قسم الفيزياء - كلية العلوم — الفيزياء النووية II — العلامة : 100

جامعة دمشق — سنة رابعة - فيزياء

المدة $11^{30}-13^{30}$ — الفصل الثاني 28.07.2025

سلم تصحيح

5 + 5 + 10 = 20 LDM : Q1

$$\mathcal{E}(A,Z)= Zm_\pi c^2 + A m_\nu c^2 - Z m_\nu c^2 - \xi_{vol.} A + \xi_{surf.} A^{2/3} + \xi_{coul.} Z(Z-1)A^{-1/3} + \xi_{asym.}(A-2Z)^2 A^{-1} + \delta(A)$$ 5 .1

$$= (m_\pi c^2 A - \xi_{vol.} A + \xi_{surf.} A^{2/3} + \xi_{asym.} A) + (m_\pi c^2 - m_\nu c^2 - \xi_{coul.} A^{-1/3} - 4\xi_{asym.})Z + (\xi_{coul.} A^{-1/3} + 4\xi_{asym.} A^{-1})Z^2 + \delta(A) = \xi_0 + \xi_1 Z + \xi_2 Z^2 + \delta(A)$$

$$\xi_0 = m_\pi c^2 A - \xi_{vol.} A + \xi_{surf.} A^{2/3} + \xi_{asym.} A \,, \quad \xi_1 = m_\pi c^2 - m_\nu c^2 - \xi_{coul.} A^{-1/3} - 4\xi_{asym.}$$

$$\xi_2 = \xi_{coul} A^{-1/3} + 4\xi_{asym} A^{-1}$$

$$\left.\frac{\partial \mathcal{E}(A,Z)}{\partial Z}\right|_{A=const} = 0 \Rightarrow Z_{Stab.} = -\frac{\xi_1}{2\xi_2} \Rightarrow Z_{Stab.} = \frac{\xi_{coul} A^{-1/3} + 4\xi_{asym}}{2(\xi_{coul.} A^{-1/3} + 4\xi_{asym.} A^{-1})}$$ 5 .2

$$\Rightarrow Z_{Stab.}(A=101) = \frac{(0.714)(101)^{-1/3} + 4(23.20)}{2[0.714(101)^{-1/3} + 4(23.20)(101)^{-1}]} \simeeq 43$$

$$S_n = Zm_\pi c^2 + (A-1-Z)m_\nu c^2 - E_B(A-1,Z) + m_\nu c^2 - Zm_\pi c^2 - (A-Z)m_\pi c^2 + E_B(A,Z)$$ 10 .3

$$= E_B(A,Z) - E_B(A-1,Z) = \frac{\partial E_B}{\partial A}\Delta A \,, \text{ but } \Delta A = 1 \Rightarrow$$

$$S_n = \frac{\partial E_B}{\partial A} = \frac{\partial}{\partial A}\left[\xi_{vol} A - \xi_{surf.} A^{2/3} - \xi_{coul}(Z^2 - Z)A^{-1/3} - \xi_{asym.}(A^2 - 4AZ + 4Z^2)A^{-1}\right]$$

$$= \xi_{vol} - \frac{2}{3}\xi_{surf.} A^{-1/3} + \frac{1}{3}\xi_{coul.} A^{-4/3} Z^2 - \frac{1}{3}\xi_{coul.} Z A^{-4/3} - \xi_{asym.} + 4\xi_{asym.} A^{-2} Z^2$$

$$\Rightarrow S_n = (\xi_{vol.} - \xi_{asym.} - \frac{2}{3}\xi_{surf.} A^{-1/3}) - (\frac{1}{3}\xi_{coul} A^{-4/3})Z + (\frac{1}{3}\xi_{coul.} A^{-4/3} + 4\xi_{asym.} A^{-2})Z^2$$

$6+4+12+8 = 30$ pts NSM

$^{45}_{21}Sc_{24}$: $j^{\pi}_{unpaired} = 7/2$, unpaired π in(f) -1 6

2 for f: $\ell = 3 \Rightarrow \xi = (-1)^3 = -1 \Rightarrow I^{\xi} = \frac{7}{2}^-$

$^{83}_{36}Kr_{44}$: unpaired ν in $g_{9/2}$, $j^{\nu}_{unpaired} = 9/2$, for $\ell = 4 \Rightarrow \xi = (-1)^4 = +1$

