2025 - 2 - ف 2] تصحيح

[1]

I - العزم الحركي السبيني ، التمثيل المصفوفي للمؤثرات (35ن) $= 40$

-1

$$[\hat S_z , \hat S_+] = [\hat S_z , \hat S_x + i.\hat S_y] = [\hat S_z , \hat S_x] + i.[\hat S_z , \hat S_y]$$

$$[\hat S_x , \hat S_y] = i.\hbar.\hat S_z \quad ; \quad [\hat S_z , \hat S_y] = -i.\hbar.\hat S_x$$

$$[\hat S_z , \hat S_+] = i.\hbar.\hat S_y + i.(-i.\hbar).\hat S_y \Rightarrow [\hat S_z , \hat S_+] = \hbar.\hat S_+$$

$$\hat S_z.\hat S_+ - \hat S_+.\hat S_z = \hbar.\hat S_+ \Rightarrow \hat S_z.\hat S_+ = \hbar.\hat S_+ + \hat S_+.\hat S_z$$

-2

$$\hat S_+ = \hat S_x + i.\hat S_y \quad ; \quad \hat S_- = \hat S_x - i.\hat S_y \Rightarrow$$

$$\hat S_-.\hat S_+ = (\hat S_x - i.\hat S_y).(\hat S_x + i.\hat S_y)$$

$$\hat S_-.\hat S_+ = \hat S_x^2 + \hat S_y^2 + i.[\hat S_x , \hat S_y] = \hat S_x^2 + \hat S_y^2 + i.(i.\hbar).\hat S_z$$

$$\hat S^2 = \hat S_x^2 + \hat S_y^2 + \hat S_z^2$$

$$\hat S_-.\hat S_+ = \hat S^2 - \hat S_z^2 - \hbar.\hat S_z$$

-3

$$\hat S_z.\hat S_+|s,m\rangle = (\hbar.\hat S_+ + \hat S_+.\hat S_z)|s,m\rangle$$

$$= \hbar.\hat S_+|s,m\rangle + \hat S_+.\hat S_z|s,m\rangle$$

$$\hat S_z.\hat S_+|s,m\rangle = \hbar.\hat S_+|s,m\rangle + m.\hbar.\hat S_+|s,m\rangle$$

$$\hat S_z.\hat S_+|s,m\rangle = (m+1).\hbar.\hat S_+|s,m\rangle$$

$$\hat S_z[\hat S_+|s,m\rangle] = (m+1).\hbar.[\hat S_+|s,m\rangle]$$

إذن $\hat S_+|s,m\rangle$ شعاع خاص للمؤثر $\hat S_z$. نتيجة نرمز لها بـ

$$\hat S_z|s,m+1\rangle = (m+1).\hbar|s,m+1\rangle$$

نلاحظ إذن القيمة الخاصة $(m+1)\hbar$ شعاع خاص

2

$$|s,m+1\rangle \longleftarrow (m+1)\cdot\hbar \longrightarrow \hat{S}_+|s,m\rangle$$

$$\Downarrow$$

$$\hat{S}_+|s,m\rangle = c\cdot|s,m+1\rangle \Rightarrow [\hat{S}_+|s,m\rangle]^\dagger = [c\cdot|s,m+1\rangle]^\dagger$$

$$\langle s,m|\hat{S}_+^\dagger = c^*\cdot\langle s,m+1|$$

$$\hat{S}_+^\dagger = (\hat{S}_x + i\cdot\hat{S}_y)^\dagger = \hat{S}_x^\dagger - i\cdot\hat{S}_y^\dagger = \hat{S}_x - i\cdot\hat{S}_y = \hat{S}_-$$

$$\langle s,m|\hat{S}_- = c^*\cdot\langle s,m+1|$$

ضرب داخلی:

$$\text{if } \vec{a}=\vec{b} \Rightarrow \vec{a}^2 = \vec{b}^2 \Leftrightarrow \vec{a}\cdot\vec{a} = \vec{b}\cdot\vec{b} \Rightarrow$$

$$[\langle s,m|\hat{S}_-]\cdot[\hat{S}_+|s,m\rangle] = [\langle s,m+1|\cdot c^*]\cdot[c\cdot|s,m+1\rangle]$$

