \hat{P}_1 |\phi_2\rangle = \frac{1}{\sqrt{34}} \cdot |a_1\rangle$$

$$\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle = \frac{1}{\sqrt{34}} \cdot \langle \phi_2 | a_1 \rangle = \frac{1}{\sqrt{34}} \cdot \overline{0 \quad 0 \quad 1} \cdot \frac{1}{\sqrt{34}} \begin{vmatrix}4\\-\sqrt{17}\\1\end{vmatrix}$$

$$\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle = \frac{1}{34} \cdot (1) = \frac{1}{34} \Rightarrow \sqrt{\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle} = \frac{1}{\sqrt{34}}$$

$$|\Psi_{After}\rangle = \frac{\hat{P}_1 |\Psi_{Before}\rangle}{\sqrt{\langle \Psi_{Before} | \hat{P}_1 | \Psi_{Before} \rangle}} = \frac{\hat{P}_1 |\phi_2\rangle}{\sqrt{\langle \phi_2 | \hat{P}_1 | \phi_2 \rangle}}$$

$$|\Psi_{After}\rangle = \frac{1}{\sqrt{34}} \cdot \frac{|a_1\rangle}{\frac{1}{\sqrt{34}}} = \frac{1}{\sqrt{34}} \cdot |a_1\rangle \cdot \sqrt{34}$$

$$|\Psi_{After}\rangle = |a_1\rangle$$

الاحتمال $P_2(a_2)$: $|\langle a_2|\phi_2\rangle|^2$

$$a_2 = 0 \longrightarrow |a_2\rangle \longrightarrow P_2(a_2) = |\langle a_2 | \phi_2 \rangle|^2$$

$$P_2(a_2) = \left| \frac{1}{\sqrt{17}} \cdot \overline{1 \quad 0 \quad -4} \cdot \begin{vmatrix}0\\0\\1\end{vmatrix} \right|^2 = \frac{16}{17}$$

شكل الحالة بعد القياس:

$$|\Psi_{After}\rangle = \frac{\hat{P}_2|\Phi_2\rangle}{\sqrt{\langle\Phi_2|\hat{P}_2|\Phi_2\rangle}}$$

$$\hat{P}_2|\Phi_2\rangle = |a_2\rangle\cdot\langle a_2|\Phi_2\rangle = \langle a_2|\Phi_2\rangle\cdot|a_2\rangle$$

$$\langle a_2|\Phi_2\rangle = \frac{1}{\sqrt{17}}\cdot\begin{pmatrix}1 & 0 & -4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = -\frac{4}{\sqrt{17}}$$

$$\hat{P}_2|\Phi_2\rangle = -\frac{4}{\sqrt{17}}\cdot|a_2\rangle$$

$$\langle\Phi_2|\hat{P}_2|\Phi_2\rangle = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\cdot\left[-\frac{4}{\sqrt{17}}\cdot|a_2\rangle\right]$$

$$= \left(-\frac{4}{\sqrt{17}}\right)\cdot\begin{pmatrix}0 & 0 & 1\end{pmatrix}\cdot\frac{1}{\sqrt{17}}\cdot\begin{pmatrix}1\\0\\-4\end{pmatrix} = -\frac{4}{17}\cdot(-4) = \frac{16}{17}$$

$$\langle\Phi_2|\hat{P}_2|\Phi_2\rangle = \frac{16}{17}$$

$$|\Psi_{After}\rangle = \frac{\sqrt{17}}{4}\cdot\left(-\frac{4}{\sqrt{17}}\right)\cdot|a_2\rangle = -|a_2\rangle$$

