قسم الفيزياء - كلية العلوم
جامعة دمشق

الالكتروديناميك
السنة الثالثة فيزياء
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المدة ٩ - ١١
العلامة 100

سلم تصحيح

Q I 20

$$F(x,y,z) = \begin{pmatrix} y\cos(xy) \\ x\cos(xy) + z \\ y \end{pmatrix}$$

$$\left.\begin{aligned} \frac{\partial F_z}{\partial y} &= \frac{\partial F_y}{\partial z} = 1 \\ \frac{\partial F_x}{\partial z} &= \frac{\partial F_z}{\partial x} = 0 \\ \frac{\partial F_y}{\partial x} &= \frac{\partial F_x}{\partial y} = \cos(xy) - xy\sin(xy) \end{aligned}\right\} \Rightarrow \vec{\nabla}\times\vec{F} = \vec{0} \Leftrightarrow \exists\,\Phi(x,y,z) : \vec{F}(x,y,z) = \vec{\nabla}\Phi(x,y,z) \quad 5$$

$$\begin{pmatrix} y\cos(xy) \\ x\cos(xy)+z \\ y \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \Phi(x,y,z)$$

10

2 $\frac{\partial \Phi}{\partial x} = y\cos(xy) \Rightarrow \Phi(x,y,z) = \sin(xy) + g(y,z)$

2 $\frac{\partial \Phi}{\partial y} = x\cos(xy) + \frac{\partial g(y,z)}{\partial y} \equiv x\cos(xy) + z$

2 $\Rightarrow \frac{\partial g}{\partial y} = z \Rightarrow g(y,z) = zy + h(z)$

$\Rightarrow$ 2 $\Phi(x,y,z) = \sin(xy) + zy + h(z)$

2 $\frac{\partial \Phi}{\partial z} = y + \frac{\partial h}{\partial z} \equiv y \Rightarrow \frac{\partial h}{\partial z} = 0 \Rightarrow h = k \,.\, k = \text{const}$

$$\Rightarrow \boxed{\Phi(x,y,z) = \sin(xy) + zy + k} \quad 5 \quad - \; k : \text{const}$$

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$$\frac{\partial \Phi}{\partial r} = -2E_0 \frac{\cos\theta}{r^3} - E_0 \cos\theta - 2\Phi_0 \sin\theta \cos\phi \quad 2$$

$$\frac{\partial \Phi}{\partial \theta} = -\frac{E_0}{r^2}\sin\theta + E_0 r \sin\theta + \Phi_0 \frac{R^2}{r^2}\cos\theta \cos\phi \quad 2$$

$$\frac{\partial \Phi}{\partial \phi} = -\Phi_0 \frac{R^3}{r^2} \sin\theta \sin\phi \qquad 2$$

$$\Rightarrow \vec{E} = \begin{pmatrix} -2E_0 \frac{\cos\theta}{r^3} - E_0\cos\theta - 2\Phi_0 \sin\theta\cos\phi \\ -\frac{E_0}{r^2}\sin\theta + E_0 r\sin\theta + \Phi_0 \frac{R^2}{r^2}\cos\theta\cos\phi \\ -\frac{\Phi_0}{r^2}R^3 \sin\theta \sin\phi \end{pmatrix}$$

الحقل الكهربائي

$$q_{lm} = \sqrt{\frac{4\pi}{2l+1}} \int d^3r\, \rho(\vec{r})\, r^l\, Y^*_{lm}(\theta, \phi)$$

30^{-2}

$l=0 \Rightarrow m=0$

4 $$q_{00} = \lambda_0 \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\pi} \sin\theta \frac{\delta(\theta-\frac{\pi}{2})}{\sin\theta} d\theta}_{1} \underbrace{\int_0^{\infty} dr\, r^2 \frac{\delta(r-R)}{r}}_{R} = 2\pi\lambda_0 R \Rightarrow \boxed{q_{00} = 2\pi\lambda_0 R}$$

