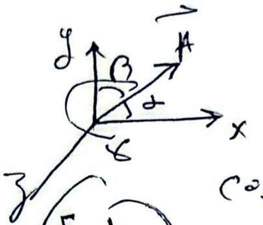
سلم تصحيح امتحان تحليل المتجهات ومبادئ الهندسة التفاضلية

الفصل 2 — 2024 - 2025

السؤال الأول :

(1)



$\vec{A} = x\vec{i} + y\vec{j} + z\vec{k}$

(5+)

$\cos\alpha = \frac{x}{|\vec{A}|}$, $\cos\beta = \frac{y}{|\vec{A}|}$, $\cos\gamma = \frac{z}{|\vec{A}|}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2+y^2+z^2}{|\vec{A}|^2} = 1$

(2)

(5+)

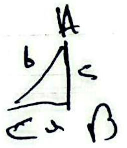
$\vec{a}\cdot(\vec{b}+\vec{c}) = |\vec{a}|\,\mathrm{proj}(\vec{b}+\vec{c})$

$= |\vec{a}|(\mathrm{proj}\,\vec{b} + \mathrm{proj}\,\vec{c})$

$= |\vec{a}|\,\mathrm{proj}\,\vec{b} + |\vec{a}|\,\mathrm{proj}\,\vec{c}$

$= \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

(3)



$A\hat{B}C = B\hat{C}A = C\hat{A}B$

$\frac{1}{2}|\vec{AB}\wedge\vec{BC}| = \frac{1}{2}|\vec{BC}\wedge\vec{CA}| = \frac{1}{2}|\vec{CA}\wedge\vec{AB}|$

(5+)

$= |\vec{AB}||\vec{BC}|\sin B = |\vec{BC}||\vec{CA}|\sin C = |\vec{CA}||\vec{AB}|\sin A$

$c\cdot a\sin B = a b\sin C = b c\sin A$

نقسم على abc

(5+)

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(4)

$\Leftarrow d\vec{OM} = \vec{T}\,ds$

$\int \vec{F}\,d\vec{OM} = \int \vec{F}\cdot\vec{T}\,ds$

السؤال الثاني :

(1)

$\vec{OM} = r\vec{f}$

$\vec{OM}' = r'\vec{f} + r\theta'\vec{g}$

$\vec{OM}'' = (r'' - r\theta'^2)\vec{f} + (2r'\theta' + r\theta'')\vec{g}$

(5+)

$\rho = \frac{|\vec{M}'|^3}{|\vec{M}'\wedge\vec{M}''|}$ (5+)

$= \frac{(r'^2 + r^2\theta'^2)^{3/2}}{|2r'^2\theta' + r r''\theta' + r^2\theta'^3 ...|}$

(5+)

(2)

$\vec{OM} = r\vec{f}$

$\vec{OM}' = r'\vec{f} + r\varphi'\vec{g} + r\theta'\sin\varphi\,\vec{h}$

(5+) $r' = 5 \quad \theta' = 0 \quad \varphi' = 6t$

$\vec{OM}' = 5\vec{f} + 12\vec{g} + 0\vec{h}$

$|\vec{OM}'| = \sqrt{25+144}$

(3)

(5+)

$x = 2\cos t$

$y = 3\sin t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

(5+)

$X = x - y'\frac{x'^2+y'^2}{x'y'' - y'x''}$

$Y = y + x'\frac{x'^2+y'^2}{x'y'' - y'x''}$

$x' = -2\sin t$

$x'' = -2\cos t$

$y' = 3\cos t \qquad y'' = -3\sin t$

يعوض

4

$\vec{OM'} \neq \vec{0}$

$$S = \int_0^{10} \sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\, dt$$

$\dot{x} = 3-3t^2$

$\dot{y} = 3+3t^2$ (5+)