القيمة (-1) يمكن إهمالها مساوٍ لو إهمال عدم الأهمية. اذن:

$$|\Psi_{After}\rangle = |a_2\rangle$$

الاحتمال : $P_3(a_3)$

$$a_3 = \sqrt{17}\cdot a \rightarrow |a_3\rangle \rightarrow P_3(a_3) = |\langle a_3|\Phi_2\rangle|^2$$

$$P_3(a_3) = \left|\frac{1}{\sqrt{34}}\cdot\begin{pmatrix}4 & \sqrt{17} & 1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right|^2 = \left|\frac{1}{\sqrt{34}}\cdot(1)\right|^2 = \frac{1}{34}$$

$$a_3 = \sqrt{17}\cdot a \rightarrow |a_3\rangle \rightarrow \hat{P}_3 = |a_3\rangle\cdot\langle a_3|$$

$$|\Psi_{Before}\rangle = |\Phi_2\rangle ; |\Psi_{After}\rangle = \frac{\hat{P}_3|\Phi_2\rangle}{\sqrt{\langle\Phi_2|\hat{P}_3|\Phi_2\rangle}}$$

$$\hat{P}_3|\Phi_2\rangle = |a_3\rangle\cdot\langle a_3|\Phi_2\rangle = \langle a_3|\Phi_2\rangle\cdot|a_3\rangle$$

3.

$$\langle a_3|\phi_2\rangle = \frac{1}{\sqrt{34}}\cdot\begin{bmatrix} 4 & \sqrt{17} & 1\end{bmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{\sqrt{34}}$$

$$\hat{P}_3|\phi_2\rangle = \frac{1}{\sqrt{34}}\cdot|a_3\rangle$$

$$\langle\phi_2|\hat{P}_3|\phi_2\rangle = \frac{1}{\sqrt{34}}\cdot\langle\phi_2|a_3\rangle$$

$$= \frac{1}{\sqrt{34}}\cdot\left[\begin{matrix}0 & 0 & 1\end{matrix}\right]\cdot\left[\frac{1}{\sqrt{34}}\cdot\begin{pmatrix}4\\\sqrt{17}\\1\end{pmatrix}\right] = \frac{1}{34}$$

$$|\psi_{After}\rangle = \sqrt{34}\cdot\frac{1}{\sqrt{34}}\cdot|a_3\rangle = |a_3\rangle$$

9- الارتياب: لماذا القيمة عند قياس $\hat{A}$ كانت في الحالة $|\phi_2\rangle$ إذن:

$$(\Delta\hat{A})_{|\phi_2\rangle} = \left[\langle\phi_2|\hat{A}^2|\phi_2\rangle - \langle\phi_2|\hat{A}|\phi_2\rangle^2\right]^{1/2}$$

$$\langle\phi_2|\hat{A}|\phi_2\rangle = \left[\begin{matrix}0 & 0 & 1\end{matrix}\right]\cdot\left[a\cdot\begin{pmatrix}0&4&0\\4&0&1\\0&1&0\end{pmatrix}\right]\cdot\left[\begin{pmatrix}0\\0\\1\end{pmatrix}\right] = 0$$

$$\langle\phi_2|\hat{A}^2|\phi_2\rangle = \left[\begin{matrix}0 & 0 & 1\end{matrix}\right]\cdot\left[a\cdot\begin{pmatrix}0&4&0\\4&0&1\\0&1&0\end{pmatrix}\cdot a\cdot\begin{pmatrix}0&4&0\\4&0&1\\0&1&0\end{pmatrix}\right]\cdot\left[\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

$$\langle\phi_2|\hat{A}^2|\phi_2\rangle = a^2 \Rightarrow$$

$$(\Delta\hat{A})_{|\phi_2\rangle} = a$$

مبرهنة Ehrenfest :

$$i\hbar\frac{\partial}{\partial t}\langle\hat{A}\rangle_{|\phi\rangle}=\langle[\hat{A},\hat{H}]\rangle_{|\phi\rangle}+i\hbar\langle\frac{\partial\hat{A}}{\partial t}\rangle_{|\phi\rangle}$$