$$\langle s,m|\hat{S}_-\cdot\hat{S}_+|s,m\rangle = |c|^2\cdot\langle s,m+1|s,m+1\rangle$$

$$\langle s,m+1|s,m+1\rangle = 1 \Rightarrow \langle s,m|\hat{S}_-\cdot\hat{S}_+|s,m\rangle = |c|^2$$

$$|c|^2 = \langle s,m|\hat{S}_-\cdot\hat{S}_+|s,m\rangle = \langle s,m|\hat{S}^2 - \hat{S}_z^2 - \hbar\cdot\hat{S}_z|s,m\rangle$$

$$|c|^2 = \hbar^2\cdot[s\cdot(s+1) - m\cdot(m+1)]$$

$$|c| = \hbar\cdot\sqrt{s\cdot(s+1) - m\cdot(m+1)}$$

$$\hat{S}_+|s,m\rangle = c\cdot|s,m+1\rangle$$

$$\hat{S}_+|s,m\rangle = \hbar\cdot\sqrt{s\cdot(s+1) - m\cdot(m+1)}\cdot|s,m+1\rangle$$

-4

$$\langle s',m'|s,m\rangle = \delta_{ss'}\cdot\delta_{mm'}$$

$$\sum_s \sum_{m=-s}^{+s} |s,m\rangle\cdot\langle s,m| = I$$

$$\sum_{m=-1}^{+1} |1,m\rangle\cdot\langle 1,m| = I$$

$\boxed{3}$:

$\boxed{5}$ -5 $\boxed{3}$

$$\hat{S}_z |s,m\rangle = m.\hbar.|s,m\rangle$$

$$\hat{S}_z^2 |s,m\rangle = m^2.\hbar^2 |s,m\rangle$$

$$\frac{a}{\hbar^2}.\hat{S}_z^2 |s,m\rangle = \frac{a}{\hbar^2}.m^2.\hbar^2.|s,m\rangle$$

$$\hat{H}^0 |s,m\rangle = E_m^0.|s,m\rangle \; ; \; E_m^0 = a.m^2$$

$$\langle s',m' | \hat{H}^0 | s,m\rangle = E_m^0.\langle s',m' | s,m\rangle$$

$$\langle s,m' | \hat{H}^0 | s,m\rangle = \delta_{m'm}.E_m^0$$

$$[\hat{H}^0]_{B_1} = a.\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{S}_x = \frac{1}{2}.(\hat{S}_+ + \hat{S}_-) \; ; \; \hat{S}_y = \frac{1}{2.i}.(\hat{S}_+ - \hat{S}_-)$$

$$\hat{S}_x^2 = \hat{S}_x.\hat{S}_x = \frac{1}{4}.(\hat{S}_+ + \hat{S}_-).(\hat{S}_+ + \hat{S}_-)$$

$$\hat{S}_x^2 = \frac{1}{4}.(\hat{S}_+^2 + \hat{S}_+.\hat{S}_- + \hat{S}_-.\hat{S}_+ + \hat{S}_-^2)$$

$$\hat{S}_y^2 = -\frac{1}{4}.(\hat{S}_+^2 + \hat{S}_-^2 - \hat{S}_+.\hat{S}_- - \hat{S}_-.\hat{S}_+)$$

$$(\hat{S}_x^2 - \hat{S}_y^2) = \frac{1}{2}.(\hat{S}_+^2 + \hat{S}_-^2)$$

$$\hat{H}' = \frac{b}{\hbar^2}.(\hat{S}_x^2 - \hat{S}_y^2) \Rightarrow \hat{H}' = \frac{b}{2.\hbar^2}.(\hat{S}_+^2 + \hat{S}_-^2)$$

$[\hat{S}_+]_{B_1}$ و $[\hat{S}_-]_{B_1}$ معروفة . لذ

$$[\hat{S}_+^2] = 2.\hbar^2.\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\hat{H}']_{B_1} = b.\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$[\hat{S}_-^2] = 2\hbar^2.\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$[H]_{B_1} = [\hat{H}^0]_{B_1} + [\hat{H}']_{B_1}$$