$\dot{z} = 6t$

$\dot{x}^2 = 9+9t^4-18t^2$

$\dot{y}^2 = 9+9t^4+18t^2$

$\dot{z}^2 = 36t^2$

$\dot{x}^2+\dot{y}^2+\dot{z}^2 = 18t^4+36t^2+18$

$= 18(t^4+2t^2+1)$

$= 18(t^2+1)^2$

$$S = \int_0^{10} \sqrt{18}(t^2+1)\, dt$$

$ds = \sqrt{18}(t^2+1)$

$\vec{T} = \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}$ (5+)

$\alpha = \frac{3(1-t^2)}{\sqrt{18}(t^2+1)}$ $\beta = \frac{3(1+t^2)}{\sqrt{18}(t^2+1)}$

$\gamma = \frac{6t}{\sqrt{18}(t^2+1)}$

$\vec{N} = \frac{d\alpha}{ds}, \frac{d\beta}{ds}, \frac{d\gamma}{ds}$

$d\alpha = \frac{3}{\sqrt{18}}\left(\frac{-2t(t^2+1)-2t(1-t^2)}{(t^2+1)^2}\right)$

$d\beta =$

$d\gamma =$

$ds = \sqrt{dx^2+d\beta^2+d\gamma^2}$

$\vec{B} = \vec{T} \wedge \vec{N}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \alpha & \beta & \gamma \\ \alpha_1 & \beta_1 & \gamma_1 \end{vmatrix}$$ (5+)

$\frac{X-x}{\dot{x}} = \frac{Y-y}{\dot{y}} = \frac{Z-z}{\dot{z}}$

- $\dot{x}(X-x)+\dot{y}(Y-y)+\dot{z}(Z-z) = 0$ (1er)

$$\begin{vmatrix} X-x & Y-y & Z-z \\ \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \end{vmatrix} = 0$$ (5+)

(5+)

$\cos\theta = \frac{\vec{B}\cdot\vec{T}}{1} = \alpha_2$ (5+)

- $\rho = \frac{1}{\text{courbure}}$

$\vec{rot}\, \vec{f} = \vec{0}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = 0$$ (10+)

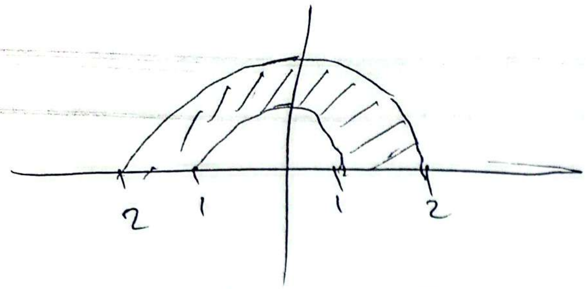
$\varphi = \int P\,dx + \int Q\,dy + R\,dz$

$= -2y\cos^2 x - z^2 x + y\cos^2 x + 2y - x^2 z + yz + K$

$\phi = -2y\cos^2 x - 2x^2 + c$

$-2 + 0 = c$

(3+)



$x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

(5+)

$E = \int_0^{\pi}\int_{1}^{2} r^2 \sin\theta \, dr\, d\theta$

$\vec{N} = \frac{\vec{grad}\,\phi}{|grad\,\phi|} = \frac{\vec{e}}{|\vec{e}|}$

$\vec{AB} = (0, 2, 2)$

$|\vec{AB}| = \sqrt{4+4} = \sqrt{8}$

$\vec{e} = 0, \frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}}$

$\frac{d\phi}{da} = l\frac{d\phi}{dx} + m\frac{d\phi}{dy} + n\frac{d\phi}{dz}$

(5+)

$E = \phi(B) - \phi(A)$

$\iint \left(\frac{d\phi}{dy} - \frac{d\phi}{dx}\right) dx\,dy$

$\frac{d\phi}{dx} = y \quad \frac{d\phi}{dy} = 2y$

$\iint y\,dx\,dy$

(5+)

(1) الماده العلمية تامه ومتناسقة مع المحتوى العلمي

(2) الى حد ما طول الاسئلة تتناسب مع وقت الامتحان

عدد الاسئلة المقررة في الاجابة على كل المطلوب

التصحيح تام

أ.م.د / محمد

(توقيع)