قسم الفيزياء . كلية العلوم
جامعة دمشق

التحليل المتجهي
سنة ثانية . فيزياء
فصل ثاني 31/7/2025
سلم تصحيح

العلامة 100
المدة 14-16

Q1 $\quad 10+18+8=\boxed{36}$

1- يجب اثبات أن $\vec{\nabla}\times\vec{F}=0$ 10

$$\vec{\nabla}\times\vec{F}=\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix}\times\begin{pmatrix}F_x\\ F_y\\ F_z\end{pmatrix}=\begin{pmatrix}\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\\ \frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\\ \frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\end{pmatrix}=\begin{pmatrix}-x+x\\ 3-y-3+y\\ -z+z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

3

$$\Rightarrow \exists\ \Phi(x,y,z)\ :\ \vec{F}(x,y,z)=\vec{\nabla}\Phi(x,y,z)$$

ايجاد $\Phi(x,y,z)$

7

$$\begin{pmatrix}F_x(x,y,z)\\ F_y(x,y,z)\\ F_z(x,y,z)\end{pmatrix}=\begin{pmatrix}\frac{\partial\Phi(x,y,z)}{\partial x}\\ \frac{\partial\Phi(x,y,z)}{\partial y}\\ \frac{\partial\Phi(x,y,z)}{\partial z}\end{pmatrix}$$

$$\Rightarrow 2x+3z-yz=\frac{\partial\Phi(x,y,z)}{\partial x}\Rightarrow$$

1

$$\Phi(x,y,z)=\int(2x+3z-yz)\,dx=x^2+3zx-yzx+B(y,z)$$

1

$$-2y-xz=\frac{\partial\Phi}{\partial y}=\frac{\partial}{\partial y}\left(x^2+3zx-yzx+B(y,z)\right)$$

1

$$\Rightarrow -2y-xz=-zx+\frac{\partial B(y,z)}{\partial y}\Rightarrow B(y,z)=\int(-2y)\,dy=-y^2+A(z)$$

1

$$\Rightarrow \Phi(x,y,z)=x^2+3zx-yzx-y^2+A(z)$$

1

$$2+3x-xy=\frac{\partial\Phi}{\partial z}=3x-yx+\frac{\partial A}{\partial z}\Rightarrow A(z)=\int 2\,dz=2z+C$$

1

$$\Rightarrow \boxed{\Phi(x,y,z)=x^2-y^2+3zx-yzx+2z+C}$$

$$\vec{\nabla}\cdot\vec{\Psi} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\,\Psi_\rho) + \frac{1}{\rho}\frac{\partial\Psi_\phi}{\partial\phi} + \frac{\partial\Psi_z}{\partial z}$$ $-\frac{4}{18}$

$$\frac{\partial}{\partial\rho}(\rho\Psi_\rho) = \frac{\partial}{\partial\rho}\left[\rho\,\frac{1}{\rho}\,(\cos\phi - 2\sin\phi)\right] = 0$$ 2

$$\frac{\partial\Psi_\phi}{\partial\phi} = \frac{\partial}{\partial\phi}\left[\frac{1}{\rho}(\sin\phi + 2\cos\phi)\right] = \frac{1}{\rho}(\cos\phi - 2\sin\phi)$$ 2

$$\Rightarrow \frac{1}{\rho}\frac{\partial\Psi_\phi}{\partial\phi} = \frac{1}{\rho^2}(\cos\phi - 2\sin\phi)$$ 8

$$\frac{\partial\Psi_z}{\partial z} = \frac{\partial}{\partial z}\left(\frac{z^2}{\rho^2}\right) = \frac{2z}{\rho^2}$$ 2

$$\Rightarrow \vec{\nabla}\cdot\vec{\Psi} = \frac{1}{\rho^2}(\cos\phi - 2\sin\phi) + \frac{2z}{\rho^2}$$ 2

10 $\underline{\vec{\nabla}\times\vec{\Psi}}$: $\frac{\partial\Psi_z}{\partial\phi} = 0$ 1, $\frac{\partial\Psi_\phi}{\partial z} = 0$ 1, $\frac{\partial\Psi_\rho}{\partial z} = 0$ 1, $\frac{\partial\Psi_z}{\partial\rho} = -\frac{z^2}{\rho^2}$