6- القيم الذاتية المنتظرة لـ $\hat{H}^0$: $E_m^0 = a.m^2$

4

4 القيم الذاتية لـ $\hat{H}$:

$$P(\lambda) = \det\left\{ [\hat{H}]_{\mathbb{B}_1} - \lambda \cdot [I] \right\} = 0$$

$$\begin{vmatrix} a-\lambda & 0 & b \\ 0 & \lambda & 0 \\ b & 0 & a-\lambda \end{vmatrix} = 0$$

<u>Resolution</u>, $\lambda = 0$; $\lambda = a \pm b$

بعض أخذ القيم الذاتية الحقيقية لـ $\hat{H}$:

$$E_0 = 0 \; ; \; E_+ = a+b \equiv E_1 \; ; \; E_- = a-b \equiv E_2$$

-7

$$E^1_{m=0} = \langle 1,0 | \hat{H}' | 1,0 \rangle = \frac{b}{\hbar^2} \cdot \langle 1,0 | \hat{S}_x^2 - \hat{S}_y^2 | 1,0 \rangle$$

$$= \frac{b}{2 \cdot \hbar^2} \cdot \langle 1,0 | \hat{S}_+^2 + \hat{S}_-^2 | 1,0 \rangle$$

$$E^1_0 = \frac{b}{2 \cdot \hbar^2} \cdot \left\{ \langle 1,0 | \hat{S}_+^2 | 1,0 \rangle + \langle 1,0 | \hat{S}_-^2 | 1,0 \rangle \right\}$$

$$|1,0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; ; \; [\hat{S}_+^2]_{\mathbb{B}_1} = 2 \cdot \hbar^2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\hat{S}_-^2]_{\mathbb{B}_1} = [\hat{S}_-]_{\mathbb{B}_1} \cdot [\hat{S}_-]_{\mathbb{B}_1}$$

إما نستعمل طريقة ضرب المصفوفات أو طريقة تطبيق المؤثرات. نحصل على:

$$E^1_0 = 0$$

-8

$$W = \langle S, m' | \hat{H}' | S, m \rangle \quad m = \pm 1 \,,\; m' = \pm 1$$

$$W = \frac{b}{\hbar^2} \langle S, m' | \hat{S}_x^2 - \hat{S}_y^2 | S, m \rangle$$

$$W = \frac{b}{2 \cdot \hbar^2} \langle S, m' | \hat{S}_+^2 + \hat{S}_-^2 | S, m \rangle$$

$$E_+ = a+b \rightarrow |1,1\rangle = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

valeurs et vecteurs propres exacts.

$$E_0 = 0 \rightarrow |1,0\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$E_- = a-b \rightarrow |1,-1\rangle = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

5

(.>24/-1-

$$\hat{S}^2 |s, m_s\rangle = s.(s+1).\hbar^2.|s, m_s\rangle$$

$$\hat{S}_z |s, m_s\rangle = m_s.\hbar.|s, m_s\rangle$$

$$s = \frac{1}{2} \Rightarrow m_s = \pm\frac{\hbar}{2} \quad ; \quad \frac{\hbar}{2} = a$$

$$[\hat{S}_z] = a.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad ; \quad [\hat{S}^2] = 3.a^2.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{S}_x = \frac{1}{2}.(\hat{S}_+ + \hat{S}_-) \; ; \; \hat{S}_y = \frac{1}{2.i}.(\hat{S}_+ - \hat{S}_-) \Rightarrow$$

$$[\hat{S}_x] = \frac{1}{2}.\{[\hat{S}_+] + [\hat{S}_-]\}$$

$$[\hat{S}_y] = \frac{1}{2.i}.\{[\hat{S}_+] - [\hat{S}_-]\}$$

باستعمال المصفوفتين $[S_+]$ ، $[S_-]$ نجد بأن:

$$[\hat{S}_x] = a.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad ; \quad [S_y] = a.\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

تم الحصول على $[\hat{S}^2]$ باستعمال:

$$[\hat{S}^2] = [\hat{S}_x^2] + [\hat{S}_y]^2 + [\hat{S}_z]^2$$

أي بجمع المصفوفات.