2 $\Rightarrow I^{\xi} = 9/2^+$

$^{122}_{51}Sb_{71}$: unpaired π in $1g_{7/2}$, $j^{\pi}_{unpaired} = 7/2$, unpaired ν in $1h_{11/2}$, $j^{\nu}_{unpaired} = 11/2$

2 $\xi = \xi_{\pi}\xi_{\nu} = (-1)^4(-1)^5 = -1$

$|j^{\nu}_{unpaired} - j^{\pi}_{unpaired}| \leq J \leq j^{\nu}_{unpaired} + j^{\pi}_{unpaired}| \Rightarrow J = 2^-, 3^-, 4^-, 5^-, 6^-, 7^-, 8^-, 9^-$

$\mu(Kr)$: $j^{\nu}_{unpaired} = 9/2$, $\ell = 4$, $j = 4 + 1/2$, $g^{\nu}_{\ell} = 0$, $g^{\nu}_{s} = -3.826$ -2 4

$\Rightarrow \mu(Kr) = [(j - 1/2)g^{\nu}_{\ell} + 1/2\, g^{\nu}_{s}]\mu_N = \frac{1}{2}g^{\nu}_{s}\mu_N = -\frac{3.826}{2}\mu_N = -1{,}91\mu_N$ 2

$\mu(Sc)$: $j^{\pi}_{unpaired} = 7/2$, $\ell = 3$, $j = 3 + 1/2$, $g^{\pi}_{\ell} = 1$, $g^{\pi}_{s} = 5{,}5856$

$\Rightarrow \mu(Sc) = [(j - 1/2)g^{\pi}_{\ell} + 1/2\, g^{\pi}_{s}]\mu_N = [(\frac{7}{2} - \frac{1}{2})(1) + \frac{1}{2}(5{,}5856)]\mu_N$

$= 5{,}2928\,\mu_N$ 2

for 1h: $\ell = 5 \Rightarrow \begin{cases} j = 5 + 1/2 = 11/2 \rightarrow h_{11/2} & 1 \\ j = 5 - 1/2 = 9/2 \rightarrow h_{9/2} & 1 \end{cases}$ $-i$ 4 $.3$ 12

[illegible] $2j+1 \Rightarrow h_{11/2} \rightarrow 12$, $h_{9/2} \rightarrow 10$ 1 1 1

$\hat{j} = \hat{\ell} + \hat{s} \Rightarrow \hat{\ell}\cdot\hat{s} = \frac{1}{2}[\hat{j}^2 - \hat{\ell}^2 - \hat{s}^2]$ $-ii$

$\langle \hat{\ell}\cdot\hat{s} \rangle = \frac{\hbar^2}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]$ 2 8

for $j = \dots$ 2

$$\Rightarrow \langle \hat{l}\cdot\hat{s}\rangle = \frac{\hbar^2}{2}\left[j(j+1)-(j-1/2)(j+1/2)-3/4\right]$$
$$= \frac{\hbar^2}{2}\left[j^2+j-j^2+\frac{1}{4}-\frac{3}{4}\right] = \frac{\hbar^2}{2}\left(j-\frac{1}{2}\right) = \frac{\hbar^2}{2}\left(\frac{11}{2}-\frac{1}{2}\right) = \frac{5}{2}\hbar^2$$

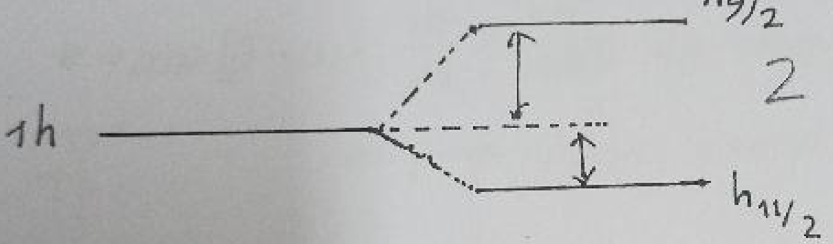
$$\Rightarrow \langle \hat{l}\cdot\hat{s}\rangle = \frac{5}{2}\hbar^2 \text{ for } j = l+1/2,\ j = 11/2$$

for $j = l - 1/2 \Rightarrow l = j + 1/2$

$$\Rightarrow \langle \hat{l}\cdot\hat{s}\rangle = \frac{\hbar^2}{2}\left[j(j+1)-(j+1/2)(j+3/2)-\frac{3}{4}\right]$$
$$= \frac{\hbar^2}{2}\left[j^2+j-j^2-2j-\frac{3}{4}-\frac{3}{4}\right] = \frac{\hbar^2}{2}\left[-j-\frac{3}{2}\right] = \frac{\hbar^2}{2}\left(-\frac{9}{2}-\frac{3}{2}\right)$$