1 $\frac{\partial}{\partial\rho}(\rho\Psi_\phi) = 0$, 1 $\frac{\partial\Psi_\rho}{\partial\phi} = \frac{1}{\rho}(-\sin\phi - 2\cos\phi)$

$$\Rightarrow \vec{\nabla}\times\vec{\Psi} = \begin{pmatrix} 0 \\ \frac{z^2}{\rho^2} \\ \frac{1}{\rho^2}(\sin\phi + 2\cos\phi) \end{pmatrix}$$ 4

$$f(x,y,z) = z\ln(y+x)$$ $-\frac{3}{8}$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$ 2

$$= \frac{yz}{y+x}dx + \frac{xz}{y+x}dy + \ln(y+x)\,dz$$

2 2 2

$12+18+10+24=64 \leftarrow$ Q1

$f(x,y,z)=x+y\,,\quad z=F(x,y)=4-x^2$ $\frac{-1}{12}$

$B_2=[0,2]\times[-5,5]$

$\frac{\partial F(x,y)}{\partial x}=-2x$ 1 , $\frac{\partial F(x,y)}{\partial y}=0$ 1

$\iint_F f\,d\sigma=\int_0^2\int_0^2 (x+2)\sqrt{4x^2+1}\,dx\,dy$ 1

$=\int_{-5}^{5}dy\int_0^2 x\sqrt{4x^2+1}\,dx+\int_{-5}^{+5}dy\,y\int_0^2\sqrt{4x^2+1}\,dx$ 1 1

$t(x)=4x^2+1\Rightarrow dt=8x\,dx$

$\int_0^2 x\sqrt{4x^2+1}\,dx=\frac{1}{8}\int\sqrt{t}\,dt=\frac{1}{8}\frac{2}{3}\left[t^{3/2}\right]_1^{17}=\frac{1}{12}(\sqrt{17^3}-1)$ 3

$\int_{-5}^{5}dy=y\Big|_{-5}^{5}=10$ 1 , $\int_{-5}^{5}y\,dy=\frac{1}{2}y^2\Big|_{-5}^{5}=0$ 1

$\Rightarrow \iint_F f\,d\sigma=(10)\frac{1}{12}(\sqrt{17^3}-1)+0=\frac{5}{6}(\sqrt{17^3}-1)$ 2

$\Phi=\iint_F \vec{B}\cdot d\vec{\sigma}=\iint_{B_2}\vec{B}(\vec{r}(\rho,\phi))\cdot\vec{n}(\rho,\phi)\,d\rho\,d\phi$ 1 $\frac{-2}{18}$

$\vec{n}(\rho,\phi):=\frac{\partial\vec{r}(\rho,\phi)}{\partial\rho}\times\frac{\partial\vec{r}(\rho,\phi)}{\partial\phi}$ 1

المعادلات البارامترية للسطح 1

$\vec{r}(\rho,\phi)=\begin{pmatrix}\rho\cos\phi\\ \rho\sin\phi\\ 0\end{pmatrix}$ 2 , $\rho\in[0,R]$ 1 , $\phi\in[0,2\pi]$

$\frac{\partial\vec{r}}{\partial\rho}=\begin{pmatrix}\cos\phi\\ \sin\phi\\ 0\end{pmatrix}$ 1 , $\frac{\partial\vec{r}}{\partial\phi}=\begin{pmatrix}-\rho\sin\phi\\ \rho\cos\phi\\ 0\end{pmatrix}$ 1

$$\vec{n} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} \times \begin{pmatrix} -\rho\sin\phi \\ \rho\cos\phi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \rho \end{pmatrix} \; 2 \quad , \quad \vec{B}(\vec{r}(\rho,\phi)) = \begin{pmatrix} -1 \\ 1 \\ \rho^2\cos^2\phi \end{pmatrix} \; 1$$

$$\Phi = \int_0^{2\pi}\int_0^{R} \begin{pmatrix} -1 \\ 1 \\ \rho^2\cos^2\phi \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \rho \end{pmatrix} d\rho\, d\phi = \int_0^{2\pi}\int_0^{R} \rho^3 \cos^2\phi\, d\rho\, d\phi \quad 3$$

$$= \int_0^{2\pi} \cos^2\phi\, d\phi \int_0^{R} \rho^3 d\rho$$

$$\int_0^{R} \rho^3 d\rho = \frac{1}{4}\rho^4 \Big|_0^R = \frac{R^4}{4} \quad 1$$