$$[\hat{H}] = \alpha.[\hat{S}]^2 + \beta_x.[\hat{S}_x] + \beta_y.[\hat{S}_y] + \beta_z.[\hat{S}_z]$$

$$[H] = 3.\alpha.a^2.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta_x.a.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \beta_y.a.\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} +$$

$$\beta_z.a.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[H] = \begin{bmatrix} 3.\alpha a^2 + \beta_x.a + a.\beta_z & \beta_x.a - i.\beta_y.a \\ \beta_x.a + i.\beta_y.a & 3.\alpha.a^2 - a.\beta_z \end{bmatrix}$$

بيانات :

6

$$|\beta_z| >> \sqrt{\beta_x^2+\beta_y^2}$$

إذن يجب الهاملتوني الغير مضطرب :

$$\hat{H}^0 = \alpha . \hat{S}^2 + \beta_z . \hat{S}_z$$

$$[H^0] = \alpha . 3 . a^2 . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta_z . a . \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[\hat{H}^0] = \begin{bmatrix} 3.\alpha.a^2 + a.\beta_z & 0 \\ 0 & 3.\alpha.a^2 - a.\beta_z \end{bmatrix}.$$

الاضطراب :

$$\hat{H}^1 = \beta_x . \hat{S}_x + \beta_y . \hat{S}_y$$

$$[H^1] = \beta_x . a . \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \beta_y . a . \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$[H^1] = \begin{bmatrix} 0 & +\beta_x . a - i.\beta_y . a \\ \beta_x . a + i . \beta_y . a & 0 \end{bmatrix}$$

$$[H^1] = a . \begin{bmatrix} 0 & \beta_x - i.\beta_y \\ \beta_x + i.\beta_y & 0 \end{bmatrix}$$

$$[\hat{H}] = [\hat{H}^0] + [\hat{H}^1]$$

$$[H] = a . \begin{bmatrix} 3.\alpha.a + \beta_z & \beta_x - i.\beta_y \\ \beta_x + i.\beta_y & 3.\alpha.a - \beta_z \end{bmatrix}.$$

3- الحل الدقيق : إيجاد القيم الذاتية الدقيقة :

$$P_H(\lambda) = det([H] - \lambda . I_2) = 0$$

على شكل الطاقات الدقيقة التالية:

$$E_{\pm} = 3\cdot\alpha\cdot a^2 \pm a\cdot\beta \quad ; \quad \beta = \sqrt{\beta_x^2+\beta_y^2+\beta_z^2}$$

الأشعة الذاتية المرافقة:

$$|\psi_{\pm}\rangle = \begin{bmatrix} \beta_z \pm \beta \\ \beta_x + i\cdot\beta_y \end{bmatrix}$$

4- الحل التقريبي باستعمال نظرية الاضطراب:

الهاملتوني الغير مضطرب:

$$[H^0] = \begin{bmatrix} E^0_+ & 0 \\ 0 & E^0_- \end{bmatrix} = \begin{bmatrix} 3\cdot\alpha\cdot a + \beta_z & 0 \\ 0 & 3\cdot\alpha\cdot a - \beta_z \end{bmatrix}$$

الأشعة الذاتية المرافقة هي $|\pm\rangle$.

هاملتوني الاضطراب:

$$[H^1] = a\cdot\begin{bmatrix} 0 & \beta_x - i\cdot\beta_y \\ \beta_x + i\cdot\beta_y & 0 \end{bmatrix}$$

تصحيح الطاقة من المرتبة الأولى:

$$E^1_{\pm} = \langle\pm|\hat{H}^1|\pm\rangle = 0$$

لأن $\hat{H}^1$ مصفوفة قطرية عناصر قطرها الأساسي معدومة.

