

① سلم تصحيح مادة التكامل

قسم الفيزياء

السؤال الأول:

1) $\frac{x^4}{x-1} = x^3 + x^2 + x + 1 + \frac{1}{x-1}$ ⑤ ①

$\int \frac{x^4}{x-1} = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1|$ ⑤

2) $u = \ln x \Rightarrow \int \frac{u}{u^2+1} du$ ⑤

$\int = \frac{1}{2} \ln|u^2+1| + C$ ⑤

$\int = \frac{1}{2} \ln(\ln^2 x + 1) + C$ ②

3) $u^6 = 2x+1 \Rightarrow 6u^5 du = 2dx$ ③

$\int = \int \frac{3u^5}{u^6 - u^3} du = 3 \int \frac{u^2}{u-1} du$ ②

$= 3 \int \left(u + 1 + \frac{1}{u-1} \right) du$ ②

$= 3 \left(\frac{u^2}{2} + u + \ln|u-1| \right)$ ②

توجد قيمة u ①

4) $\int = \tan x - 1$ ⑤

$= -\ln(\cos x) - x + C$ ⑤

5) ③ $\int = x^2 \arctan x - \int \frac{x^2}{x^2+1} dx$ (الجزء الثاني)

⑤ $\int \frac{x^2}{x^2+1} dx = x - \arctan x + C$

$$I = x^2 \arctan x - x + \arctan x + C \quad (2)$$

السؤال الثاني:

(1) تفريق الكسور:

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1} \quad (2)$$

$$A = -1, B = \frac{3}{2}, C = -\frac{1}{2}$$

$$I = \left[-\ln|x| + \frac{3}{2} \ln|x-3| - \frac{1}{2} \ln|x+1| \right]_5^{10} \quad (2)$$

$$= -\ln 10 + \frac{3}{2} \ln 7 - \frac{1}{2} \ln 11 + \ln 5 - \frac{3}{2} \ln 2 + \frac{1}{2} \ln 6 \quad (2)$$

$$(2) I = \int_0^{\infty} \frac{2u}{u^2+1} \quad (u = e^x) \quad (-2-)$$

$$(3) I = \lim_{a \rightarrow \infty} \left[\arctan u \right]_0^a$$

$$(2) I = \frac{\pi}{2}$$

$$(3) I = \int_{-2}^2 \frac{x^5 + x^3}{x^3 + x} + \int_{-2}^2 \frac{7x^4 - 5x^2 - 2}{x^3 + x} \quad (-3-)$$

$$(2) \int_{-2}^2 \frac{7x^4 - 5x^2 - 2}{x^3 + x} = 0$$

$$I = \int_{-2}^2 \frac{x^5 + x^3}{x^3 + x} = \int_{-2}^2 x^2 = \left[\frac{x^3}{3} \right]_{-2}^2 = \frac{16}{3} \quad (2)$$

سؤال الثالث

$$(3) I = \int_{\ln 8}^1 e^y (y-1) dy$$

$$= [(y-1)e^y - e^y]_{\ln 8}^1 = 8(\ln 8 - 16 + 1)$$

$$I = \int_1^2 \frac{1}{2} \pi dx \quad (3)$$

$$I = \frac{3}{4} \quad (4)$$

السؤال الثالث :

$$S = \int_0^{2\pi} 2 \cdot x' dt = 9 \int_0^{2\pi} (1 - \cos t)^2 dt \quad (5) \quad (1)$$

$$= 9 \int_0^{2\pi} \left(1 - 2\cos t + \frac{1 + \cos 2t}{2} \right) dt \quad (3)$$

$$= 27\pi \quad (5)$$