$\hat{Z}$ = Time-independent operator $\Rightarrow \langle\frac{\partial}{\partial t}\hat{Z}\rangle_{|\phi\rangle}=\hat{0} \Rightarrow$

$$i\hbar\frac{\partial}{\partial t}\langle\hat{Z}\rangle_{|\phi\rangle}=\langle[\hat{Z},\hat{H}]\rangle_{|\phi\rangle}$$

$$[\hat{Z},\hat{H}]=[\hat{Z},\hat{T}+\hat{V}]=[\hat{Z},\hat{T}]+[\hat{Z},\hat{V}]$$

$$[\hat{Z},\hat{H}]=[\hat{Z},\frac{\hat{P}_Z^2}{2m}]+[\hat{Z},\frac{1}{2}m.w^2.\hat{Z}^2]$$

$$[\hat{Z},\hat{H}]=\frac{1}{2.m}[\hat{Z},\hat{P}_Z^2]+\frac{1}{2}m.w^2.[\hat{Z},\hat{Z}^2]=\frac{1}{2.m}.[\hat{Z},\hat{P}_Z^2]$$

$$[\hat{X}_i,\hat{P}_j^n]=\delta_{ij}.n.i.\hbar.\hat{P}_j^{n-1}\Rightarrow[\hat{X}_i,\hat{P}_i^n]=n.i.\hbar.\hat{P}_i^{n-1}\Rightarrow$$

$$[\hat{Z},\hat{P}_Z^2]=2.i.\hbar.\hat{P}_Z\Rightarrow[\hat{Z},\hat{H}]=\frac{i.\hbar}{m}.\hat{P}_Z$$

$$i\hbar\frac{\partial}{\partial t}\langle\hat{Z}\rangle_{|\phi\rangle}=\langle\phi|[\hat{Z},\hat{H}]|\phi\rangle=\frac{i.\hbar}{m}.\langle\phi|\hat{P}_Z|\phi\rangle\Rightarrow$$

$$\frac{\partial}{\partial t}\langle\hat{Z}\rangle_{|\phi\rangle}=\frac{1}{m}.\langle\hat{P}_Z\rangle_{|\phi\rangle}$$

11 (1 من المعطيات:

$$[\hat{A},[\hat{A},\hat{B}]]=0 \Rightarrow \hat{A}\cdot[\hat{A},\hat{B}]=[\hat{A},\hat{B}]\cdot\hat{A}$$

$$[\hat{B},\hat{A}]\cdot\hat{A}^n=\underbrace{[\hat{B},\hat{A}]\cdot\hat{A}}\cdot\hat{A}^{n-1}=\hat{A}\cdot[\hat{B},\hat{A}]\cdot\hat{A}^{n-1}$$

$$=\hat{A}\cdot\underbrace{[\hat{B},\hat{A}]\cdot\hat{A}}\cdot\hat{A}^{n-2}=\hat{A}^2\cdot[\hat{B},\hat{A}]\cdot\hat{A}^{n-2}=\cdots=\hat{A}^n\cdot[\hat{B},\hat{A}]$$

إذن:

$$[\hat{B},\hat{A}]\cdot\hat{A}^n=\hat{A}^n\cdot[\hat{B},\hat{A}]$$

(2

$$[\hat{A},\hat{B}\cdot\hat{C}]=[\hat{A},\hat{B}]\cdot\hat{C}+\hat{B}\cdot[\hat{A},\hat{C}]$$

$$[\hat{B},\hat{A}^n]=[\hat{B},\hat{A}^{n-1}\cdot\hat{A}]=[\hat{B},\hat{A}^{n-1}]\cdot\hat{A}+\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]$$

$$[\hat{B},\hat{A}^n]=\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-1}]\cdot\hat{A} \qquad (1)$$

نستعمل العلاقة (1) لتحديد المبدل $[\hat{B},\hat{A}^{n-1}]$ الموجود في (1):

يكفي إجراء الاستبدال $n\to n-1$:

$$[\hat{B},\hat{A}^n]=\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\{\hat{A}^{n-2}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-2}]\cdot\hat{A}\}\cdot\hat{A}$$

$$[\hat{B},\hat{A}^n]=\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\hat{A}^{n-2}\cdot[\hat{B},\hat{A}]\cdot\hat{A}+[\hat{B},\hat{A}^{n-2}]\cdot\hat{A}^2$$