التصحيح من المرتبة الثانية:

$$E^2_+ = \frac{|\langle-|\hat{H}^1|+\rangle|^2}{E^0_+ - E^0_-} \quad ; \quad E^2_- = \frac{|\langle+|H^1|-\rangle|^2}{E^0_- - E^0_+} = -E^2_+$$

استعملنا العلاقة العامة:

$$E^2_{\pm} = \sum_{m=\pm} \frac{|\langle m|\hat{H}^1|\pm\rangle|^2}{E^0_{\pm} - E^0_m}$$

$$\langle-|H^1|+\rangle = a\cdot(\beta_x + i\cdot\beta_y)$$

$$|\langle-|\hat{H}^1|+\rangle|^2 = a^2\cdot(\beta_x^2 + \beta_y^2)$$

$\boxed{8}$ $E_+^0 - E_-^0 = \hbar \cdot \beta_z$

$$E_\pm^2 = \pm \frac{a^2 \cdot (\beta_x^2 + \beta_y^2)}{\hbar \cdot \beta_z} = \pm \frac{a}{2} \cdot \frac{\beta_x^2 + \beta_y^2}{\beta_z}$$

عبارة الطاقة الثانية:

$$E_\pm = E_\pm^0 + E_\pm^2$$

5- الشعاع الذاتي المصحح: $|\tilde{+}\rangle$:

$$|\tilde{+}\rangle = |+\rangle + \frac{\langle -|\hat{H}^1|+\rangle}{E_+^0 - E_-^0} \cdot |-\rangle$$

$$|\tilde{+}\rangle = |+\rangle + \frac{a \cdot (\beta_x + i \cdot \beta_y)}{\hbar \cdot \beta_z} \cdot |-\rangle$$

$$|\tilde{+}\rangle = |+\rangle + \frac{\beta_x + i \cdot \beta_y}{2 \cdot \beta_z} \cdot |-\rangle$$

6-1 التحقق: نشر القيمة الدقيقة E_+ بالنسبة لـ β_x و β_y إلى الرتبة الثانية:

$$E_+ = 3 \cdot \alpha \cdot a^2 + a \cdot \beta = 3 \cdot \alpha \cdot a^2 + a \cdot \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2}$$

$$E_+ = 3 \cdot \alpha \cdot a^2 + a \cdot \beta_z \cdot \sqrt{1 + \frac{\beta_x^2 + \beta_y^2}{\beta_z^2}}$$

$$\sqrt{1+x} \simeq 1 + \frac{1}{2} \cdot x$$

$$E_+ \simeq 3 \cdot \alpha \cdot a^2 + a \cdot \beta_z \cdot \left(1 + \frac{\beta_x^2 + \beta_y^2}{2 \cdot \beta_z^2}\right)$$

$$E_+ \simeq 3 \cdot \alpha \cdot a^2 + a \cdot \beta_z + a \cdot \frac{\beta_x^2 + \beta_y^2}{2 \cdot \beta_z}$$

$$E_+ \simeq 3 \cdot \alpha \cdot a^2 + a \cdot \beta_z + \frac{\hbar}{4} \cdot \frac{\beta_x^2 + \beta_y^2}{\beta_z} = E_+^0 + E_+^2$$

وهذا يدل على توافق بين الحل الدقيق والحل الناتج عن نظرية الاضطراب.

$\boxed{9}$ $\quad \mathcal{F}_S = \mathcal{F}_S(1) \otimes \mathcal{F}_S(2)$ $\quad$ -1

$$d_S = 2\times 2 = 4$$

$$|+\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \;;\; |-\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

يتم بناء أشعة قاعدة الفضاء $\mathcal{F}_S$ بإجراء الجداء التنسوري لأشعة القاعدة لقاعدتي الفضائين $\mathcal{F}_S(1)$ و $\mathcal{F}_S(2)$:

$$|i,\pm\rangle \otimes |j,\pm\rangle \;,\; i,j \in \{+,-\}$$

$$|++\rangle = |1,+\rangle \otimes |2,+\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|+-\rangle = |1,+\rangle \otimes |2,-\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|-+\rangle = |1,-\rangle \otimes |2,+\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$|--\rangle = |1,-\rangle \otimes |2,-\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat S_{1z} = \hat S_z \otimes I_2 \;,\; \hat S_{2z} = I_2 \otimes \hat S_z \qquad -3$$

$\hat S_{1z}$ تعمل على السبين الأول و S_{2z} تعمل على السبين الثاني

$$S_{1z} = S_z \otimes I_2 = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{1z} = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$S_{2z} = I_2 \otimes S_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\boxed{10}$

$$S_{2z} = \frac{\hbar}{2} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$S_{1z} + S_{2z} = \frac{\hbar}{2} \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$H^1 = \beta_z \cdot (S_{1z} + S_{2z})$$

$$H^1 = \frac{\hbar}{2} \cdot \beta_z \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

IV - (26 ن.)