2

$$\Rightarrow \langle \hat{l}\cdot\hat{s}\rangle = -3\hbar^2 \text{ for } j = l+1/2,\ j = 9/2$$



$h_{9/2}$ $\alpha\ -3\hbar^2 V_{os}$ $V_{os}<0$

2

$h_{11/2}$ $\alpha\ \frac{5}{2}\hbar^2 V_{os}$ $V_{os}<0$

$$V(r) = \frac{1}{2}m(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2) - c\,\hat{l}\cdot\hat{s} + D\hat{l}^2$$

:4 8

axial symmetry $\omega_1 = \omega_2 \neq \omega_3$

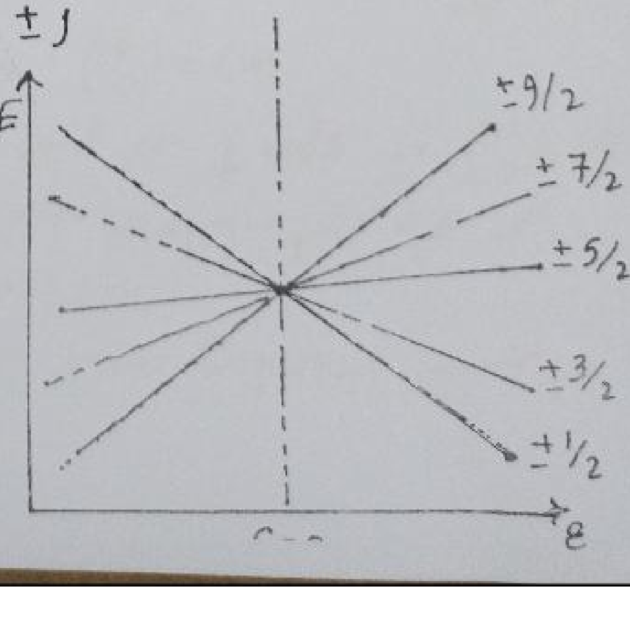
5

$$\omega_1 = \omega_2 = \omega_0\left(1+\frac{\varepsilon}{3}\right),\quad \omega_3 = \omega_0\left(1-\frac{2}{3}\varepsilon\right)$$

يؤدي توالد السويات عند $\varepsilon=0$ وعدد السويات المتوالدة $\frac{2j+1}{2}$ والعدد الكوانتي

$$\Omega = \pm 1/2, \pm 3/2, \dots, \pm j$$

$g_{9/2}$: $\frac{2\cdot 9/2+1}{2} = 5$ سويات متوالدة



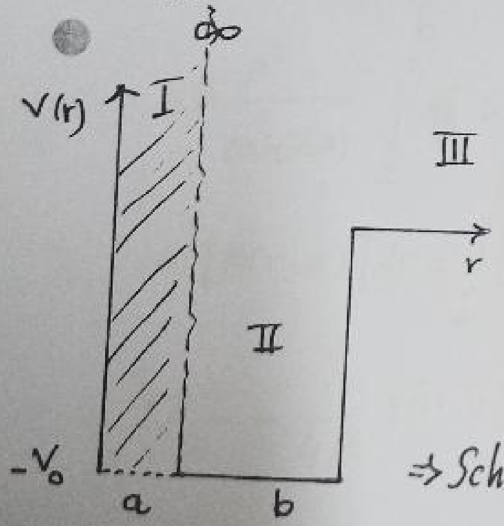
3

1- عدد كوانتي داخلي يستخدم لتصنيف الجسيمات اقترحه Heisenberg لتفسير أن شدة التفاعل المتبادل الشديد مستقلة عن الشحنة 8

حيث فرض Heisenberg أن البروتون والنترون هما حالتان لجسيم واحد سمي nucleon واقترح أن البروتون مشابه لجسيم ذو spin ↑ أي $|p\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ 3