5 $$q_{10} = \lambda_0 \sqrt{\frac{4\pi}{3}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \int_0^{\pi} d\theta \sin\theta \frac{\delta(\theta-\pi/2)}{\sin\theta} \int_0^{\infty} dr\, r^2 \frac{\delta(r-R)}{r} \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$= 2\pi\lambda_0 \underbrace{\int_0^{\pi} d\theta \cos\theta\, \delta(\theta-\pi/2)}_{\cos\pi/2 = 0} \underbrace{\int_0^{\infty} dr\, r\, \delta(r-R)}_{R} \Rightarrow \boxed{q_{10} = 0}$$

$$q_{11} = \sqrt{\frac{4\pi}{3}} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{\infty} dr\, r^2 \lambda_0 \frac{\delta(r-R)}{r} \frac{\delta(\theta-\pi/2)}{\sin\theta} \left(-\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\phi}\right)$$

5 $$= -\frac{\lambda_0}{\sqrt{2}} \int_0^{2\pi} d\phi\, e^{i\phi} \underbrace{\int_0^{\pi} d\theta \sin\theta\, \delta(\theta-\frac{\pi}{2})}_{1} \underbrace{\int_0^{\infty} dr\, r\, \delta(r-R)}_{R}$$

$$\int_0^{2\pi} d\phi\, e^{i\phi} = \frac{1}{i}\left[e^{2\pi i} - 1\right] = \frac{1}{i}\left[1-1\right] = 0 \Rightarrow \boxed{q_{11} = 0}$$

25+30=55

-1
25

$r>R$: $r\to\infty$ $\vec{E}=E_0\hat{e}_z$

$$E_0=-\frac{\partial\Phi}{\partial z}\Rightarrow \Phi(r\to\infty,\theta,\phi)=-E_0 z$$

$$=-E_0 r\cos\theta$$

7

$$=-E_0 r\sqrt{\frac{4\pi}{3}}\,Y_{10}(\theta,\phi)$$

$$\Rightarrow \xi_{10}=-E_0\sqrt{\frac{4\pi}{3}},\quad \xi_{lm}=0 \text{ for } (l,m)\neq(1,0)$$

الشرط الحدي :

$$\Phi(R,\theta,\phi)=\xi_{10}RY_{10}(\theta,\phi)+\frac{\eta_{10}}{R^2}Y_{10}(\theta,\phi)+\frac{\eta_{11}}{R^2}Y_{11}(\theta,\phi)+\frac{\eta_{1-1}}{R^2}Y_{1-1}(\theta,\phi)$$

5

$$=-\frac{\Phi_0}{2}\sqrt{\frac{8\pi}{3}}\left(Y_{11}(\theta,\phi)-Y_{1-1}(\theta,\phi)\right)$$

$$\xi_{10}R+\frac{\eta_{10}}{R^2}=0\Rightarrow \eta_{10}=E_0\sqrt{\frac{4\pi}{3}}R^3,\quad \eta_{l\neq,m}=0$$

3

$$\frac{\eta_{11}}{R^2}=-\frac{\Phi_0}{2}\sqrt{\frac{8\pi}{3}}\Rightarrow \eta_{11}=-\frac{\Phi_0}{2}\sqrt{\frac{8\pi}{3}}R^2$$

$$\frac{\eta_{1-1}}{R^2}=-\frac{\Phi_0}{2}\sqrt{\frac{8\pi}{3}}\Rightarrow \eta_{1-1}=\frac{\Phi_0}{2}\sqrt{\frac{8\pi}{3}}R^2$$

$$\Phi(r,\theta,\phi)=-E_0\left(\frac{4\pi}{3}\right)^{1/2}r\left(\frac{3\pi}{4}\right)^{1/2}\cos\theta+\frac{E_0R^3}{r^2}\left(\frac{4}{3\pi}\right)^{1/2}\left(\frac{3\pi}{4}\right)^{1/2}\cos\theta+$$

$$\frac{\Phi_0}{2}\left(\frac{8\pi}{3}\right)^{1/2}\frac{R^2}{r^2}\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{i\phi}+\frac{\Phi_0}{2}\left(\frac{8\pi}{3}\right)^{1/2}\frac{R^2}{r^2}\left(\frac{3}{8\pi}\right)^{1/2}e^{-i\phi}$$