نظرية التغايرات لجسيم في الحالة الأساسية في فضاء أحادي البعد

$$\psi(x,b)=\langle x|\psi(b)\rangle = A\cdot e^{-b\cdot x^2}\ ,\ b\in\mathbb{R}^+$$

1- اثبات التنظيم: (5)

$$\langle\psi|\psi\rangle = 1 = \int_{-\infty}^{+\infty}\left(A\cdot e^{-b\cdot x^2}\right)^*\cdot\left(A\cdot e^{-b\cdot x^2}\right)\cdot dx$$

$$1 = |A|^2\cdot\int_{-\infty}^{+\infty} dx\cdot e^{-2\cdot b\cdot x^2} = |A|^2\cdot\sqrt{\frac{\pi}{2\cdot b}} \Rightarrow \boxed{A = \left(\frac{2\cdot b}{\pi}\right)^{1/4}}$$

$$\langle T\rangle_\psi = \langle\psi|\hat{T}|\psi\rangle = \int_{-\infty}^{+\infty} dx\cdot\left(A\cdot e^{-b\cdot x^2}\right)^*\cdot\left(-\frac{\hbar^2}{2\cdot m}\cdot\frac{d^2}{dx^2}\right)\cdot\left(A\cdot e^{-b\cdot x^2}\right)$$ -2

$3\times(5)=\boxed{15}$

$$\langle T\rangle_\psi = -|A|^2\cdot\frac{\hbar^2}{2\cdot m}\cdot\int_{-\infty}^{+\infty} dx\cdot e^{-b\cdot x^2}\cdot\frac{d^2}{dx^2}e^{-b\cdot x^2}$$

$$\frac{d}{dx}e^{-b\cdot x^2} = -2\cdot b\cdot x\cdot e^{-b\cdot x^2}$$

$$\frac{d^2}{dx^2}e^{-b\cdot x^2} = \frac{d}{dx}\left(-2\cdot b\cdot x\cdot e^{-b\cdot x^2}\right) = -2\cdot b\cdot\left[e^{-b\cdot x^2} + x\cdot(-2\cdot b\cdot x)\cdot e^{-b\cdot x^2}\right]$$

$$\frac{d^2}{dx^2}e^{-b\cdot x^2} = -2\cdot b\cdot e^{-b\cdot x^2} + 4\cdot b^2\cdot x^2\cdot e^{-b\cdot x^2}$$

$$\langle T\rangle_\psi = -|A|^2\cdot\frac{\hbar^2}{2\cdot m}\cdot\int_{-\infty}^{+\infty} dx\cdot\left[-2\cdot b\cdot e^{-b\cdot x^2} + 4\cdot b^2\cdot x^2\cdot e^{-b\cdot x^2}\right]\cdot e^{-b\cdot x^2}$$

$$\langle T\rangle_\psi = -|A|^2\cdot\frac{\hbar^2}{2\cdot m}\cdot\left\{-2\cdot b\cdot\int_{-\infty}^{+\infty} dx\cdot e^{-2\cdot b\cdot x^2} + 4\cdot b^2\int_{-\infty}^{+\infty} dx\cdot x^2\cdot e^{-b\cdot x^2}\right\}$$

$$\langle T\rangle_\psi = -\left(\frac{2\cdot b}{\pi}\right)^{1/2}\cdot\left(\frac{\hbar^2}{2\cdot m}\right)\left[-2\cdot b\cdot\sqrt{\frac{\pi}{2\cdot b}} + 4\cdot b^2\cdot\frac{1}{2\cdot(2\cdot b)}\cdot\sqrt{\frac{\pi}{2\cdot b}}\right]$$

$$\langle T\rangle_\psi = \frac{\hbar^2}{2\cdot m}\cdot\left[2\cdot b\cdot\sqrt{\frac{2\cdot b}{\pi}}\cdot\sqrt{\frac{\pi}{2\cdot b}} - b\cdot\sqrt{\frac{2\cdot b}{\pi}}\cdot\sqrt{\frac{\pi}{2\cdot b}}\right] = \frac{\hbar^2}{2\cdot m}\cdot(2\cdot b - b)$$

$$\boxed{\langle T\rangle_\psi = \langle\psi|\hat{T}|\psi\rangle = \frac{\hbar^2\cdot b}{2\cdot m}}$$