والنيوترون مشابه لجسيم ذو spin ↓ أي $|n\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$$\left.\begin{array}{l} \hat{H}_{int}\hat{T}_z^{+}|T,T_z\rangle = \hat{H}_{int}|T,T_z+1\rangle = E|T,T_z+1\rangle \\ \hat{T}_z^{+}\hat{H}_{int}|T,T_z\rangle = \hat{T}_z^{+}E|T,T_z\rangle = E|T,T_z+1\rangle \end{array}\right\} \Rightarrow [\hat{H}_{int},\hat{T}_z^{+}] = 0$$ 5



2-

i- النكليونات تحافظ على مسافات ثابتة بينها باعتبار كثافة النوى ثابتة أي هناك مسافات أصغرية لاقتراب النكليونات من بعضها. 2+3=5

ii- 12

ground state $\Rightarrow l = 0$

$\Rightarrow$ Schrödinger Eq. $\dfrac{d^2u(r)}{dr^2} - \dfrac{2\mu}{\hbar^2}[V(r) - E]\,u(r) = 0$

region I: $V(r) = \infty \Rightarrow u_I(r) = 0$ 2

region II: $a < r < b$: $V(r) = -V_0$, $E = -E_B \Rightarrow$

$$\frac{d^2u_{II}}{dr^2} + \beta^2 u_{II}(r) = 0, \quad \beta^2 = \frac{2\mu}{\hbar^2}[V_0 - E_B]$$

Solution: $u_{II}(r) = A\sin(\beta r) + B\cos(\beta r)$

$$u_{II}(r) \xrightarrow[r \to a]{} 0 \Rightarrow 0 = A\sin(\beta a) + B\cos(\beta a) \Rightarrow B = -\frac{\sin(\beta a)}{\cos(\beta a)}A$$

$$\Rightarrow u_{II}(r) = A\sin(\beta r) - A\frac{\sin(\beta a)}{\cos(\beta a)}\cos(\beta r)$$

$$\Rightarrow u_{II}(r) = \frac{1}{\cos(\beta a)}\left[A\sin(\beta r)\cos(\beta a) - A\sin(\beta a)\cos(\beta r)\right]$$

$$\Rightarrow u_{II}(r) = \frac{A}{\cos(\beta a)} \sin[\beta(r-a)]$$ 5

región III: $r > b$: $V_0 = 0$, $E = -E_B \Rightarrow$

Schrödinger Eq. $\frac{d^2 u_{III}}{dr^2} - \gamma^2 u_{III}(r) = 0$, $\gamma^2 = \frac{2\mu E_B}{\hbar^2}$

solution: $u_{III}(r) = C e^{-\gamma r} + D e^{\gamma r}$

$$u_{III}(r) \xrightarrow[r \to \infty]{} 0 \Rightarrow D = 0 \Rightarrow u_{III}(r) = C e^{-\gamma r}$$ 5

normalization cond. $4\pi \int_0^\infty |u(r)|^2 dr = 1$ .iii 8

$u_{II}(r)$: $4\pi \frac{A^2}{\cos^2(\beta a)} \int_a^b \sin^2[\beta(r-a)] dr = 1$

$$I = \int_a^b \sin^2[\beta(r-a)] dr = \int_a^b \left[\frac{1}{2} - \frac{1}{2}\cos[2\beta(r-a)]\right] dr$$

$$= \frac{b-a}{2} - \frac{1}{4\beta} \sin 2\beta(r-a)\Big]_a^b = \frac{b-a}{2} - \frac{1}{4\beta}\sin 2\beta(b-a) \Rightarrow$$

$$\frac{4\pi A^2}{\cos^2(\beta a)}\left[\frac{1}{2}(b-a) - \frac{1}{4\beta}\sin 2\beta(b-a)\right] = 1 \Rightarrow$$

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$$A = \frac{\cos(\beta a)}{(4\pi)^{1/2}\left[\frac{1}{2}(b-a) - \frac{1}{4\beta}\sin 2\beta(b-a)\right]^{1/2}}$$

$u_{III}(r)$: $4\pi C^2 \int_b^\infty e^{-2\gamma r} dr = 1 \Rightarrow 4\pi C^2 \left[-\frac{1}{2\gamma} e^{-2\gamma r}\right]_b^\infty = 1$

$$\Rightarrow 4\pi C^2 \frac{e^{-2\gamma b}}{2\gamma} = 1 \Rightarrow C = \left(\frac{2\gamma}{4\pi}\right)^{1/2} e^{\gamma b}$$ 3

$$f_B(\theta,\phi) = -\frac{2\mu}{q\hbar^2}\int_0^\infty dr\, V(r)\, r \sin(qr)$$ - 3