ولكن:

$$[\hat{A},[\hat{A},\hat{B}]]=\hat{0} \Rightarrow \hat{A}\cdot[\hat{A},\hat{B}]=[\hat{A},\hat{B}]\cdot\hat{A}$$

$$[\hat{B},\hat{A}^n]=\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\hat{A}^{n-2}\cdot\hat{A}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-2}]\cdot\hat{A}^2$$

$$[\hat{B},\hat{A}^n]=\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-2}]\cdot\hat{A}^2$$

$$[\hat{B},\hat{A}^n]=2\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-2}]\cdot\hat{A}^2$$

لتحديد $[\hat{B},\hat{A}^{n-2}]$ نستعمل (1) ويكفي استبدال $n\to n-2$:

$$[\hat{B},\hat{A}^n]=2\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\{\hat{A}^{n-3}\cdot[\hat{B},\hat{A}]+[\hat{B},\hat{A}^{n-3}]\cdot\hat{A}\}\cdot\hat{A}^2$$

$$[\hat{B},\hat{A}^n]=2\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]+\hat{A}^{n-3}\cdot[\hat{B},\hat{A}]\cdot\hat{A}^2+[\hat{B},\hat{A}^{n-3}]\cdot\hat{A}^3.$$

نستعمل:

$$[\hat{B},\hat{A}]\cdot\hat{A}^n=\hat{A}^n\cdot[\hat{B},\hat{A}]$$

$$[\hat{B},\hat{A}^n] = 2\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}] + \hat{A}^{n-1}\cdot[\hat{B},\hat{A}] + [\hat{B},\hat{A}^{n-3}]\cdot\hat{A}^3$$

$$[\hat{B},\hat{A}^n] = 3\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}] + [\hat{B},\hat{A}^{n-3}]\cdot\hat{A}^3$$

بمتابعة نفس المراحل m مرة:

$$[\hat{B},\hat{A}^n] = m\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}] + [\hat{B},\hat{A}^{n-m}]\cdot\hat{A}^m$$

عندما نصل إلى $m=n$ نحصل على:

$$[\hat{B},\hat{A}^n] = n\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}] + \underbrace{[\hat{B},\hat{A}^0]}_{=0}\cdot\hat{A}^n \Rightarrow$$

$$[\hat{B},\hat{A}^n] = n\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}] \quad (2)$$

if $[\hat{A},[\hat{A},\hat{B}]] = \hat{0}$, Then: $[\hat{B},\hat{A}^n] = n\cdot\hat{A}^{n-1}\cdot[\hat{B},\hat{A}]$ $\quad n\in\mathbb{N}^*$

3) نعلم أن:

$$[\hat{X}_k,\hat{P}_l] = \delta_{kl}\cdot i\hbar\cdot\hat{I}$$

if $l=k \Rightarrow [\hat{X}_k,\hat{P}_k] = i\hbar\cdot\hat{I}$

if $l\neq k \Rightarrow [\hat{X}_k,\hat{P}_{l\neq k}] = \hat{0}$

لنحسب: $[\hat{X}_i,[\hat{X}_j,\hat{P}_k]]$

من أجل ذلك نستعمل: $[\hat{X}_j,\hat{P}_k] = \delta_{jk}\cdot i\hbar\cdot\hat{I}$ ، $[\hat{X}_i,\hat{I}] = 0$

بالتالي:

$$[\hat{X}_i,[\hat{X}_j,\hat{P}_k]] = [\hat{X}_i,\delta_{jk}\cdot i\hbar\cdot\hat{I}] = \delta_{jk}\cdot i\hbar\cdot[\hat{X}_i,\hat{I}] = \hat{0}$$

$$[\hat{P}_i,[\hat{X}_j,\hat{P}_k]] = [\hat{P}_i,\delta_{jk}\cdot i\hbar\cdot\hat{I}] = \delta_{jk}\cdot i\hbar\cdot[\hat{P}_i,\hat{I}] = \hat{0}$$

إذن:

$$[\hat{X}_i,[\hat{X}_j,\hat{P}_k]] = \hat{0} \quad , \quad [\hat{P}_i,[\hat{X}_j,\hat{P}_k]] = \hat{0}$$

إذن تحقق المؤثرات $\hat{X}_i$ و $\hat{P}_j$ العلاقة $[\hat{A},[\hat{A},\hat{B}]] = \hat{0}$.

END

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