2)

$$\langle V \rangle_\psi = \langle \psi | \hat{V} | \psi \rangle = \int_{-\infty}^{+\infty} dx \cdot \left(A \cdot e^{-b \cdot x^2}\right)^* \cdot (\alpha \cdot |x|) \cdot \left(A \cdot e^{-b x^2}\right) \quad \boxed{9}$$

$$\langle V \rangle_\psi = \alpha \cdot |A|^2 \cdot \int_{-\infty}^{+\infty} dx \cdot |x| \cdot e^{-2 \cdot b \cdot x^2} = 2 \cdot \alpha \cdot |A|^2 \cdot \int_0^{\infty} dx \cdot x \cdot e^{-2 \cdot b \cdot x^2}$$

$$|x| = \begin{cases} +x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\langle V \rangle_\psi = \frac{2 \cdot \alpha \cdot |A|^2 \cdot (0!)}{2 \cdot (2 \cdot b)} = \frac{4 \cdot \alpha}{4 \cdot b} \cdot \sqrt{\frac{2 \cdot b}{\pi}} \Rightarrow \boxed{\langle \hat{V} \rangle_\psi = \frac{\alpha}{\sqrt{2 \cdot \pi \cdot b}}}$$

$$\boxed{\langle \hat{H} \rangle_\psi = E = \langle T \rangle_\psi + \langle V \rangle_\psi = \frac{\hbar^2}{2 \cdot m} \cdot b + \frac{\alpha}{\sqrt{2 \cdot \pi}} \cdot \frac{1}{\sqrt{b}}}$$

5

3- يمثل b في التابع الموجي التغايري البارامتر التغايري، نبحث عن قيمة b ولتكن b_0، التي تعطي الطاقة الصغرى الحدية. يجب إذن أن نبحث عن النهاية الحدية للطاقة بالنسبة للبارامتر b :

$$\frac{\partial}{\partial b} E = \frac{\partial}{\partial b}\left[\frac{\hbar^2}{2 \cdot m} \cdot b + \frac{\alpha}{\sqrt{2 \cdot \pi}} \cdot b^{-1/2}\right] = \frac{\hbar^2}{2 \cdot m} - \frac{1}{2} \cdot \frac{\alpha}{\sqrt{2 \cdot \pi}} \cdot b^{-3/2} = 0 \Rightarrow$$

$$b_0^{3/2} = \frac{\alpha}{2 \cdot \sqrt{2 \cdot \pi}} \cdot \frac{2 \cdot m}{\hbar^2} = \frac{\alpha \cdot m}{\hbar^2 \cdot \sqrt{2 \cdot \pi}} \Rightarrow b_0 = \left(\frac{\alpha \cdot m}{\hbar^2 \cdot \sqrt{2 \cdot \pi}}\right)^{2/3}$$

$$E(b_0) = \frac{\hbar^2}{2 \cdot m} \cdot b_0 + \frac{\alpha}{\sqrt{2 \cdot \pi}} \cdot \frac{1}{\sqrt{b_0}} = \frac{3}{2} \cdot \left(\frac{\hbar^2 \cdot \alpha^2}{2 \cdot \pi \cdot m}\right)^{1/3}$$

$$\psi(x, b_0) = \langle x | \psi(b_0) \rangle = A \cdot e^{-b_0 \cdot x^2}$$

$$A = \left(\frac{2 \cdot b_0}{\pi}\right)^{1/4} ; \quad b_0 = \left(\frac{\alpha \cdot m}{\hbar^2 \cdot \sqrt{2 \cdot \pi}}\right)^{2/3}$$