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$$\Rightarrow f_B(\theta,\phi) = -\frac{2\mu}{q\hbar^2}\frac{g^2}{(2\pi)^3}\int_0^\infty dr\, \frac{e^{-r\xi}}{r}\, r\sin(qr)$$

$$= -\frac{2\mu}{q\hbar^2}\frac{g^2}{(2\pi)^3}\int_0^\infty dr\, e^{-r\xi}\sin(qr) \quad , \quad \sin(qr) = \frac{e^{iqr}-e^{-iqr}}{2i}$$

$$I = \int_0^\infty e^{-r\xi}\frac{e^{iqr}-e^{-iqr}}{2i}dr = \frac{1}{2i}\left[\int_0^\infty e^{-r\xi}e^{iqr}dr - \int_0^\infty e^{-r\xi}e^{-iqr}dr\right]$$

$$= \frac{1}{2i}\left[\int_0^\infty e^{-(iq+\xi)r}dr - \int_0^\infty e^{-(iq+\xi)r}dr\right]$$

$$= \frac{1}{2i}\left[\frac{-1}{iq-\xi}e^{-(qi+\xi)r}\Big|_0^\infty + \frac{-1}{iq+\xi}e^{-(iq+\xi)r}\Big|_0^\infty\right]$$

$$= \frac{1}{2i}\left[\frac{1}{-iq+\xi} + \frac{1}{iq+\xi}\right] = \frac{1}{2i}\frac{2iq}{q^2+\xi^2} = \frac{q}{q^2+\xi^2}$$

$$\Rightarrow f_B(\theta,\phi) = -\frac{2\mu g^2}{\hbar^2(2\pi)^3}\frac{1}{q^2+\xi^2}$$ 5 scattering amplitude

diff. cross. section

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_B = |f_B^{\xi}(\theta)|^2 = \frac{4\mu^2 g^4}{\hbar^4(2\pi)^6}\frac{1}{(q^2+\xi^2)^2}$$ 2

$$\sigma_B^{tot} = \int\frac{d\sigma_B(\theta)}{d\Omega}d\Omega = \frac{4\mu^2 g^4}{\hbar^4(2\pi)^6}\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\sin\theta\,\frac{1}{(q^2+\xi^2)^2} \quad , \quad q = 2k\sin\frac{\theta}{2}$$

$$= \frac{4\mu^2 g^4}{\hbar^4(2\pi)^6}(2\pi)\int_0^{\pi}\frac{d\theta\,\sin\theta}{\left[4k^2\sin^2\frac{\theta}{2}+\xi^2\right]^2} \quad , \quad \sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$$

$$I_\theta = \int_0^{\pi}\frac{d\theta\,\sin\theta}{\left[\xi^2+2k^2(1-\cos\theta)\right]^2} = \int_0^{\pi}\frac{du}{(\xi^2+2k^2u)^2} = -\frac{1}{2k^2}\frac{1}{\xi^2+2k^2u}\Big|_0^{\pi}$$

$$= -\frac{1}{2k^2}\frac{1}{\xi^2+2k^2(1-\cos\theta)}\Big|_0^{\pi}$$

حيث فرضنا $1-\cos\theta = u$

$$\Rightarrow I_\theta = -\frac{1}{2k^2}\left[\frac{1}{\xi^2+4k^2} - \frac{1}{\xi^2}\right]$$

$$= -\frac{1}{2k^2}\,\frac{\xi^2-\xi^2-4k^2}{\xi^2(\xi^2+4k^2)} = \frac{\cancel{4}}{\cancel{2k^2}}\frac{2}{\xi^2(\xi^2+4k^2)}$$

$$\Rightarrow \sigma_B^{tot} = \frac{4\mu^2 g^4}{\hbar^4(2\pi)^6}(2\pi)\frac{2}{\xi^2(\xi^2+4k^2)} \Rightarrow$$

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$$\sigma_B^{tot} = \frac{16\mu^2 g^4 \pi}{\hbar^4(2\pi)^6}\,\frac{1}{\xi^2(\xi^2+4k^2)}$$