$$\int_0^{2\pi} \cos^2\phi\, d\phi = \int_0^{2\pi} \left[\frac{1}{2} + \frac{1}{2}\cos 2\phi\right] d\phi = \pi + 0 = \pi \quad 2$$

$$\Rightarrow \Phi = \frac{1}{4}\pi R^4 \quad 1$$

3 - $\frac{}{10}$ أ - نص مبرهنة Gauß : إن التكامل السطحي لحقل شعاعي مستمر وقابل للتفاضل في الفضاء على سطح مغلق يساوي التكامل الحجمي لتفرق الحقل الشعاعي على الحجم الفراغي المحدود بذلك السطح المغلق. 2 5

$$\oint\!\!\!\oint_F \vec{F}\cdot d\vec{\sigma} = \iiint_{B_3} (\vec{\nabla}\cdot\vec{F})\, d\tau \quad 1$$

F السطح المغلق ، B_3 حجم الفراغ المحدود بـ F ، $d\vec{\sigma}$ عنصر السطح ، $d\tau$ عنصر الحجم
$\vec{F}\cdot d\vec{\sigma}$ تدفق الحقل الشعاعي $\vec{F}$ ، $\vec{\nabla}\cdot\vec{F}$ تفرق الحقل الشعاعي $\vec{F}$. 2

ب. 5

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0 \Rightarrow \iiint_{B_3} \frac{\partial\rho}{\partial t} d\tau + \iiint_{B_3} (\vec{\nabla}\cdot\vec{J})\, d\tau = 0 \quad 1$$

$$\Rightarrow \frac{\partial}{\partial t}\iiint_{B_3} \rho\, d\tau + \oint\!\!\!\oint_F \vec{J}\cdot d\vec{\sigma} = 0 \quad 1 \quad \Longleftrightarrow \quad \frac{\partial q(t)}{\partial t} + I(t) = 0 \quad 1$$

إن تغير كمية الشحنة الكهربائية $q(t)$ في منطقة B_3 تساوي كمية الشحنة الكهربائية التي تمر عبر السطح F للمنطقة B_3. 2

$$\vec{F}(x,y,z) = \begin{pmatrix} z \\ xz \\ xyz \end{pmatrix}$$

$$0 \le \rho \le 2, \quad \frac{\pi}{4} \le \phi \le \frac{\pi}{2}, \quad \rho\cos\phi \le z \le \rho\sin\phi$$

$$2 \quad \Phi := \oint\!\!\oint_{F} \vec{F}\cdot d\vec{\sigma} = \iiint_{B_3} (\vec{\nabla}\cdot\vec{F})\, d\tau \quad , \; d\tau = dx\,dy\,dz \;\; 1 \quad 1.$$

$$= \rho\, d\rho\, d\phi\, dz$$

$$1. \; x = \rho\cos\phi, \; y = \rho\sin\phi \quad 1$$

$$\vec{\nabla}\cdot\vec{F} = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \cdot \begin{pmatrix} z \\ xz \\ xyz \end{pmatrix} \overset{2}{=} xy = \rho^2 \cos\phi \sin\phi \quad 2$$

$$\Rightarrow \Phi := \int_{\pi/4}^{\pi/2} d\phi \int_0^2 \rho\, d\rho \int_{\rho\cos\phi}^{\rho\sin\phi} dz\,(\rho^2 \cos\phi \sin\phi) \quad 2$$

$$= \int_{\pi/4}^{\pi/2} d\phi \int_0^2 \rho\, d\rho\, (\rho^2 \cos\phi \sin\phi)(\rho\sin\phi - \rho\cos\phi)$$

$$= \int_{\pi/4}^{\pi/2} d\phi \int_0^2 \rho^4 d\rho\, (\cos\phi \sin^2\phi - \cos^2\phi \sin\phi)$$

$$= \frac{1}{5}\rho^5 \Big]_0^2 \int_{\pi/4}^{\pi/2} (\cos\phi \sin^2\phi - \cos^2\phi \sin\phi)\, d\phi \qquad 12$$

$$= \frac{1}{5}\rho^5 \Big]_0^2 \left[\frac{1}{3}\sin^3\phi + \frac{1}{3}\cos^3\phi \right]_{\pi/4}^{\pi/2}$$

$$= \frac{2^5}{2}\left[\frac{1}{3} + 0 \quad - \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3 - \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3\right]$$

$$= \frac{16}{15}(2 - \sqrt{2})$$