4

$$\Rightarrow \boxed{\Phi(r,\theta,\phi)=E_0\left(\frac{R^3}{r^3}-1\right)r\cos\theta+\Phi_0R^2\frac{1}{r^2}\sin\theta\cos\phi},\quad \cos\phi=\frac{e^{i\phi}+e^{-i\phi}}{2}$$

$$\begin{pmatrix}E_r\\E_\theta\\E_\phi\end{pmatrix}=\begin{pmatrix}\frac{\partial\Phi}{\partial r}\\ \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\\ \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\end{pmatrix}$$

$$F_D(v) = 6\pi\eta^{\nu} v^{\xi} R^{\epsilon}$$

10+20+10 = 40 Pts Q I

$$[\eta] = \frac{MLT^{-2}T}{L^2} = ML^{-1}T^{-1}$$

.1 10

$$[F_D(v)] = [6\pi][ML^{-1}T^{-1}]^{\nu}[LT^{-1}]^{\xi}[L]^{\epsilon}$$

$$MLT^{-2} \equiv 1\, M^{\nu} L^{-\nu} T^{-\nu}, \xi, -\xi, \epsilon$$

$$q_{1-1} = \frac{\lambda_0}{\sqrt{2}} \underbrace{\int_0^{2\pi} d\phi\, e^{-i\phi}}_{0} \underbrace{\int_0^{\pi} d\theta \sin\theta\, \delta(\theta - \tfrac{\pi}{2})}_{1} \underbrace{\int_0^{\infty} dr\, \delta(r-R)}_{R}$$

$$\Rightarrow \boxed{q_{1-1} = 0}$$

4

$$q_{20} = \sqrt{\frac{4\pi}{5}}\, \lambda_0 \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dr\, r^2 \frac{\delta(\theta - \pi/2)}{\sin\theta} \frac{\delta(r-R)}{r} \left[\sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)\right] r^2$$

$$= 2\pi \frac{\lambda_0}{2} \int_0^{\pi} d\theta\, \delta(\theta - \tfrac{\pi}{2})(3\cos^2\theta - 1) \int_0^{\infty} dr\, r^3 \delta(r-R) = -\pi \lambda_0 R^3$$

وبنفس الطريقة يمكن اثبات أن

4 $\quad q_{21} = 0,\quad q_{2-1} = 0,\quad q_{22} = 0,\quad q_{2-2} = 0$ (1, 1, 1, 1)

5 $$\Rightarrow \Phi(r,\theta,\phi) = \frac{2\pi R \lambda_0}{r} - \frac{1}{\sqrt{2}} \frac{\pi \lambda_0 R^3}{r^3} (3\cos^2\theta - 1)$$

Q III 25

1 $$\vec{J} = J(\rho)\hat{e}_z \Rightarrow \vec{A} = A(\rho,\phi,z)\hat{e}_z$$

1 $$\vec{A} = A(\rho)\hat{e}_z$$ بسبب التناظر الدوراني والانسحابي فإن

4

$$\rho \le R: \quad \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho \frac{\partial}{\partial\rho}\right) A_\rho = -\frac{4I}{cR^2}$$

$$\Rightarrow \frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)A_\rho = -\frac{4I}{cR^2}\rho \Rightarrow \rho\frac{\partial A_\rho}{\partial\rho} = -\frac{I}{cR^2}\rho^2 + c_1 \quad 2$$

$$\Rightarrow \frac{\partial A_\rho}{\partial\rho} = -\frac{2I}{cR^2}\rho + \frac{c_1}{\rho} \Rightarrow \quad 2$$

3 $$A(\rho) = -\frac{1}{c}\frac{I}{R^2}\rho^2 + c_1 \ln\frac{\rho}{R} + d_1$$

5 $$\rho > R: \quad \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)A_\rho = 0 \Rightarrow A(\rho) = c_2 \ln\frac{\rho}{R} + d_2$$

2 3

$c_1 = 0$, $d_2 = 0$, $d_1 = \frac{I}{c}$, $c_2 = -\frac{2I}{c}$

1 1 1 1

2 $$A(\rho) = \frac{I}{c} \begin{cases} 1 - \frac{\rho^2}{R^2} & \rho \leq R \\ -2 \ln \frac{\rho}{R} & \rho > R \end{cases}$$

والحقل المغنطيسي : $\vec{B} = \vec{\nabla} \times \vec{A}$, $\vec{A} = \underbrace{A(\rho)}_{A_z} \hat{e}_z$

5 $$\Rightarrow \vec{B} = -\frac{\partial A(\rho)}{\partial \rho} \hat{e}_\phi = \frac{2I}{c} \hat{e}_\phi \begin{cases} \frac{\rho}{R^2} & \rho \leq R \\ \frac{1}{\rho} & \rho > R \end{cases}$$

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